THEORETICAL AND APPLIED MECHANICS
Volume 42 (2015) Issue 3, 151–166 DOI: 10.2298/TAM1503151M

MODE SHAPE IDENTIFICATION OF AN EXISTING THREE-STORY FLEXIBLE STEEL STAIRWAY AS A CONTINUOUS DYNAMIC SYSTEM

George D. Manolis, Triantafyllos K. Makarios, Vasiliki Terzi, and Ioanna Karetsou

Abstract. The issue of modal shape identification for a flexible steel stairway located within the building complex comprising the Civil Engineering department of Aristotle University in Thessaloniki, Greece is presented here. The aforementioned stairway is a system with continuous distribution of mass and stiffness, a fact that makes structural identification challenging as compared to structures where lumping of these two basic parameters can be observed. More specifically, this stairway was instrumented using a local multi-channel network of accelerometers. Two 12 bit-nominal resolution, digital uniaxial accelerometers of the type KUOWA-PCD-30A, connected by cables and with 'common time' and 'common start' characteristics were installed on the stairway. The dominant modes of vibration of the stairway were computed by the 'modal response acceleration time history methodology'. In parallel, a detailed finite element method model of the stairway was constructed and calibrated according to the ambient vibration results. We note that the identification procedure used for the dynamic characteristics of spatial structures yields results that can be used to develop a family of numerical models for the stairway ranging from the simple single-degree-of-freedom system to highly detailed multiple-degree-of-freedom models. Finally, some useful information on the theoretical procedure for the identification of modal shapes is included herein.

1. Introduction

In the last few years, issues such as 'monitoring' and 'structural integrity' of structures is being developed rather vigorously by employing suitable, multi-channel networks for measuring acceleration response time-histories, followed by appropriate data processing techniques. If the structural response remains within the linear elastic range, then the recorded acceleration response time-histories contain the structure's modal response.

It is well known that a clear deterministic relationship between ambient vibration loading or ground excitations as input and the ensuing structural response as

2010 *Mathematics Subject Classification:* Primary 74H05; Secondary 65P05.
Key words and phrases: identification, mode shapes, modal response, acceleration time histories, finite element models.

output does not exist. Furthermore, the methodologies used to identify the dynamic characteristics (i.e., natural frequencies, mode shapes and modal damping ratios) of a structure must take into account the instrumentation configuration that is used to monitor that structure. Thus, many methodologies are adapted to investigate dynamic structural response on a case-by-case basis (buildings, bridges, towers, dams, and also other categories of structures such as aircraft frames).

Nevertheless, the main objective in all cases is to identify the dynamic characteristics of the structure through an analytic processing of the measured response. In the past, various deterministic and stochastic methods have been proposed, e.g., Basseville et al. [1] , Brincker et al. [2], Peeters and De Roeck [3], Wenzel and Pichler [4], Overschee and De Moor [5], Makarios [6, 7], and finally Manolis et al. [8]. Although there are many techniques available for structural identification, as well as commercial software packages for general use, the structural engineering community has yet to agree on a unique, well-documented procedure for identifying the dynamic characteristics of structures and structural systems through a detailed investigation of their measured response. Of course, different types of structures (e.g., tall buildings, bridges, silos, etc.) require specialized equipment configurations and techniques. The present work is but one more contribution in this direction, whereby a minimal equipment configuration can be used to successfully identify the dynamic characteristics of steel stairways.

Steel stairways are used mainly as emergency exits in hotels, hospitals and public buildings. The experimental and numerical study of their static and dynamic behaviour is important for various reasons. For instance, steel stairways are usually attached to a pre-existing structure made of different structural material, which is usually reinforced concrete. Furthermore, they are open-air structural elements and thus their loads are mainly generated from environmental conditions. Moreover, in case of an emergency, humans ascend and descend at a pace higher than walking, which results in a high frequency loading. In many cases, steel stairways behave as slender structures, which makes them vulnerable to vibrations. Therefore, any information concerning the dynamic properties in terms of eigenmodes and eigenfrequencies of the stairway is important, not only for design, but also for maintenance of existing ones.

Despite significant advances in analysis provided by various numerical methods, there still exist challenges regarding the prediction of the actual dynamic behaviour of stairways. Some of the most crucial uncertainties concerning the behaviour of an existing stairway refer to the lack of information on the as-built structure (architectural plans, material properties, foundation design, etc.), the boundary conditions and the effects of non-structural elements, see Belver et al. [9]. The gap between real structural behaviour and numerical model prediction can be bridged by performing various modal tests on the existing structure, see Zivanovic et al. [10]. More specifically, various researchers have studied the behaviour of steel stairways numerically, see Arbitrio [11], Howes et al. [12], Howes and Gordon [13] and Huntington and Mooney [14]. Other researchers combine both experimental data and numerical results, see Davis and Murray [15], Setareh and Jin [16], Keinner et al. [17], Capellini et al. [18], Eid et al. [19], Kim et al. [20], Modak et al.

[21] and Bradand and Thomas [22]. All comparison studies between numerical and experimental results are in terms of modal properties and this process leads to the verification of the structural model, and to possible methods for upgrading it by using various updating techniques, see Brownjohn and Xia [23].

