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EFFECT OF COUPLE STRESSES ON HYDROMAGNETIC EYRING–POWELL FLUID FLOW THROUGH A POROUS CHANNEL

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Abstract. In this paper, the flow of hydromagnetic non-Newtonian fluid under couple stresses through a porous channel is investigated using the Eyring–Powell model. The fluid is driven by an axial constant pressure gradient. Approximate solutions of the nonlinear dimensionless equations governing the fluid flow are obtained using a new modification of Adomian decomposition method (ADM). The effects of the variation of various flow parameters on both the velocity and temperature fields are deduced and discussed including surface-fluid interface friction and rate of heat transfer.

Nomenclature

u	dimensionless velocity
μ	is the fluid velocity
V_0	is the uniform suction/injection velocity
ρ	is the fluid density
P'	is the fluid pressure
yν	is the kinematic viscosity
σ	is the electrical conductivity of the fluid
yB_0	is the magnetic field strength of the Lorentz force
η	is the co-efficient of couple stresses
C_p	is the specific heat capacity
T	is the fluid temperature
κ	is the thermal conductivity of the fluid
Re	is the suction/injection Reynolds number
Pe	is the Peclet number
Br	is the Brinkman number
Ha^2	Hartmann number
a^2	is the couple stresses inverse

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1. Introduction

The striking feature of couple stress fluid is the introduction of the size dependent effect that is usually neglected in the classical continuum mechanics. Over the years, tremendous achievements have been made on non-Newtonian fluid flow under couple stresses. Some of these studies include Eldabe et al. [1] in which the effect of couple stresses on the oscillatory unsteady non-Newtonian hydromagnetic fluid flow through a porous channel using the Eyring–Powell model was investigated. In [2], Rathod and Tanveer considered the pulsatile flow of blood through a porous medium and under the influence of periodic body acceleration in the presence of a magnetic field. Zueco and Bég [3] applied the Network Simulation Method (NSM) and the Eyring–Powell rheological non-Newtonian model to simulate the pulsatile flow of a couple stresses fluid in a two-dimensional channel with wall transpiration. Adesanya and Ayeni [4] applied the Adomian decomposition method to prove the existence and uniqueness of the solution to the pulsatile couple stress fluid flowing through a porous medium using the Eyring–Powell rheological non-Newtonian model.

All the studies are valid in the hydrodynamical case in which the effect of temperature on the couple stress non-Newtonian fluid flow has been neglected. In reality, the effect of temperature on the couple stress fluid flow cannot be neglected in many industrial processes since it is the basis for the design and development of modern products. Common examples can be found in polymer extrusions, flow of liquid steel, solidification of liquid crystals, cooling of liquid metallic plate, lubricating fluids and many more. In fact, the literature is very rich on the thermodynamical properties of the couple stress fluid. For example, Adesanya and Makinde [5] considered the heat transfer to the unsteady pulsatile hydromagnetic flow of non-Newtonian fluid through a porous channel under couple stresses. Srinivasacharya and Kaladhar [6] studied the steady incompressible couple stress fluid flow between parallel disks and they showed that the presence of couple stresses in the fluid decreases both the fluid velocity and temperature and also that an increase in the Prandtl number leads to the decrease in fluid temperature.

In a related study, Srinivasacharya and Kaladhar [7] investigated the Hall and ion-slip effects on fully developed, electrically conducting couple stress fluid flow between vertical parallel plates in the presence of a temperature dependent heat source. From this result, the authors pointed out that as the magnetic parameter increases, the velocity in the direction of the flow and the temperature are decreased while the presence of couple stresses in the fluid decreases the velocity and temperature among other parameters. Moreover, Srinivasacharya et al. [8] studied the steady flow of incompressible couple stress fluid flow between parallel porous plates maintained at constant non-uniform temperature. They showed that the fluid velocity decreases with an increase in the couple stress parameter while an increase in the couple stress parameter increases the fluid temperature within the channel. It was also shown that an increase in the fluid Prandtl number increases the fluid temperature. Other important articles on couple stress fluid flow under different geometries can be found in [9–11] and references therein.

