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EARTHQUAKE RESPONSE OF ADJACENT STRUCTURES WITH VISCOELASTIC AND FRICTION DAMPERS

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Abstract. We study the seismic response of two adjacent structures connected with a dry friction damper. Each of them consists of a viscoelastic rod and a rigid block, which can slide without friction along the moving base. A simplified earthquake model is used for modeling the horizontal ground motion. Energy dissipation is taken by the presence of the friction damper, which is modeled by the set-valued Coulomb friction law. Deformation of viscoelastic rods during the relative motion of the blocks represents another way of energy dissipation. The constitutive equation of a viscoelastic body is described by the fractional Zener model, which includes fractional derivatives of stress and strain. The problem merges fractional derivatives as non-local operators and theory of set-valued functions as the non-smooth ones. Dynamical behaviour of the problem is governed by a pair of coupled multi-valued differential equations. The posed Cauchy problem is solved by use of the Grünwald–Letnikov numerical scheme. The behaviour of the system is analyzed for different values of system parameters.

1. Introduction

Various types of seismic control systems are used to dissipate energy reducing destroying effects of ground motion or wind excitation on structures. Collisions between neighboring structures, or pounding, during a major earthquake may cause considerable damage to the buildings if they are not on appropriate distance one from another. This phenomenon was investigated by many authors who found interconnection between adjacent structures with a damper as an effective way against pounding. For example, Luco and De Barros [14] analyzed adjacent structures connected with viscous dampers, while Bhaskararao and Jangid used a friction damper between two adjacent single-degree-of-freedom structures, see [5]. They applied the Kelvin–Voight as well as the Coulomb friction model for describing dynamical behaviour of the structures. Pounding between adjacent bridge segments is also evident and it is analyzed in numerous papers, see [6] and [17].

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Accurate models are required for describing such a complex phenomenon as pounding including elastic and plastic deformation, friction and impact. The use of fractional derivatives as non-local operators for describing the behavior of viscoelastic dampers should be justified for the sake of reliability of rheological model.

2. The model

The system under consideration consists of two rigid blocks representing the basis of adjacent buildings, Figure 1a. Another two blocks of masses m_1 and m_2 can slide without friction along the lower ones under the action of ground motion. The system is at rest at the beginning. Suddenly, the bases start oscillatory motion in the same way (coordinate u) during a simplified earthquake excitation. This causes upper blocks to move translatory relative to the bases with relative coordinates x_1 and x_2, while energy dissipates by deformation of the viscoelastic rods. If the conditions for relative motion between the upper blocks are met there will be energy dissipation within a friction damper, too. Lengths of the viscoelastic rods in undeformed state are denoted by l_α and l_β . Lengths L and a_1 are constants.

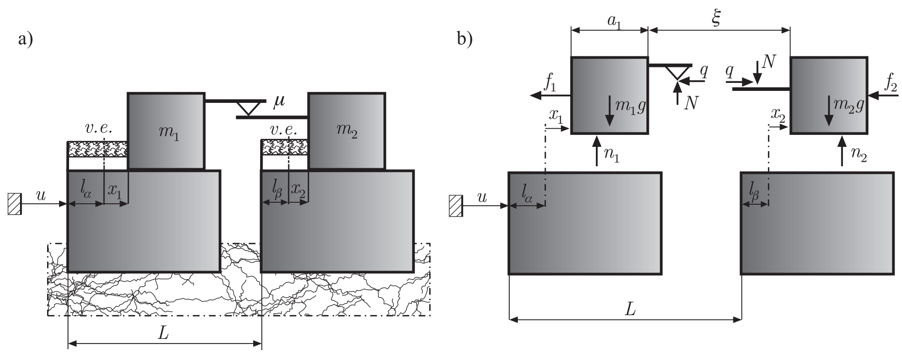

FIGURE 1. System under consideration and a free body diagram.