The present work identifies the dynamic characteristics of a specialized type of steel stairway with continuous mass distribution and stiffness. These stairways serve as escape routes from high-rise buildings, are placed externally to the building and can be classified as being very flexible structures. Also, since low amplitude ambient vibrations are used as the external excitation, the assumption of viscously damped modes is correct. The analysis procedure presented in this work is adapted to the particular instrumentation configuration used on the steel stairway, supplemented by the method of 'modal time-histories'. More specifically, the present paper is based on an extension of a previous project, see Makarios et al. [24], Karetsou [25] and Papanikolaou [26], where ambient vibrations induced by ordinary, daily use of the stairway by pedestrians were used as the source of excitation to measure the stairway response. Within this framework, a parallel modeling procedure using the finite element method (FEM) based SAP 2000 [27] software was developed and the results obtained were calibrated against the experimental data. The FEM models were then reconfigured and used to reproduce details in the dynamic response of the stairway. This way, the numerical models help extend the results of the experimental effort and can be used to trace the ageing and deterioration phenomena expected during the useful service life of the stairway.

2. The steel stairway and mathematic analysis

The steel stairway under study is located in the courtyard behind the main building of the Civil Engineering Department, which is part of the Faculty of Engineering at Aristotle University, Thessaloniki, Greece (see Figure 1). The stairway was constructed in 1982 and comprises three spans which terminate at three landing levels of a low rise annex building housing laboratories.

The central span is supported by two beams (IPN140 cross section), while the beams supporting the other two end spans, as well as the landings, are IPN100 cross sections. The columns supporting the beams at the second landing, as well as the main column of the third landing, are IPN140 cross sections. All remaining columns of the structure are IPN100 cross sections. As illustrated in Figure 1, the IPN140 columns are connected by cross diagonal bars with an RHS 60×40 cross section. These cross bars reinforce the lateral stiffness of the stairway and reduce the buckling length in the columns. The vertical stiffness of the landing is reinforced by the contribution of T50 beams on the first landing and IPN beams on the other two. Finally, the landings, the treads and the risers are made of sheet metal plates of 3mm thickness. The same material is used as protective sideway cover for the beams. Next, the instrumentation scheme used during a two week time period for conducting the measurements comprised three uniaxial accelerometers, see Figure 2.

We note that the stairway under study cannot be modeled as a lumped mass system, since there is no detectable concentration of mass at any floor level. Therefore, the structural system is strongly continuous in terms of the mass and stiffness

FIGURE 1. The steel stairway in front (left) and side (right) views.

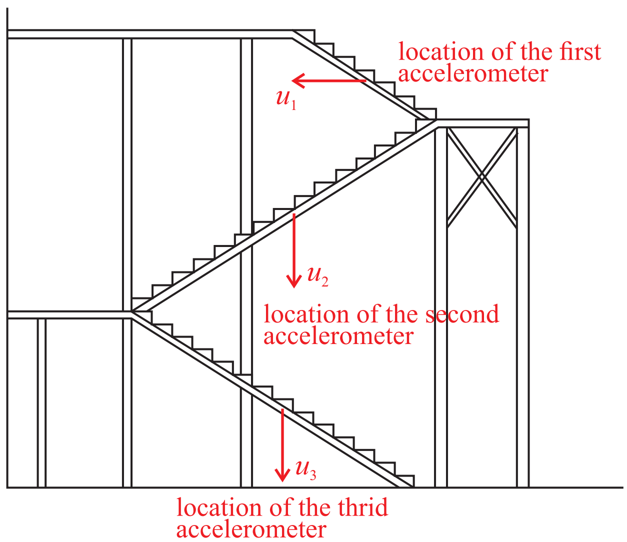

FIGURE 2. Location of accelerometers at key points in the structure.

distributions, and this causes difficulties in the identification of the eigenmodes. In accordance to the accelerometer placement scheme, the vector **u** corresponds to three basic degrees of freedom (DOF), and the generalized matrices for the mass **M**, the stiffness **K** and the damping **C** of the steel stairway are fully populated in the form shown below.

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad \mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \quad \mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

The equation of motion of the steel stairway subjected to environmental dynamic loads is now the following:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{W}(t) \tag{2.1}$$

In the above, $\mathbf{W}$(t) is the loading vector of the unknown environmental dynamic loading on the specified degrees of freedom. For this case, the solution of Eq. (2.1) is of the following form:

$$u(t) = \boldsymbol{\phi}_1 q_1(t) + \boldsymbol{\phi}_2 q_2(t) + \boldsymbol{\phi}_3 q_3(t) = \begin{bmatrix} \boldsymbol{\phi}_1 & \boldsymbol{\phi}_2 & \boldsymbol{\phi}_3 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \end{bmatrix} = \boldsymbol{\Phi}\mathbf{q}(t) \tag{2.2}$$

where

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\phi}_1 & \boldsymbol{\phi}_2 & \boldsymbol{\phi}_3 \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} \quad \mathbf{q}(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \end{bmatrix}$$

More specifically, ϕ_1, ϕ_2, ϕ_3 are the basic three eigenmodes; Φ is the modal matrix of the steel stairway and $q_i(t)$, $i = 1, 2, 3$ are the corresponding time functions of vibration of each eigenmode.