In view of the fact that there is no single constitutive equation that can adequately describe the rheological properties of all non-Newtonian fluids, the present paper is focused on the flow and heat transfer to a hydromagnetic non-Newtonian fluid under couple stresses considering the effect of Joule dissipation. The present analysis uses the Eyring–Powell rheological non-Newtonian model, which to the best of authors' knowledge, has not been previously accounted for in the literature. The problem presented here is strongly nonlinear with no known exact solution. An approximate solution of the problem will be obtained using the Adomian decomposition method. This method has been used in the literature to obtain solutions to a wide variety of nonlinear problems some of which can be found in [**12–27**]. To establish the accuracy of the solution, the approximate solution of the linearized problem is compared with its exact solution where the computed result compared favorably with the exact solution. In the rest of our paper, Section 2 presents the formulation of the problem, while in Section 3, analytical solutions of the problem are obtained. Results are discussed in Section 4. In Section 5, we present several concluding remarks.

2. Mathematical Model

For a non-Newtonian fluid flow through a channel, the Eyring–Powell constitutive model can be written as

$$\tau_{xy} = \mu\frac{du'}{dy'} + \frac{1}{\beta}\sin h^{-1}\Big(\frac{1}{c}\frac{du'}{dy'}\Big). \tag{2.1}$$

The first term represents the viscosity effect while latter part represents the elastic part. The second term in the right hand side of (2.1) can be expanded as

$$\sin h^{-1}\Big(\frac{1}{c}\frac{du'}{dy'}\Big) \cong \frac{1}{c}\frac{du'}{dy'} - \frac{1}{6}\Big(\frac{1}{c}\frac{du'}{dy'}\Big)^3 + \frac{3}{40}\Big(\frac{1}{c}\frac{du'}{dy'}\Big)^5 - \frac{5}{112}\Big(\frac{1}{c}\frac{du'}{dy'}\Big)^7 + \mathcal{O}(.)^9,$$
$$\Big|\frac{1}{c}\frac{du'}{dy'}\Big| < 1,$$

where $\mathcal{O}(.)$ represents other terms, retaining the first and second terms of the approximations and neglecting higher order terms, we obtain

$$\sin h^{-1}\Big(\frac{1}{c}\frac{du'}{dy'}\Big) \cong \frac{1}{c}\frac{du'}{dy'} - \frac{1}{6}\Big(\frac{1}{c}\frac{du'}{dy'}\Big)^3, \tag{2.2}$$

where β, c are the characteristics of the Eyring-Powell model, and μ is the fluid viscosity.

2.1. Description of the Problem. Consider the couple stresses fluid flowing steadily with temperature T_1 between two infinite horizontal non-conducting plates located at $y = L$ apart with uniform wall temperatures T_0. By assuming a very small magnetic Reynolds number the induced magnetic field can be safely neglected. Take a Cartesian coordinate system (x, y) where the x-axis lies along the centre of the channel, y is the distance measured along the normal direction. The hot fluid is injected into the lower wall at $y = 0$ and extracted with the same constant velocity at the upper wall $y = L$. By neglecting the electric field, the momentum equation for the flow can be written as [**1**, **6**]

$$V_0 \frac{du'}{dy'} = -\frac{1}{\rho}\frac{dP'}{dx'} + \frac{d\tau_{xy}}{dy'} - \frac{\sigma B_0^2 u}{\rho} - \frac{\eta'}{\rho}\frac{d^4u'}{dy'^4}, \tag{2.3}$$