A free body diagram is presented in Figure 1b where f_1 and f_2 stand for the forces in viscoelastic rods, n_1 and n_2 are normal reaction forces between the blocks, while q and N represent the friction force and the normal contact force within the friction damper, respectively. According to the chosen type of the friction damper it can be assumed that the force N can be mono-directional with a constant intensity and that it will alliterate the positions where the forces n_1 and n_2 act what is not in the concern of this paper because the analyzed motion goes in horizontal direction only. The normal contact force N ensures significant clamping of interaction pairs necessary to provide appropriate friction force, see [7], [19], [4]. The distance between the upper blocks reads $\xi = L + l_\beta + x_2 - l_\alpha - x_1 - a_1$. Dynamics of the blocks with masses m_1 and m_2 is governed by the Fundamental axiom of dynamics

$$m_1\big(u^{(2)} + x_1^{(2)}\big) = -f_1 - q, \tag{2.1}$$

$$m_2\big(u^{(2)} + x_2^{(2)}\big) = -f_2 + q, \tag{2.2}$$

constitutive equations are presented by the fractional Zener model, see [1]

$$f_1 + \tau_{f\alpha} f_1^{(\alpha)} = \frac{E_\alpha A_\alpha}{l_\alpha}\big(x_1 + \tau_{x\alpha} x_1^{(\alpha)}\big), \tag{2.3}$$

$$f_2 + \tau_{f\beta} f_2^{(\beta)} = \frac{E_\beta A_\beta}{l_\beta}\big(x_2 + \tau_{x\beta} x_2^{(\beta)}\big), \tag{2.4}$$

where $(\cdot)^{(\kappa)} = d^\kappa(\cdot)/dt^\kappa$, A_α, A_β, E_α, E_β are cross-sectional areas and moduli of elasticity of viscoelastic rods, respectively. Constants $\tau_{f\gamma}$, $\tau_{x\gamma}$ with dimensions time$^\gamma$, and E_γ must satisfy conditions $\tau_{x\gamma} > \tau_{f\gamma}$, $\tau_{f\gamma} > 0$, and $E_\gamma > 0$, according to the second law of thermodynamics, where $0 < \gamma < 1$, $\gamma \in \{\alpha, \beta\}$. Fractional derivatives of the Riemann–Liouville type $f_i^{(\gamma)}$ and $x_i^{(\gamma)}$, $(i = 1, 2)$ are used in equations (2.3) and (2.4), see [16]

$$[z(t)]^{(\gamma)} \equiv \frac{d^\gamma z(t)}{dt^\gamma} = \frac{d}{dt}\left[\frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{z(\tau)}{(t-\tau)^\gamma} d\tau\right].$$

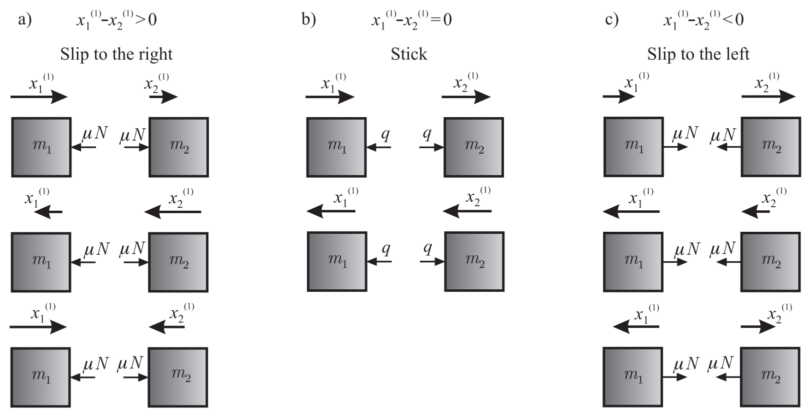

FIGURE 2. Combinatorial analysis of relative motion of upper blocks regarding to the relative sliding velocity within the dry friction damper, with directions of the friction force.

Friction force q within the friction damper connecting the upper blocks is given by the Coulomb friction law taken in the set-valued form by (2.5), see [8] and [13],

$$q \in \begin{cases} \mu N, & x_1^{(1)} - x_2^{(1)} > 0 \\ \mu N[-1, 1], & x_1^{(1)} - x_2^{(1)} = 0 \\ -\mu N, & x_1^{(1)} - x_2^{(1)} < 0 \end{cases} \tag{2.5}$$

where q is set-valued when the relative velocity between the upper blocks equals zero and the friction coefficient is denoted by μ. One should note that there are three different motion phases depending on relative sliding velocity, not between upper blocks and bases, but within the friction damper: slip to the right $(x_1^{(1)} - x_2^{(1)} > 0)$, slip to the left $(x_1^{(1)} - x_2^{(1)} < 0)$ and a stick phase $(x_1^{(1)} - x_2^{(1)} = 0)$. The upper blocks can move both to the right, both to the left or in opposite directions. Also, their relative velocities can be different or equal. All the cases are classified into three classes regarding to the relative sliding velocity within the friction damper. The classes and characteristic cases are shown in Figure 2a.