It can been proven that the system of Eq. (2.1), which is particular to the placement scheme of the three accelerometers, is equivalent to a single degree of freedom (SDOF) vibrator response for each eigen-mode i and is described below (see Makarios [6, 7]) as follows:

$$\ddot{q}_i(t) + 2\xi\omega\dot{q}_i(t) + \omega_1^2 q_i(t) = \frac{\boldsymbol{\phi}_i^T \boldsymbol{W(t)}}{\boldsymbol{\phi}_i^T \boldsymbol{M}\boldsymbol{\phi}_i} \tag{2.3}$$

Equation (2.3) shows that any solution for the calculation of the time varying modal functions $q_i(t)$ requires a known environmentally-induced dynamic loading vector $\mathbf{W}$(t), which is of course impossible to estimate a priori. However, it is clear that its exact solution is a superposition of the solution $q_{o,i}(t)$ to the homogeneous equation and of the particular solution $q_{w,i}(t)$ which in turn depends on the dynamic loading $\mathbf{W}$(t):

$$q_i(t) = q_{o,i}(t) + q_{w,i}(t)$$

Moreover, it is known that for free vibrations, the exact solution $q_{o,i}(t)$ for the homogeneous equation coincides with the final solution. Consequently, suppose a theoretical situation where the structure vibrates, then the action of the dynamic loading $\mathbf{W}$(t) ceases and its free vibration response is now being recorded. In this case, the condition that the solution $q_{o,i}(t)$ of the homogeneous equation is also the final solution of the problem is fulfilled:

$$q_i(t) = q_{o,i}(t)$$

More specifically, each term in the summation of Eq. (2.2) represents the modal response movements $u_i(t)$ for each $i = 1, 2, 3$ eigenmode:

$$\boldsymbol{u}_i(t) = \boldsymbol{\phi}_i(t) q_i(t) \Rightarrow \begin{bmatrix} u_{1i}(t) \\ u_{2i}(t) \\ u_{2i}(t) \end{bmatrix} = \begin{bmatrix} \phi_{1i} q_i(t) \\ \phi_{2i} q_i(t) \\ \phi_{3i} q_i(t) \end{bmatrix} \tag{2.4}$$

From the combination of Eqs. (2.4) and (2.5), the following conclusion is drawn: *If the dissociation of the modal responses $\phi_i q_i(t)$ for each DOF of the system was feasible, then the direct estimation of the components of each one of the eigenmodes by the elimination of the time function $q_i(t)$, which is common for all the degrees of freedom of the same eigenmode, would also be feasible.* Indeed, referring at the instant of the simultaneous extreme widths of vibration (a, b, c) for all DOF (u_1, u_2, u_3) as shown in Figure 3, the components of the ith-eigenmode are directly described by the following equation:

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{a}{a} \\ \frac{b}{a} \\ \frac{c}{a} \end{bmatrix} = \begin{bmatrix} \phi_{1i} \\ \phi_{2i} \\ \phi_{3i} \end{bmatrix} = \begin{bmatrix} 1 \\ \phi_{2i} \\ \phi_{3i} \end{bmatrix} \tag{2.5}$$

The uncoupling of the modal response is achieved numerically by truncation of the harmonic having a frequency equal to the ith-eigenfrequency of the structure. This truncation is achieved by the use of an appropriate digital filter applied to the initial recordings, which permits only the transmission of one specific frequency, that equals the eigenfrequency of the structure, so long as the latter is already known. The eigenfrequency of the structure can be determined by the combination of the "peak-picking" technique applied to the Fast Fourier Transform (FFT) diagrams of the recordings, followed by an examination of the phase difference between the extreme response values. This last point must take into account that a phase difference of 0 or π (rad) between different recordings corresponds to an eigenmode. The aforementioned method is known as the 'modal time-history method' and has been recently developed in Makarios [6, 7]. Thus, in the present work, the method is applied to the steel stairway, which is a system with infinite DOF having a continuous mass and stiffness distribution. This makes the identification of the important eigenfrequencies quite difficult.

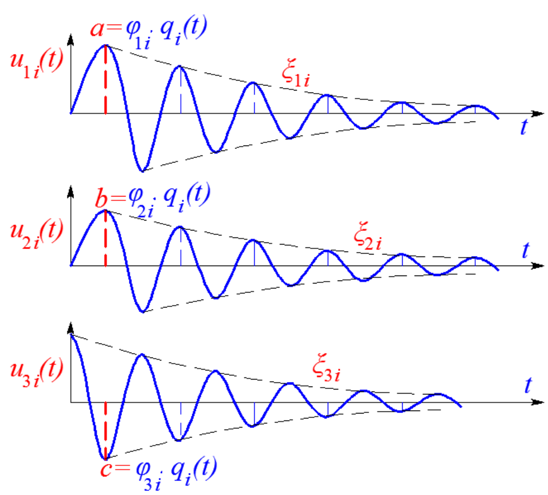

FIGURE 3. Modal time-histories of the three DOF system.