subject to the following boundary conditions

$$u'(0) = \frac{d^2u'}{dy'^2}(0) = \frac{d^2u'}{dy'^2}(L) = u'(L) = 0.$$

Substituting (2.1) and (2.2) into (2.3) yields

$$\rho V_0 \frac{du'}{dy'} = -\frac{dP'}{dx'} + \mu\frac{d^2u'}{dy^{2'}} + \frac{1}{\beta c}\frac{d^2u'}{dy'^2} - \frac{1}{2\beta c^3}\Big(\frac{du'}{dy'}\Big)^2\frac{d^2u'}{dy'^2} - \sigma B_0^2 u' - \eta'\frac{d^4u'}{dy'^4},$$

together with the appropriate boundary conditions

$$u'(0) = \frac{d^2u'}{dy^2}(0) = \frac{d^2u'}{dy^2}(1) = u'(1) = 0.$$

The temperature distribution within the channel can be described using the energy equation in the form

$$\rho C_p V_0 \frac{dT}{dy'} = k\frac{d^2T}{dy'^2} + \mu\Big(\frac{du'}{dy'}\Big)^2 + \eta\Big(\frac{d^2u'}{dy'^2}\Big)^2 + \sigma B_0^2 u'^2,$$

together with the appropriate wall boundary conditions $T(0) = T_0 = T(L)$. Introducing the following dimensionless variables and parameters

$$y=\frac{y'}{L},\ x=\frac{x'}{L},\ u=\frac{u'}{V_0},\ \theta=\frac{(T-T_0)}{T_1-T_0},\ P=\frac{P'}{\rho V_0^2},\ S=\frac{V_0 L}{\nu},\ \kappa=1+M,\ M=\frac{1}{\beta c\mu},$$

$$b=\frac{V_0^2}{\mu\beta c^3 L^2}, a^2=\frac{L^2\mu}{\eta'},\ H^2=\frac{\sigma B_0^2 L^2}{\mu},\ Pe=\frac{\rho C_p V_0 L}{k},\ Br=\frac{V_0^2\mu}{k(T_1-T_0)},\ \lambda=-\frac{dP}{dx},$$

we obtain

$$S\frac{du}{dy} = S\lambda + \kappa\frac{d^2u}{dy^2} - \frac{b}{2}\Big(\frac{du}{dy}\Big)^2\frac{d^2u}{dy^2} - H^2u - \frac{1}{a^2}\frac{d^4u}{dy^4}, \tag{2.4}$$

$$Pe\frac{d\theta}{dy} = \frac{d^2\theta}{dy^2} + Br\Big(\Big(\frac{du}{dy}\Big)^2 + \frac{1}{a^2}\Big(\frac{d^2u}{dy^2}\Big)^2 + H^2u^2\Big), \tag{2.5}$$

subject to the following boundary conditions

$$u(0) = \frac{d^2u}{dy^2}(0) = \frac{d^2u}{dy^2}(1) = u(1) = 0, \tag{2.6}$$

$$\theta(0) = 0 = \theta(1). \tag{2.7}$$

Equations (2.4) to (2.7) represent the system of two-coupled nonlinear boundary valued problems with a mixed set of Dirichlet and Neumann-like boundary conditions that will be subsequently solved by ADM.

3. Analysis of the Method

Consider the inhomogeneous nonlinear differential equation in Adomian's operator-theoretic form

$$Lw + Rw + Nw = g. \tag{3.1}$$

In (3.1), w is the unknown function or system output, which is to be determined by recursion, L is the invertible linear space operator that is usually the highest order derivative, R is a linear remainder, $[R] < [L]$ differential operator whose order is less than L, N represents the nonlinear terms whose order is also less than L, $[N] < [L]$ while g is the source term or system input. Applying the inverse operator L^{-1} to both side of (3.1) and using the given initial or boundary conditions, we obtain

$$w = v - L^{-1}(Rw) - L^{-1}(Nw),$$

where v represents the terms arising from integrating the source term g and from the auxiliary conditions. The standard ADM defines the solution w by the decomposition series

$$w = \sum_{n=0}^{\infty} w_n,$$

and the nonlinear term comprises the series of the Adomian polynomials,

$$Nw = \sum_{n=0}^{\infty} A_n,$$

where the A_n are the Adomian polynomials defined by the relation Adomian and Rach [21]