During sliding phase, when the upper blocks slip to the left or to the right the friction force equals μN, where its direction is shown in Figure 2a and Figure 2c, while for the stick case it can be any value between $-\mu N$ and μN, see (2.5). Ground acceleration is described by the simplified earthquake model (2.6), depending on parameters k, u_0, Ω, see [20]

$$u^{(2)} = e^{-kt}[-u_0\Omega^2 \sin(\Omega t)]. \tag{2.6}$$

The systems of passive seismic protection within the analyzed adjacent structures dissipate the seismic energy in two different ways. The first one is by the deformation of viscoelastic elements during the relative motion between the upper and the lower blocks, while the second one is the energy dissipation within the dry friction damper when there is relative motion between the upper blocks. The amount of dissipated energy during time can be calculated by the following expression

$$\Delta = \int_0^t \left[f_1 x_1^{(1)} + f_2 x_2^{(1)} + \mu N \big(x_1^{(1)} - x_2^{(1)} \big) \right] dt.$$

One should note that the friction damper dissipates energy during the sliding phase only, i.e. when the system is exposed to strong excitations.

The task is to find the motion of the upper blocks caused by the simplified earthquake excitation by solving the system of equations (2.1)-(2.6) together with the restrictions which follow from the Second law of thermodynamics and with homogenous initial conditions

$$x_1(0) = x_2(0) = x_1^{(1)}(0) = x_2^{(1)}(0) = f_1(0) = f_2(0) = 0. \tag{2.7}$$

Introducing dimensionless quantities like in [20]:

$$\begin{aligned}
\bar{x}_i =& \frac{x_i E_\alpha A_\alpha}{l_\alpha m_1 g}, \quad \bar{\xi} = \frac{\xi E_\alpha A_\alpha}{l_\alpha m_1 g}, \quad \bar{t} = t\sqrt{\frac{E_\alpha A_\alpha}{l_\alpha m_1}}, \quad \bar{f}_i = \frac{f_i}{m_1 g},\\
\bar{q} =& \frac{q}{m_1 g}, \quad \bar{\tau}_{f\gamma} = \tau_{f\gamma}\Big(\frac{E_\gamma A_\gamma}{l_\gamma m_1}\Big)^{\gamma/2}, \quad \bar{\tau}_{x\gamma} = \tau_{x\gamma}\Big(\frac{E_\gamma A_\gamma}{l_\gamma m_1}\Big)^{\gamma/2},\\
\rho =& \frac{m_1}{m_2}, \quad \varepsilon = \frac{A_\beta E_\beta}{A_\alpha E_\alpha}, \quad \bar{u}_0 = \frac{u_0 E_\alpha A_\alpha}{l_\alpha m_1 g},\\
\bar{\Omega} =& \Omega\sqrt{\frac{l_\alpha m_1}{E_\alpha A_\alpha}}, \quad (i = 1, 2), \quad \gamma \in \{\alpha, \beta\},
\end{aligned}$$

as well as the limiting value of dimensionless friction force in the form $\bar{\mu} = \mu N/(m_1 g)$, and omitting the bar for the sake of simplicity the system of equations describing the posed problem (2.1)-(2.6) together with initial conditions (2.7) and restrictions to the parameters of the model in dimensionless form reads

$$x_1^{(2)} - e^{-kt} u_0 \Omega^2 \sin(\Omega t) = -f_1 - q, \tag{2.8}$$

$$
\begin{aligned}
x_2^{(2)} - e^{-kt} u_0 \Omega^2 \sin(\Omega t) &= \rho(-f_2 + q), \\
f_1 + \tau_{f\alpha} f_1^{(\alpha)} &= x_1 + \tau_{x\alpha} x_1^{(\alpha)}, \\
f_2 + \tau_{f\beta} f_2^{(\beta)} &= \varepsilon\big(x_2 + \tau_{x\beta} x_2^{(\beta)}\big), \\
q &\in \begin{cases} \mu, & x_1^{(1)} - x_2^{(1)} > 0 \\ \mu[-1,1], & x_1^{(1)} - x_2^{(1)} = 0 \\ -\mu, & x_1^{(1)} - x_2^{(1)} < 0 \end{cases} \\
x_i(0) = x_i^{(1)}(0) &= f_i(0) = 0, \quad (i = 1, 2), \\
\tau_{f\gamma} > 0, \quad \tau_{x\gamma} &> \tau_{f\gamma}, \quad 0 < \gamma < 1, \quad \gamma \in \{\alpha, \beta\}.
\end{aligned}
$$

Equations $(2.8)_{1,2}$, where non-dimensional friction force is given by $(2.8)_5$, represent a system of two coupled multi-valued differential equations which should be solved together with fractional differential equations $(2.8)_{3,4}$.