3. Instrumentation of the steel stairway

The stairway was instrumented by a system of two 12 bit-nominal resolution, digital uniaxial accelerometers of the type KYOWA-PCD-30A. One sensor (channel 1) was used for the recording of the vertical accelerations, while the second sensor (channel 2) was used for the recording of the horizontal accelerations, see Figure 4. The fastening of the sensors on the lower surface of the treads was achieved by the use of silicon adhesives. The sensor recording the horizontal accelerations (reference accelerometer) was installed in the middle of the third span, while the sensor recording the vertical accelerations was alternately installed in the middle of the first and second spans (Figure 2).

Figure 4. Accelerometer installation on the bottom side of the stairway.

The ambient vibration loading was realized during regular use, with people ascending and descending the steel stairway. People did not stay put on the stairway, they merely went up and down in a short time interval. A sufficient number of response recordings was made, and durations of two (2), five (5) and fifteen (15) minutes were considered, including the free vibration part of the response as well. The two accelerometers were connected to a logging unit for receiving data. The latter was connected to a laptop computer, see Figure 5.

Figure 5. Recording unit of accelerometer network.

Using the appropriate software, visualization and calibration of the recordings took place. By selecting a range of 10000 μm/m and a calibration factor of 0.000829, the accelerometer in the vertical vibration direction was capable of recording the acceleration of gravity g=9.81 m/sec^2 under calm conditions. The software used also offers the ability of exporting data in the .xls format. Therefore, the recordings were easily imported and processed in a spreadsheet (MS Excel program).

4. Recorded signal processing

During the processing of the recordings, multiple time windows were examined. The forced vibration part was truncated and only the part of the free vibration was taken into consideration. The noise was removed from the free vibration recordings by the use of appropriate filters, see Makarios [6, 7]. Finally, the FFT of each degree of freedom was calculated. The main observation concerning the FFT diagrams refers to the lack of the usual 'spikes' for certain eigenfrequencies due to the fact that the steel stairway does not respond as a discrete system, since it does not contain any sizeable lumped masses, see Figures 6 and 7.

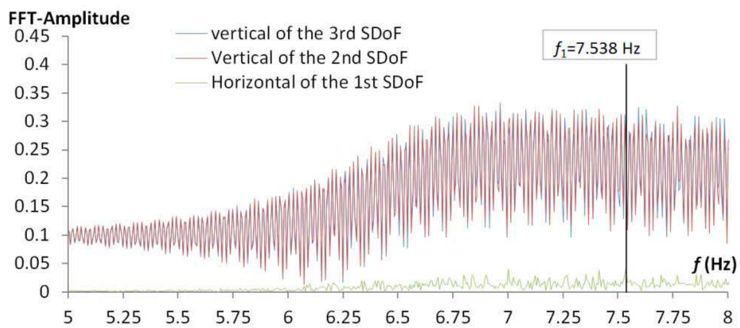

Figure 6. FFT of the three displacement components from 5 Hz to 8 Hz.

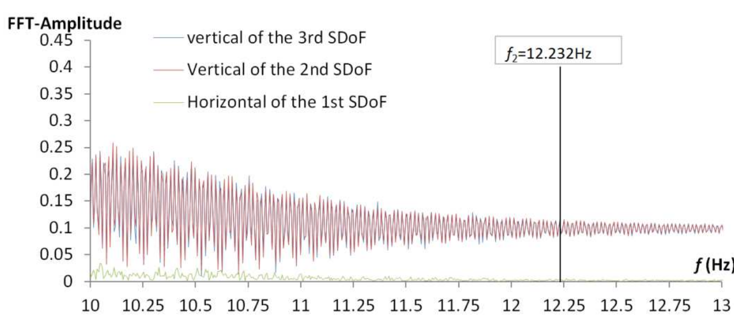

Figure 7. FFT of the three displacement components from 10 Hz to 13 Hz.

More specifically, the FFT displacement amplitudes present a rather smooth uniformity, due to the fact that that the structure performs as a system with continuous mass and stiffness distribution. Combining the 'peak-picking' technique with an estimation of the phase difference between the modal response histories, it is possible to detect whether or not the peaks observed refer to the modal response of the structure or simply stem from the external dynamic loading. This way, the peaks in the entire frequency range of FFT diagrams were sequentially examined. As a result, two frequencies, at $f_1 = 7.538\,\mathrm{Hz}$ and $f_2 = 12.232\,\mathrm{Hz}$, were estimated as being the eigenfrequencies of the structure. Indeed, the time histories corresponding to these two frequencies, present a phase difference of 0 and π rad respectively, as illustrated in Figures 8 and 9.