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left[N\Big(\sum_{i=0}^{\infty}\lambda^i w_i\Big)\right]\right]_{\lambda=0},$$

such that the Adomian polynomials are evaluated as

$$\begin{aligned}
A_0 &= f(w_0),\\
A_1 &= w_1 f^{(1)}(w_0),\\
A_2 &= w_2 f^{(1)}(w_0) + \frac{1}{2!}w_1^2 f^{(2)}(w_0),\\
A_3 &= w_3 f^{(1)}(w_0) + w_1 w_2 f^{(2)}(w_0) + \frac{1}{3!}w_1^3 f^{(3)}(w_0).\\
&\dots
\end{aligned}$$

The solution components $w_0, w_1, w_2, \dots$ are then determined by using the standard Adomian recursion relation

$$\begin{aligned}
w_0 &= v,\\
w_{k+1} &= -L^{-1}Rw_k - L^{-1}A_k, \quad k \geq 0,
\end{aligned}$$

where w_0 is referred to as the zeroth-order component. For an ordinary differential equation, the unknown constants of integration inherent in v are evaluated using the initial or boundary conditions. Then the approximate solution thus obtained is the partial sum

$$S = \sum_{i=0}^{Q} w_i,$$

where Q is the truncation point assuming that the solution converges. Next we proceed to obtain the approximate solution to Equations (2.4) to (2.7) by using the ADM solution. Re-writing (2.4), we get

$$\frac{d^4u}{dy^4} = Sa^2\lambda a^2 - H^2a^2u - Sa^2\frac{du}{dy} + \kappa a^2\frac{d^2u}{dy^2} - \frac{ba^2}{2}\Big(\frac{du}{dy}\Big)^2\frac{d^2u}{dy^2}, \tag{3.2}$$

subject to the following boundary conditions

$$u(0) = \frac{d^2u}{dy^2}(0) = 0 = u(1) = \frac{d^2u}{dy^2}(1). \tag{3.3}$$

In Adomian's operator-theoretic form, (3.2) can be written as

$$L^4u(y) = g(y) + Ru(y) + Nu(y),$$

where we have used the definitions:

$$\begin{aligned} L^4u(y) &= \frac{d^4u}{dy^4}(y) \\ g(y) &= S\lambda a^2 \\ Ru(y) &= -H^2a^2u(y) - Sa^2\frac{du}{dy}(y) + \kappa a^2\frac{d^2u}{dy^2}(y) \\ Nu(y) &= -\frac{ba^2}{2}\Big(\frac{du}{dy}(y)\Big)^2\Big(\frac{d^2u}{dy^2}(y)\Big), \end{aligned}$$

where L^4 is the linear operator to be inverted. We usually use L but use L^4 for convenience in boundary value problems. R is the linear remainder operator. N is the nonlinear operator. $u(y)$ is the system output and $g(y)$ is the system input, which in this case is constant. We now define the following Volterra integrals

$$L^{-4}w(y) = \int_0^y\int_0^y\int_0^y\int_0^y w(Y)dYdYdYdY,$$

such that

$$L^{-4}L^4u(y) = L^{-4}g(y) + L^{-4}Ru(y) + L^{-4}Nu(y). \tag{3.4}$$

As a result, Equation (3.4) becomes

$$u(y) = u(0) + y\frac{du}{dy}(0) + \frac{y^2}{2}\frac{d^2u}{dy^2}(0) + \frac{y^3}{6}\frac{d^3u}{dy^3}(0) + S\lambda a^2\frac{y^4}{24} + L^{-4}Ru(y) + L^{-4}Nu(y).$$