Different sets of differential equations are recognized in (2.8) during different motion phases of the mechanical system. At the end of one motion phase the another one starts, so it is necessary to analyze transition conditions. For $t = 0$ the system is at rest in the stick phase, because of the initial conditions $(2.8)_6$. While being in a stick phase friction force is calculated by $q = (\rho f_2 - f_1)/(\rho + 1)$ and if the condition $|q| = \mu$ is met the system changes phase from stick to slip in the following way: if $q = \mu$ there will be slip to the right, while if $q = -\mu$ slip to the left will take place. Otherwise, if the system is in the slip phase and the condition $x_1^{(1)} - x_2^{(1)} = 0$ is met, transition to slip in another direction occurs if $|(\rho f_2 - f_1)/(\rho + 1)| > \mu$, while transition from slip to stick occurs for $|(\rho f_2 - f_1)/(\rho + 1)| \leqslant \mu$. Values of the state variables of the system at the end of one motion phase represent the initial conditions for the next phase.

It is important to note that $\xi > 0$ represents the condition for moving of structures without pounding. If that condition is violated during the motion then another complex effects such as impact and plastic deformation of structure elements, which are not described by the equations above, will take place.

3. The solution

The presence of the set-valued dry friction model classifies this problem in the class of a non-smooth dynamical systems, where different sets of differential equations are valid for different motion phases. Besides that, the posed problem implements fractional derivatives as non-local operators into the non-smooth dynamical system which places it in a category of complex problems. Solutions of this kind of problems are presented in [**10**] and [**9**] where impact problems are analyzed and whose mathematical models are almost identical to the model used in this problem of oscillations of adjacent structures. Similar procedure as in mentioned papers will be applied here. Namely, for some classes of functions important for applications the Riemann–Liouville and the Grünwald–Letnikov definitions of fractional derivatives are equivalent, see [**15**]. The motion of the system described by (2.8) will be determined numerically by the use of the Grünwald–Letnikov scheme as follows.

Introducing a time step h and by the time discretization $t_m = m \cdot h$ ($m = 0, 1, 2, \dots$), the Grünwald–Letnikov fractional derivative of order γ reads

$$z^{(\gamma)} = h^{-\gamma} \sum_{j=0}^{m} \omega_j^{\gamma} z_{m-j},$$

where coefficients ω_j^{γ} are calculated using recurrence relationships

$$\omega_0^{\gamma} = 1, \quad \omega_j^{\gamma} = \Big(1 - \frac{\gamma+1}{j}\Big)\omega_{j-1}^{\gamma}, \quad (j = 1, 2, 3, \dots),$$

Both the first and the second derivative of a function can be approximated by following expressions

$$z_m^{(1)} = \frac{z_{m+1} - z_m}{h}, \quad z_m^{(2)} = \frac{z_{m+1} - 2z_m + z_{m-1}}{h^2},$$

respectively. Applying these approximations to the system of equations (2.8) we obtain numerical algorithm for calculation of contact forces as well as relative positions of upper blocks during slip phases within a friction damper

$$\begin{aligned}
f_{1m} &= \frac{1}{1 + \tau_{f\alpha} h^{-\alpha}} \Big\{ x_{1m}\Big(1 + \frac{\tau_{x\alpha}}{h^{\alpha}}\Big) \\
&\quad + h^{-\alpha} \sum\nolimits_{j=1}^{m} \big[\omega_j^{\alpha}(\tau_{x\alpha} x_{1m-j} - \tau_{f\alpha} f_{1m-j})\big] \Big\}, \\
f_{2m} &= \frac{\varepsilon}{1 + \tau_{f\beta} h^{-\alpha}} \Big\{ x_{2m}\Big(1 + \frac{\tau_{x\beta}}{h^{\beta}}\Big) \\
&\quad + h^{-\beta} \sum\nolimits_{j=1}^{m} \Big[\omega_j^{\beta}\Big(\tau_{x\beta} x_{2m-j} - \frac{\tau_{f\beta}}{\varepsilon} f_{2m-j}\Big)\Big] \Big\}, \\
x_{1m+1} &= 2x_{1m} - x_{1m-1} + h^2 \big[e^{-kmh} u_0 \Omega^2 \sin(\Omega m h) - f_{1m} - q_m\big], \\
x_{2m+1} &= 2x_{2m} - x_{2m-1} + h^2 \big[e^{-kmh} u_0 \Omega^2 \sin(\Omega m h) + \rho(-f_{2m} + q_m)\big],
\end{aligned} \tag{3.1}$$

where the friction force q_m has the value either μ or $-\mu$, see $(2.8)_5$.