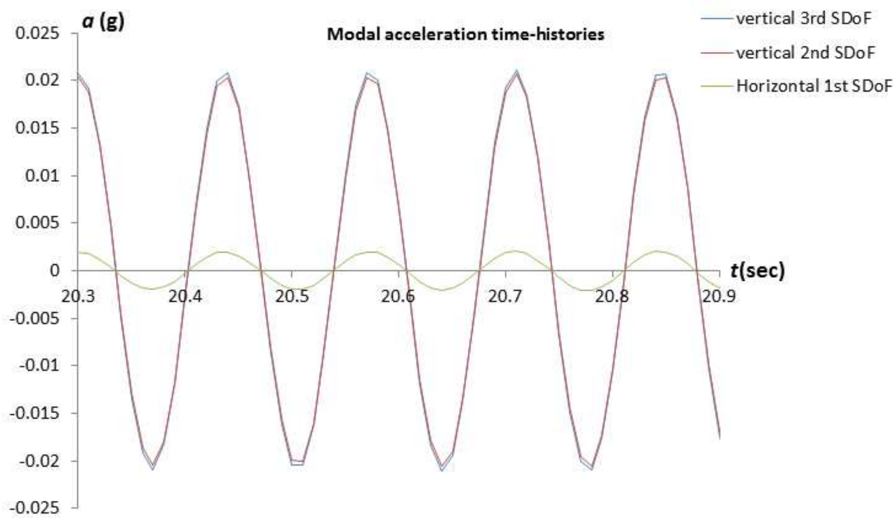

FIGURE 8. Modal acceleration time-histories for frequency $f_1 = 7.54\,\mathrm{Hz}$, where the phases between the three components are zero.

According to Figure 3 and Eq. (2.5), the modal components of the two eigenmodes of the stairway were computed by the SeismoSignal [28] data processing program and the following values were recovered:

$$\boldsymbol{\phi}_1 = \begin{bmatrix}\phi_{11}\\ \phi_{21}\\ \phi_{31}\end{bmatrix} = \begin{bmatrix}1\\ 41.405\\ 47.50\end{bmatrix} \quad \boldsymbol{\phi}_2 = \begin{bmatrix}\phi_{12}\\ \phi_{22}\\ \phi_{32}\end{bmatrix} = \begin{bmatrix}-1\\ 51.16\\ 44.00\end{bmatrix}$$

Finally, the free vibration part of the modal time histories components was used in order to calculate the equivalent modal damping ratios ξ^{mod}, by applying the equation of the logarithmic decrement in each of the three modal components. As shown in Table 1, the final averaged equivalent modal damping ξ corresponds to the mean value of the aforementioned ξ^{mod}.

5. Numerical model of the steel stairway

In the present study, the FEM modeling of the steel stairway is based only on information provided and/or estimated by appropriate engineering judgement

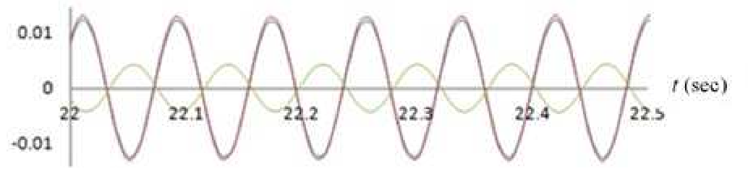

FIGURE 9. Modal acceleration time-histories for frequency $f_2 = 12.23\,\text{Hz}$, where the phase between the first component and the other two is π.

TABLE 1. Modal and equivalent viscous damping ratios for the stairway

$f_1 = 7.35\,\text{Hz}$	ξ^{mod}	$\xi_1 = 0.0045$
1^{st} DOF	0.0037	
2^{nd} DOF	0.0048	
3^{rd} DOF	0.0049	
$f_2 = 12.23\,\text{Hz}$	ξ^{mod}	$\xi_2 = 0.0070$
1^{st} DOF	0.0073	
2^{nd} DOF	0.0065	
3^{rd} DOF	0.0071	

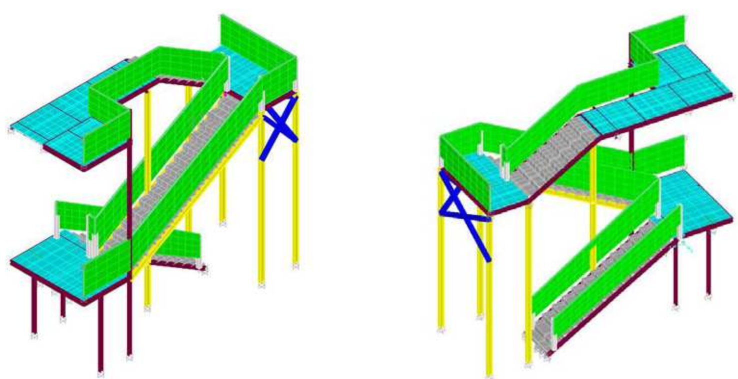

FIGURE 10. FEM model of the steel stairway using SAP2000: (left) front view showing the middle span; (right) back view showing the first and third spans.

regarding the as-built structure from architectural plans and from actual measurements. In what follows, we list all necessary assumptions regarding the built-up of the FEM model of the steel stairway (see Figure 10) using the commercial program SAP2000, version 6.1 [27].

(1) Each structural element is considered to behave as a linear, elastic and isotropic material.