Using the boundary conditions at $y = 0$, i.e,

$$u(0) = \frac{d^2u}{dy^2}(0) = 0. \tag{3.5}$$

The equivalent nonlinear Volterra integral equation for with two undetermined constants of integration as an intermediate step

$$u(y) = y\frac{du}{dy}(0) + \frac{y^3}{6}\frac{d^3u}{dy^3}(0) + S\lambda a^2\frac{y^4}{24} + L^{-4}Ru(y) + L^{-4}Nu(y). \tag{3.6}$$

We also define the Fredholm integrals

$$\begin{aligned} L_1^{-4}w(y) &= \int_0^1\int_0^y\int_0^y\int_0^y w(y)dYdYdYdY, \\ L_1^{-2}w(y) &= \int_0^1\int_0^y w(y)dYdY. \end{aligned}$$

Next, we use the remaining boundary conditions in (3.3) and appropriate algebraic manipulations to determine the unknown constants of integration by formula as shown in [25–27]

$$u(1) = 0 = \frac{du}{dy}(0) + \frac{1}{6}\frac{d^3u}{dy^3}(0) + \frac{1}{24}S\lambda a^2 + L_1^{-4}Ru(y) + L_1^{-4}Nu(y), \tag{3.7}$$

$$\frac{d^2u}{dy^2}(1) = 0 = \frac{d^3u}{dy^3}(0) + \frac{1}{2}S\lambda a^2 + L_1^{-2}Ru(y) + L_1^{-2}Nu(y). \tag{3.8}$$

Solving (3.7) and (3.8) simultaneously, we have

$$\frac{d^3u}{dy^3}(0) = -\frac{1}{2}S\lambda a^2 - L_1^{-2}Ru(y) - L_1^{-2}Nu(y), \tag{3.9}$$

$$\frac{du}{dy}(0) = \frac{1}{24}S\lambda a^2 + \frac{1}{6}L_1^{-2}Ru(y) + \frac{1}{6}L_1^{-2}Nu(y) - L_1^{-4}Ru(y) - L_1^{-4}Nu(y). \tag{3.10}$$

As the expression for the two remaining constants, substituting (3.9) and (3.10) in (3.6), we get

$$\begin{aligned} u(y) = {} & y\Big\{\frac{1}{24}S\lambda a^2 + \frac{1}{6}L_1^{-2}Ru(y) + \frac{1}{6}L_1^{-2}Nu(y) - L_1^{-4}Ru(y) - L_1^{-4}Nu(y)\Big\} \\ & + \frac{y^3}{6}\Big\{-\frac{1}{2}S\lambda a^2 - L_1^{-2}Ru(y) - L_1^{-2}Nu(y)\Big\} \\ & + S\lambda a^2\frac{y^4}{24} + L^{-4}Ru(y) + L^{-4}Nu(y). \end{aligned}$$

Or the equivalent nonlinear Fredholm–Volterra integral equation for the solution $u(y)$ without any undetermined constants of integration

$$\begin{aligned} u(y) = {} & \frac{1}{24}S\lambda a^2\{y - 2y^3 + y^4\} \\ & + y\Big\{\frac{1}{6}\big[L_1^{-2}Ru(y) + L_1^{-2}Nu(y)\big] - \big[L_1^{-4}Ru(y) + L_1^{-4}Nu(y)\big]\Big\} \\ & - \frac{y^3}{6}\big\{L_1^{-2}Ru(y) + L_1^{-2}Nu(y)\big\} + L^{-4}Ru(y) + L^{-4}Nu(y). \end{aligned}$$

Adomian decomposition series and the series of Adomian polynomials

$$\begin{aligned} u(y) &= \sum_{n=0}^{\infty} u_n(y), \\ Nu(y) &= \sum_{n=0}^{\infty} A_n(y), \\ A_n(y) &= A_n(u_0(y), \dots, u_n(y)), \end{aligned} \tag{3.11}$$

for the nonlinear term be represented by

$$A_n = \sum_{m=0}^{n} \frac{d^2u_{n-m}}{dY^2} \sum_{l=0}^{m} \Big(\frac{du_{m-l}}{dY}\Big)\Big(\frac{du_l}{dY}\Big). \tag{3.12}$$