On the other hand, for the motion phase where upper blocks move together as a unity (stick within the friction damper, $x_1^{(1)} - x_2^{(1)} = 0$) the forces in the viscoelastic rods are calculated using $(3.1)_{1,2}$, while the algorithm for calculation of the relative positions of upper blocks and the friction force is given by

$$\begin{aligned}
x_{1m+1} &= 2x_{1m} - x_{1m-1} \\
&\quad + h^2 \Big[e^{-kmh} u_0 \Omega^2 \sin(\Omega m h) - \frac{\rho}{\rho+1}(f_{1m} + f_{2m})\Big], \\
x_{2m+1} &= x_{1m+1} - c, \\
q_m &= \frac{\rho f_{2m} - f_{1m}}{\rho + 1}, \qquad (m > 0),
\end{aligned} \tag{3.2}$$

with $x_{10} = x_{20} = f_{10} = f_{20} = x_{11} = x_{21} = 0$, following from the initial conditions $(2.8)_6$, and where c=const. represents the difference between x_1 and x_2 at the end of the previous slip phase. At the beginning of the motion of the system $c = 0$.

For given homogenous initial conditions $(2.8)_6$ system starts to move in the stick phase. For certain values of system parameters it is possible that system switches motion phases during time, when transition conditions are met. Initial conditions for each motion phase are obtained from the state of the system at the and of the previous phase. Motion of the system during simplified earthquake excitation is obtained by the use of numerical algorithms (3.1) and (3.2) alternately according to the actual motion phase. During the motion of the system viscoelastic rods deform, where their elongation and the contact forces are calculated on the basis of history of deformation because of the presence of fractional derivatives as non-local operators in constitutive equations $(2.8)_{3,4}$. Although the system changes its motion phases, it is very important that history of deformation of the viscoelastic rods needs to be taken from the beginning of deformation process, actually from $t = 0$. The importance of historical effects in problems which include fractional derivatives is presented in [**11**] and [**2**], to mention just a few.

As it was mentioned above the system changes its motion phases according to inequality constraints. It is necessary to determine switching times between phases in order to know when exactly to start using the appropriate set of differential equations. An efficient way for determination of switching times of non-smooth mechanical systems is presented in the paper of Tarner [**18**], where the slack variable, which replaces time as the independent variable, was introduced. By the use of the slack variable algorithm, which was already used in problems comprising non-smooth and non-local operators, see [**10**], [**20**] and [**9**], switching times can be calculated. Motion of the system is determined for certain values of system parameters and solutions are presented in the next section.

4. Results

Taking into account system (2.8) and considerations mentioned above, the simulation of motion of adjacent structures during the simplified earthquake excitation can be performed for different values of system parameters. In this paper the behaviour of the same mechanical system is analyzed for two different excitations. In both cases the mechanical system is described by the following set of parameters: $\rho = 0.5$, $\varepsilon = 2$, $\alpha = 0.23$, $\tau_{x\alpha} = 1.183$, $\tau_{f\alpha} = 0.004$, $\beta = 0.53$, $\tau_{x\beta} = 4$, $\tau_{f\beta} = 0.2$, $\mu = 0.3$. The time step $h = 0.002$ is chosen for numerical simulations.

In the first case (strong excitation) stick-slip and slip-slip transitions occur during the motion where excitation parameters are $u_0 = 1.5$, $\Omega = 1.5$, $k = 0.5$. Graphical representation of the result is given in Figures 4-5.