(2) The structure is built of structural steel (span beams and landing beams, columns and guardrails) and of sheet metal (risers, treads and landings).
(3) Frame elements are used to model all beams, the supporting columns and the columns of the guardrails. These frame element in SAP2000 [27] correspond to the general, three-dimensional, beam-column formulation which includes the effects of biaxial bending and shear, of torsion and of axial action.
(4) Existing overlapping and/or eccentricities in the frame elements cross sections coming at a joint is taken into account by the 'end offsets' formulation provided by SAP2000, see Figure 11.
(5) Four-node shell elements are used to model all the surface elements of the stairway, such as the landings, the risers and the treads.
(6) Four-node shell elements are also used to model the guardrails, as they add stiffness to the adjacent structural elements, see Davis and Murray [15].
(7) The homogeneous shell element formulation provided by SAP2000 [27] is selected, which combines independent membrane and plate action.
(8) The selection of isotropic behavior in the aforementioned shell finite elements is based on the investigation conducted by Setareh and Jin [16] for a monumental stair. From the aforementioned study, it was concluded that orthotropic properties in the shell elements do not contribute to the possible discrepancies between the measured and computed natural frequencies.
(9) The existing welded connections between all of the parts of the stairway are modeled as fully continuous connections.
(10) The total mass of the canopy is taken into account under the assumption of lumped masses at the common nodes with the stairway beams.

One of the main modelling uncertainties are the boundary conditions, see Belver et al. [9] and Keinner et al. [17]. The steel stairway under study is supported on the ground level by eight steel columns and a concrete surface block, which is in turn connected to the lower level of the first span. Furthermore, the stairway is supported laterally by beam ends, inserted into the adjacent building walls. In order to investigate the effect of the boundary conditions on the dynamic characteristics of the stairway, three different FEM models were created. The first model considers all supports as fixed, the second considers all supports as pinned and the third considers all supports as fixed, except for the lateral ones.

5.1. Modal Analysis. In order to investigate the dynamic characteristics of the steel stairway, a modal analysis is performed to recover the undamped, free-vibration mode shapes and eigenfrequencies of the system. These natural modes (eigenmodes) provide insight into the dynamic behavior of the steel stairway, and the first three are depicted in Figure 12. All three FEM models used yield the same order of eigenmodes. In particular, the first eigenmode refers to the modal deformed shape of the second span of the stairway, the second eigenmode to the first span and the third eigenmode refers to the third span. Therefore, the boundary conditions details do not alert the order of the eigenmodes.

Figure 11. Eccentricity of beam elements at a common node: (left) actual connection; (right) FEM model representation.

On the other hand, the different boundary conditions affect the numerical values of the eigenfrequencies corresponding to each mode. These values are summarized in Table 2 for the first ten computed eigenfrequencies, as well as their variation under the different boundary condition assumptions. The first eigenfrequency values range between 7.24 Hz and 7.26 Hz, the second eigenfrequency values between 7.65 Hz and 7.81 Hz and the third eigenfrequency values between 13.27 Hz and 14.23 Hz. All aforementioned eigenfrequencies are in good agreement with the experimental data, i.e., f_1=7.53 Hz and f_2=12.23 Hz.

Figure 12. Eigenmodes of (left) second span; (center) first span; (right) third span.

The pinned support assumption in the second FEM model leads to an even more flexible steel stairway compared to that of fixed supports (the first FEM model). Also, the first two eigenfrequencies are not much affected by the actual support type, whereas the third eigenfrequency is reduced considerably by 6.74 %. Therefore, it is the dynamic behaviour of the third span that is mostly affected by the boundary conditions, whereas the dynamic behaviour of the first and second span is unaffected.

Considering some more details, the type of column support at the lower level of the first span due to the presence of the concrete block foundation is modelled

Table 2. FEM model eigenfrequencies of the stairway for three different boundary condition hypotheses

Mode	Fixed Supports	Pinned Supports	Pinned Lateral Supports	Difference between Models 2-1	Difference between Models 3-1
	f(Hz)	f(Hz)	f(Hz)	Δf(%)	Δf(%)
1	7.26	7.24	7.26	-0.22%	-0.04%
2	7.71	7.65	7.81	-0.70%	1.35%
3	14.24	13.28	13.28	-6.74%	-7.18%
4	15.22	15.09	15.09	-0.86%	-0.86%
5	15.78	15.72	15.73	-0.39%	-0.32%
6	17.80	17.63	17.63	-0.93%	-0.94%
7	19.20	19.06	19.11	-0.76%	-0.47%
8	19.47	19.44	20.40	-0.14%	4.82%
9	20.46	20.11	20.52	-1.72%	0.28%
10	21.04	20.52	20.53	-2.49%	-2.50%

as fixed, whereas the beam ends that are inserted in the adjacent building walls are modelled as pinned (third FEM model). Again, the first eigenmode is not affected at all by this support type. However, the second and the third eigenmodes are quite different from the fixed assumption. In particular, the second eigenmode, which refers to the first span, tends to be stiffer, whereas the third eigenmode, which refers to the third span, tends to be more flexible. The basic conclusions that can be drawn from the above FEM modal analysis of the three basic steel stairway models are the following:

(1) The first three eigenmodes refer to the modal deformed shape of the second, first and third span of the steel stairway, respectively.
(2) There is a good agreement between the numerical and the experimental results in terms of eigenfrequency values.
(3) The specific boundary conditions of the steel stairway mostly affect the behavior of the first and third spans, which are laterally supported on the adjacent building walls.
(4) The true behaviour of the lateral supports can be considered to be somewhere between a fully-fixed and a fully-pinned support.