Then the Adomian polynomials for the nonlinear term (3.5) can be computed as

$$
\begin{aligned}
A_0 &= \frac{d^2u_0}{dY^2}\Big(\frac{du_0}{dY}\Big)^2,\\
A_1 &= 2\frac{d^2u_0}{dY^2}\Big(\frac{du_0}{dY}\Big)\Big(\frac{du_1}{dY}\Big) + \frac{d^2u_1}{dY^2}\Big(\frac{du_0}{dY}\Big)^2,\\
A_2 &= \frac{d^2u_0}{dY^2}\Big(\frac{du_1}{dY}\Big)^2 + 2\frac{d^2u_0}{dY^2}\Big(\frac{du_0}{dY}\Big)\Big(\frac{du_2}{dY}\Big)\\
&\quad + 2\frac{d^2u_1}{dY^2}\Big(\frac{du_0}{dY}\Big)\Big(\frac{du_1}{dY}\Big)\\
&\quad + \frac{d^2u_2}{dY^2}\Big(\frac{du_0}{dY}\Big)^2,\\
&\quad \dots.
\end{aligned}
\tag{3.13}
$$

Upon substitution, we have

$$
\begin{aligned}
\sum_{n=0}^{\infty} u_n(y) &= \frac{1}{24}S\lambda a^2\{y - 2y^3 + y^4\}\\
&\quad + y\Big\{\frac{1}{6}\Big[L_1^{-2}R\sum_{n=0}^{\infty}u_n(y) + L_1^{-2}\sum_{n=0}^{\infty}A_n(y)\Big]\\
&\quad - \Big[L_1^{-4}R\sum_{n=0}^{\infty}u_n(y) + L_1^{-4}\sum_{n=0}^{\infty}A_n(y)\Big]\Big\}\\
&\quad - \frac{y^3}{6}\Big\{L_1^{-2}R\sum_{n=0}^{\infty}u_n(y) + L_1^{-2}\sum_{n=0}^{\infty}A_n(y)\Big\}\\
&\quad + L^{-4}R\sum_{n=0}^{\infty}u_n(y) + L^{-4}\sum_{n=0}^{\infty}A_n(y).
\end{aligned}
\tag{3.14}
$$

Duan–Rach modified recursion scheme in this case can be written as

$$
\begin{aligned}
u_0(y) &= \frac{1}{24}S\lambda a^2\{y - 2y^3 + y^4\},\\
u_{n+1}(y) &= y\Big\{\frac{1}{6}\big[L_1^{-2}Ru_n(y) + L_1^{-2}A_n(y)\big] - \big[L_1^{-4}Ru_n(y) + L_1^{-4}A_n(y)\big]\Big\}\\
&\quad - \frac{y^3}{6}\big\{L_1^{-2}Ru_n(y) + L_1^{-2}A_n(y)\big\}\\
&\quad + L^{-4}Ru_n(y) + L^{-4}A_n(y), \quad n \geq 0.
\end{aligned}
\tag{3.15}
$$

To ensure rapid convergence and reduce computational size, the iterative process (3.15) can be modified as follows

$$
\begin{aligned}
u_0(y) &= \frac{1}{24}S\lambda a^2 y^4,\\
u_1(y) &= -\frac{1}{12}S\lambda a^2 y^3\\
&\quad + y\Big\{\frac{1}{6}\big[L_1^{-2}Ru_0(y) + L_1^{-2}A_0(y)\big] - \big[L_1^{-4}Ru_0(y) + L_1^{-4}A_0(y)\big]\Big\}\\
&\quad - \frac{y^3}{6}\big\{\big[L_1^{-2}Ru_0(y) + L_1^{-2}A_0(y)\big]\big\} + \big[L^{-4}Ru_0(y) + L^{-4}A_0(y)\big],
\end{aligned}
$$