The relative positions of upper blocks are presented in Figure 4, while the excitation, the friction force and the difference between the relative velocities $x_1^{(1)} - x_2^{(1)}$ are shown in Figure 4. At the beginning, for a very short period of time, the system moves in the stick phase within the friction damper. During that phase the relative positions x_1 and x_2 coincide while the friction force q increases from zero to its limiting value μ. After that, slip to the right and slip to the left alternate while the friction force jumps from one limiting value to another one. In a certain time instant t^*, when the difference between the relative velocities $x_1^{(1)} - x_2^{(1)}$ becomes zero, the system enters into the stick phase again, where the relative positions x_1 and x_2 do not coincide, but they differ for a constant, i.e. the curves of x_1 and x_2

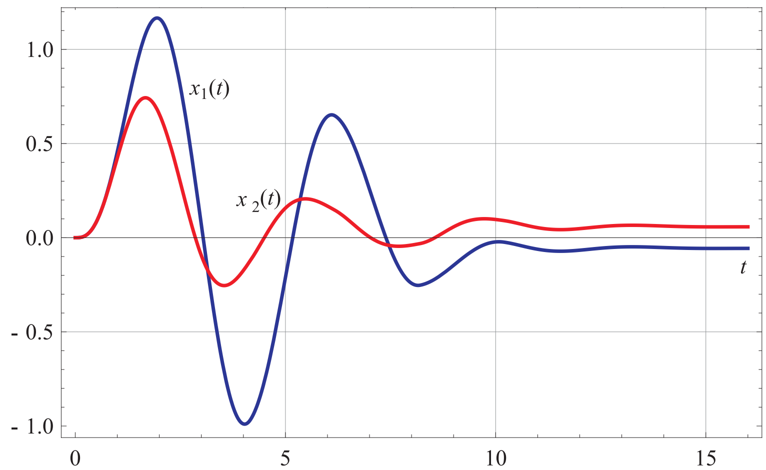

FIGURE 3. Relative position of upper blocks in the case of a strong excitation.

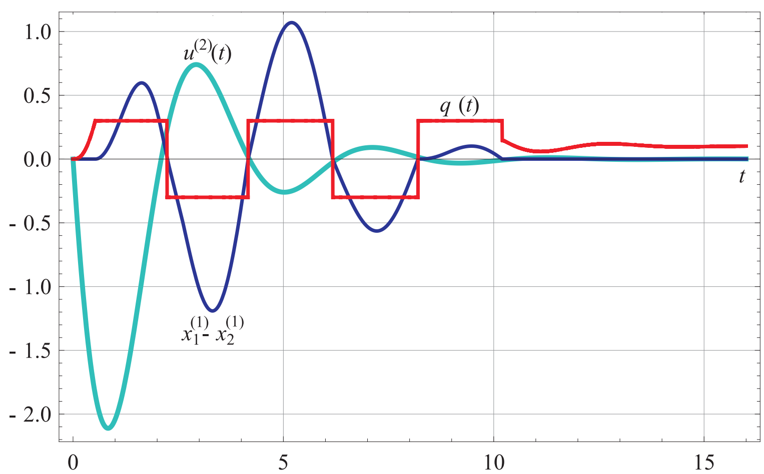

FIGURE 4. Ground acceleration, friction force and difference between velocities of upper blocks in the case of a strong excitation.

are equidistant. In the time instant t^*, when relative motion between upper blocks stops, the jump of the friction force q occurs. For $t > t^*$ the friction force q is a continuous function of time ensuring the stick phase, $q \in [-\mu, \mu]$. It is allowed by the set-valued Coulomb friction law used and can not be described if a single-valued sign function is incorporated in the dry friction model. After t^* the system remains in the stick phase because of vanishing of the earthquake excitation, as it can be seen in Figure 4.

Forces f_1 and f_2 in the viscoelastic rods are shown in Figure 5. Amplitudes of oscillations of these forces decrease during time up to a very small values at

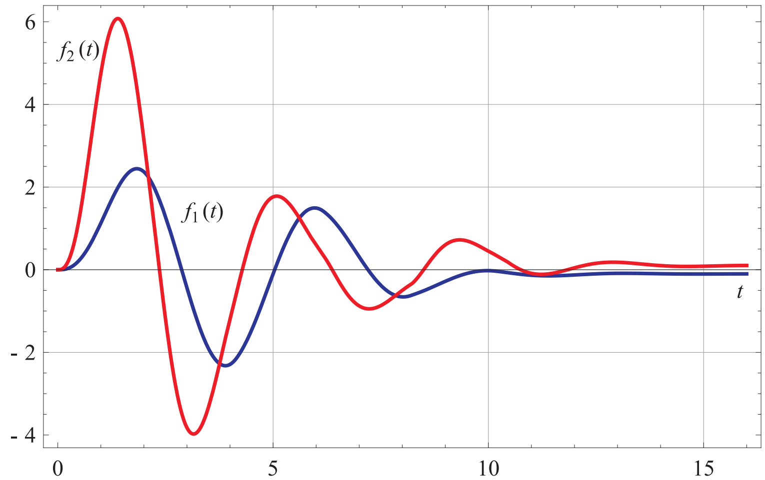

FIGURE 5. Forces in viscoelastic rods in the case of a strong excitation.

the end of the shown time interval, according to small elongations x_1 and x_2. It is important to note that during the stick phase energy dissipates by deformation of viscoelastic rods, while during slip phases there is energy dissipation within the friction damper, too.