Taking into account all available experimental data, the present FEM models can be further upgraded by means of FEM updating techniques.

6. Conclusions

We presented here an identification method for the dynamic characteristics of a flexible metal structure which responds as a continuous, in terms of mass and stiffness distribution, dynamic system to environmentally-induced loads. This is a rather difficult problem in structural dynamics, since there is a spread of adjacently grouped eigenfrequencies with equivalent response amplitudes. This results in the absence of a dominant frequency that would be detectable in terms of a bell or nail shaped bump in the FFT plots. Thus, in order to produce reliable results, the method of modal acceleration time histories was applied, which calculates the phase differences between the recorded DOF in the total time history response. Next, a discrete-parameter FEM model for the steel was developed. Even though this model was based only on available data and appropriate engineering judgement had to be used regarding the as-built structure, it succeeded in reproducing eigenfrequencies that are in good agreement with the experimentally measured ones. The ensuing parametric FEM analyses reveal the crucial effect of the boundary conditions on the dynamic response of the steel stairway. Thus, on-site recordings from the multi-channel system of accelerometers, with the appropriate analytic processing, can produce a very good estimate of the true values of the eigenfrequencies and eigenmodes, even in the case of continuous systems. It is of course acknowledged that use of FEM models contributes to the identification of the dominant eigenfrequency range, thus narrowing the search during the processing of the experimental recordings. However, the development of a numerical model is not absolutely necessary for the identification of the dynamic characteristics of the flexible structure, since the method of the modal time history accelerations method is autonomous. The true value of finite element models in our case lies elsewhere: Once calibrated, they can be used in conjunction with non-periodic dynamic response recordings to establish the structural ageing over time as the stairway remains under continuous use.

References

1. M. Basseville, A. Benveniste, M. Goursat, L. Hermans, L. Mevel, H. Auweraer, *Output-only subspace based structural identification: from theory to industrial testing practice*, J. Dyn. Syst.-T-ASME: Special Issue on Identification of Mechanical Systems **123**(4) (2001), 68–76.
2. R. Brincker, L. Zhang, P. Andersen, *Modal identification of output-only systems using frequency domain decomposition*, Smart Mater. Struct. **10** (2001), 441–451.
3. B. Peeters, G. De Roeck, *Stochastic system identification for operational modal analysis: A review*, J. Dyn. Syst. Meas. Control-ASME: Special Issue on Identification of Mechanical Systems **123**(4) (2001), 659–667.
4. H. Wenzel, D. Pichler, *Ambient Vibration Monitoring*, John Wiley & Sons, Chichester, England, 2005.
5. P. Overschee, B. De Moor, *Subspace Identification for Linear Systems*, Kluwer Academic Publishers, Dordrecht, 1996.
6. T. Makarios, *Identification of the mode shapes of spatial tall multi-storey buildings due to earthquakes: The new modal time histories method*, Struct. Des. Tall Spec. **21** (2012), 621–641.
7. A. T. Makarios, *Chapter 4: Identification of building dynamic characteristics by using the modal response acceleration time-histories in the seismic excitation and the wind dynamic loading cases*, in: P. S. de Brito Andre and H. Varum (eds.), *Accelerometers: Principles, Structure and Applications*, Nova Science Publishers, New-York, 2013, 7–113.
8. G. D. Manolis, A. Athanatopoulou, K. D. Dragos, A. Arabatzis, A. Lavdas, C. Z. Karakostas, *Identification of pedestrian bridge dynamic response through field measurements and numerical modeling: Case studies*, J. Theor. Appl. Mech., Sofia **44**(2) (2014), 3–24.

9. A. V. Belver, S. Zivanovic, H. V. Dang, M. Istrate, A. L. Iban, *Modal testing and FE model updating of a lively staircase structure*, *Topics in Modal Analysis I, Vol. 5*, Proceedings of the Society of Experimental Mechanics Conference, Series 30, 2012, 547–557.
10. S. Zivanovic, A. Pavic, P. Reynolds, *Modal testing and FE model tuning of a lively footbridge structure*, Eng. Struct. **28**(6) (2006), 857–868.
11. V. Arbitrio, *Longchamp stair optimization and vibration study*, Structure **February Issue** (2009), 10–13.
12. C. Howes, M. Krynski, S. Kordt, *The feature stair at Louis Vuitton in Crystals at City Center*, Proceedings ASCE Structures Congress **2011** (2011), 2585–2593.
13. C. Howes, E. Gordon, *The spiral stairs at the art Gallery of Ontario*, Proceedings ASCE Structures Congress **2011** (2011), 2594–2604.
14. D. J. Huntington, J. W. Mooney, *How to keep monumental stairs from vibrating*, Proceedings ASCE Structures Congress **2011** (2011), 2572–2587.
15. B. Davis, T. M. Murray, *Comparison of measured modal properties and walking accelerations with analytical predictions for a slender monumental stair*, Proceedings of the 2008 Architectural Engineering National Conference (2008).
16. M. Setareh, E. J. Jin, *Modal Analysis and FE Model Updating of a Monumental Stair*, School of Architecture and Design Research Report, Virginia Tech, Blacksburg, Virginia, 2013.
17. T. M. Keinner, J. A. Wilkinson, R. C. Fischer, *Effect of boundary conditions on the modal analysis of a staircase*, Proceedings 2004 BAQUS Users Conference (2004).
18. A. Capellini, S. Manzoni, M. Vanali, *Experimental and numerical studies of the people effects on a structure modal parameters*, Topics on the Dynamics of Civil Structures, Vol.1, Proceedings of the Society for Experimental Mechanics Conference, **26** (2012), 17–25.
19. R. Eid, M. Scisa, D. Stevenson, B. Howe, *Staircase vibrations due to human activity*, Proceedings ASCE Structures Congress **2011** (2011), 2562–2571.
20. S. Kim, Y. Lee, A. Scanlon, *Experimental assessment of vibration serviceability of stair systems*, J. Constr. Steel Res. **64**(2) (2007), 253–259.
21. S. Modak, T. Kundara, B. Nakra, *Comparative study of model updating methods using experimental data*, Comput. Struct. **80**(5–6) (2002), 437–444.
22. D. Bradand, M. Thomas, *Slender monumental stair vibration serviceability*, J. Archit. Eng. **15**(4) (2009), 111–121.
23. J. M. W. Brownjohn, P. Xia, *Dynamic assessment of curved cable-stayed bridge by model updating*, J. Struct. Eng.-ASCE **126**(2) (2000), 252–260.
24. T. K. Makarios, G. D. Manolis, I. Karetsou, M. Papanikolaou, V. Terzi, *Modeling and identification of the dynamic response of an existing three-story steel stairway*, Proceedings of COMPDYN **2015** (2015).
25. I. Karetsou, *Instrumentation of steel stairway and identification of its dynamic characteristics: Model of analysis*, MSc Thesis, Graduate Program on Earthquake Engineering and Seismic Design of Structures, Civil Engineering Department, Aristotle University, Thessaloniki, Greece, 2014.
26. M. Papanikolaou, *Identification of dynamic characteristics of steel stairway using recordings and development of suitable model*, MSc Thesis, Graduate Program on Earthquake Engineering and Seismic Design of Structures, Civil Engineering Department, Aristotle University, Thessaloniki, Greece, 2014.
27. *CSI Analysis Reference Manual for SAP2000, ETABS, SAFE and CSiBridge*, *Computers and Structures*, Berkeley, California, 2013.
28. *SeismoSignal, Version 5.1.0*, *Seismosoft Ltd*, Pavia, Italy, 2012.

ИДЕНТИФИКАЦИЈА МОДОВА ОБЛИКА ПОСТОЈЕЋЕГ ТРОСПРАТНОГ ФЛЕКСИБИЛНОГ ЧЕЛИЧНОГ СТЕПЕНИШТА КАО НЕПРЕКИДНОГ ДИНАМИЧКОГ СИСТЕМА

Резиме. У раду се разматра проблем идентификације модова облика флексибилних челичних степеница које се налазе у оквиру комплекса зграде Грађевинског одељења Аристотеловог универзитета у Солуну, Грчка. Поменуто степениште је систем са континуираном дистрибуцијом масе и крутости, чињеница која доводи до тога да је опис структуре изазов у поређењу са структурама где се могу посматрати скоковита увећања ова два основна параметра. Одређеније, ово степениште је мерено користећи локалну више-каналну мрежу мерача убрзања. На степеништу су постављена два 12-битна, номиналне резолуције, дигитална једноосна мерача убртања типа KUOWA-PCD-30A, повезаних кабловима са "заједничким временом" и "заједничким почетним" карактеристикама. Доминантни облици вибрација степеништа су израчунати помоћу "методе облика одговора убрзања у времену". Паралелно, према резултатима амбијенталних вибрација, конструисан је и калибрисан детаљан метод коначних елемената модела степеништа. Напомињемо да је идентификациона процедура коришћења за динамичке карактеристике просторних структура дала резултате који се могу користити за развој серије нумеричких модела степеништа, почев од једноставног система са једним степеном слободе до детаљних модела са више степени слободе. На крају, овде су укључене неке корисне информације о теоријској процедури за идентификацију модалних облика.

Institute of Statics and Dynamics
Civil Engineering Department
Aristotle University of Thessaloniki
Thessaloniki
Greece
gdm@civil.auth.gr

makariostr@civil.auth.gr

terziv@civil.auth.gr

joykaretsou@hotmail.com

(Received 02.07.2015)
(Revised 19.10.2015)