In the second case (weak excitation) the simplified earthquake motion is described by $u_0 = 0.05$, $\Omega = 3.5$, $k = 0.3$. In this case upper blocks move in the same way, i.e. there is the stick phase within the friction damper all the time. The relative positions x_1 and x_2 coincide so that $x_1^{(1)} - x_2^{(1)} = 0$, which is shown in Figure 6 and Figure 7.

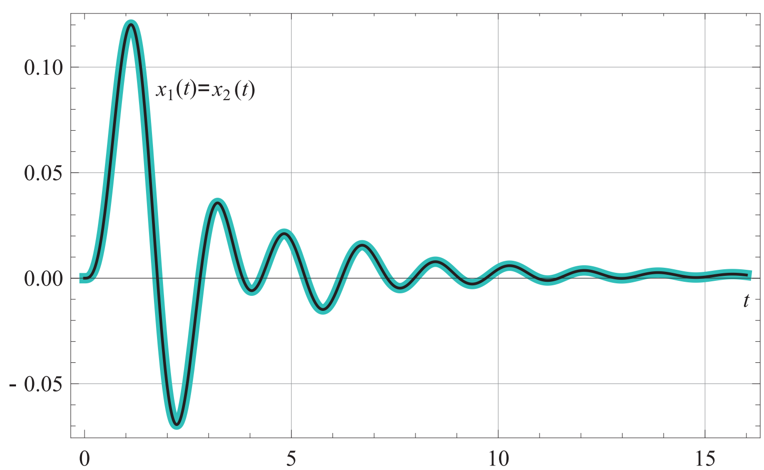

FIGURE 6. Relative position of upper blocks in the case of a weak excitation.

During the motion of the system there is energy dissipation through the deformation of viscoelastic rods only. There is no slip within the friction damper and the magnitude of the friction force is less than its limiting value μ, where the friction force q is shown in Figure 7. The graphical representation of the contact forces f_1 and f_2 is given in Figure 8.

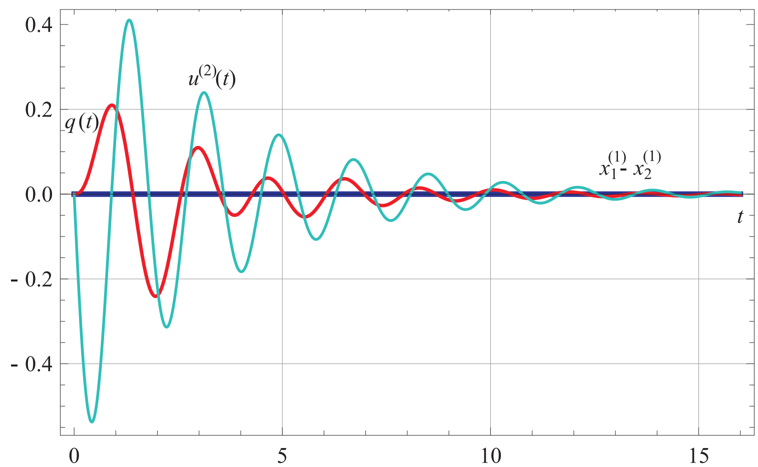

FIGURE 7. Ground acceleration, friction force and difference between velocities of upper blocks in the case of a weak excitation.

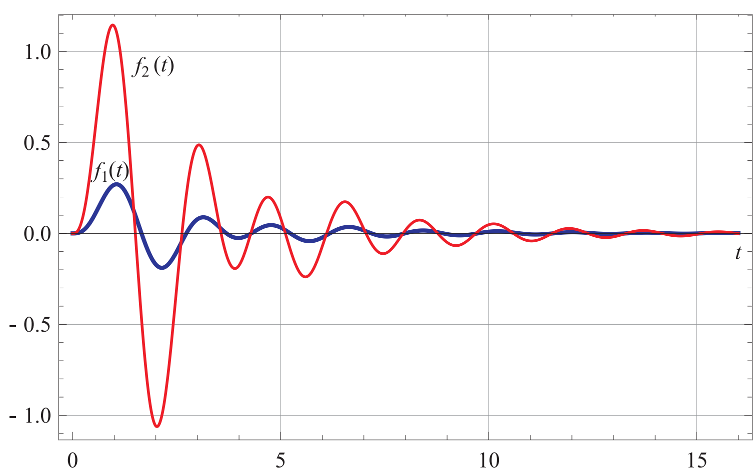

FIGURE 8. Forces in viscoelastic rods in the case of a weak excitation.

After some time, in both cases, relative motion of the upper blocks, the contact forces and energy dissipation as well become very small and the system continues to move with the stick phase within the friction damper. That is the reason why the attributes of motion are presented during the time interval $t \in [0, 16]$. In addition to this, the non-dimensional distance ξ between upper blocks in both cases is always positive which implies that the structures move without pounding. The distance ξ is calculated by using the following dimensionless values: $L = 3$, $l_\alpha = l_\beta = a_1 = 1$.

It is interesting to comment on the ceasing motion of the system as a whole. Although for the chosen excitation type the amplitudes of oscillations become very small (or negligible) after some time due to energy dissipation, adjacent structures described here will stop when $t \to \infty$. To stop the system in a finite time it is necessary to introduce the dry friction between the upper and the lower blocks in the form of the set-valued constitutive law.

5. Conslusions

The motion of the system under consideration is modeled by equations (2.1)-(2.6) with initial conditions (2.7) and restrictions to the system parameters mentioned above. Solutions of the system in dimensionless form (2.8) are obtained for two sets of values of system parameters by the use of the Grünwald–Letnikov numerical scheme. The combinatorial analysis was performed in order to find conditions which are valid during each motion phase of the non-smooth mechanical system. Non-smooth character of the system is caused by the presence of the dry friction, which is taken by the set-valued Coulomb friction law (2.5). In general case slip to the left, slip to the right and stick phase alternate during the motion. The influence of excitation parameters on the motion of structures is analyzed. For weak excitation the friction damper does not activate, i.e. there is no slip in the friction damper, the upper blocks move as a unit without relative motion between them, where energy dissipates through the deformation of the viscoelastic rods only. On the other hand, in the case of strong excitation, the upper blocks move in a different way so the energy dissipates both by deformation of the viscoelastic rods and by activation of the friction damper. On the bases of calculation of the distance ξ between upper blocks it is possible to conclude whether the pounding between the two adjacent structures will take place or not. In both cases presented in this paper the distance ξ is greater than zero so there is no collisions between the adjacent structures.

For further investigations it would be interesting to analyze influence of the parameters of the mechanical system on behaviour of the adjacent structures. Also, instead of the simplified earthquake model (2.6) it is possible to use a more complicated model or the one based on earthquake data base. Besides that, the application of some other approximation of fractional derivatives, such as for example the Atanackovic–Stankovic expansion formula presented in [**3**], used in [**12**], could reduce difficulties which appear in problems merging fractional derivatives as nonlocal operators with the theory of non-smooth set valued functions.

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СЕИЗМИЧКИ ОДГОВОР СУСЕДНИХ КОНСТРУКЦИЈА СА ВИСКОЕЛАСТИЧНИМ И ФРИКЦИОНИМ ПРИГУШИВАЧИМА

Резиме. У овом раду проучен је сеизмички одзив две суседне конструкције повезане пригушивачем који ради на принципу сувог трења. Свака од конструкција садржи и вискоеластични штап као и крути блок, који може да клизи без трења по покретној основи. Поједностављени модел земљотреса коришћен је за моделирање хоризонталног кретања тла. Дисипација енергије врши се помоћу фрикционог пригушивача, који је моделиран Кулоновим законом трења дефинисаним на скупу. Деформација вискоеластичних штапова током релативог кретања блокова представља други вид дисипације енергије. Конститутивна једначина вискоеластичног тела је представљена фракционим Зенеровим моделом, који укључује фракционе изводе напона и деформације. Проучавани проблем повезује фракционе изводе као нелокалне операторе са теоријом неглатких вишевредносих функција. Динамика постављеног проблема описана је паром спрегнутих вишевредносних диференцијалних једначина. Постављени Кошијев пролем решен је применом Грунвалд–Летниковљеве нумеричке процедуре. Понашање система је анализирано за различите вредности улазних параметара.

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