$$
\begin{aligned}
u_2(y) &= \frac{1}{24} S\lambda a^2 y \\
&\quad + y\Big\{\frac{1}{6}\big[L_1^{-2}Ru_1(y) + L_1^{-2}A_1(y)\big] - \big[L_1^{-4}Ru_1(y) + L_1^{-4}A_1(y)\big]\Big\} \\
&\quad - \frac{y^3}{6}\big\{\big[L_1^{-2}Ru_1(y) + L_1^{-2}A_1(y)\big]\big\} + \big[L^{-4}Ru_1(y) + L^{-4}A_1(y)\big] \\
u_{n+1}(y) &= y\Big\{\frac{1}{6}\big[L_1^{-2}Ru_n(y) + L_1^{-2}A_n(y)\big] - \big[L_1^{-4}Ru_n(y) + L_1^{-4}A_n(y)\big]\Big\} \\
&\quad - \frac{y^3}{6}\big\{\big[L_1^{-2}Ru_n(y) + L_1^{-2}A_n(y)\big]\big\} \\
&\quad + \big[L^{-4}Ru_n(y) + L^{-4}A_n(y)\big], \quad n \geq 2.
\end{aligned} \tag{3.16}
$$

The solution approximant can be written as

$$
\varphi_{n+1}(y) = \sum_{m=0}^{n} u_n(y), \tag{3.17}
$$

while the error remainder function gives

$$
\begin{aligned}
ER_{1,\,n+1}(y) &= \frac{d^4\varphi_{n+1}}{dy^4}(y) - S\lambda a^2 + H^2 a^2 \varphi_{n+1}(y) + Sa^2\frac{d\varphi_{n+1}}{dy}(y) \\
&\qquad - \kappa a^2 \frac{d^2\varphi_{n+1}}{dy^2}(y) + \frac{ba^2}{2}\Big(\frac{d\varphi_{n+1}}{dy}(y)\Big)^2\Big(\frac{d^2\varphi_{n+1}}{dy^2}(y)\Big),
\end{aligned} \tag{3.18}
$$

the maximal error remainder parameter

$$
MER_{1,n+1} = \max_{0 \leq y \leq 1} |ER_{1,n+1}(y)|. \tag{3.19}
$$

Similarly,

$$
\frac{d^2\theta}{dy^2}(y) = Pe\frac{d\theta}{dy}(y) - Br\Big\{\Big(\frac{du}{dy}(y)\Big)^2 + \frac{1}{a^2}\Big(\frac{d^2u}{dy^2}(y)\Big)^2 + H^2u^2(y)\Big\}, \tag{3.20}
$$

$$
\theta(0) = 0.\theta(1) = 0. \tag{3.21}
$$

We define

$$
L^2\theta(y) = Q\theta(y) + Mu(y), \tag{3.22}
$$

where L^2 is the linear operator to be inverted, Q is the linear remainder operator (we usually use R). M is the nonlinear operator (we usually use N). $\theta(y)$ is the system output and $M(y)$ is the system input in this case.

$$
L^2\theta(y) = \frac{d^2\theta}{dy^2}(y), Q\theta(y) = Pe\frac{d\theta}{dy}(y), L^{-2}w(y) = \int_0^y \int_0^y w(y)dYdY, \tag{3.23}
$$

where

$$
Mu(y) = -Br\Big\{\Big(\frac{du}{dy}(y)\Big)^2 + \frac{1}{a^2}\Big(\frac{d^2u}{dy^2}(y)\Big)^2 + H^2u^2(y)\Big\}. \tag{3.24}
$$

Following the classical Adomian decomposition, we take

$$
L^{-2}L^2\theta(y) = L^{-2}Q\theta(y) + L^{-2}Mu(y), \tag{3.25}
$$

To obtain

$$\theta(y) = \theta(0) + y\frac{d\theta}{dy}(0) + L^{-2}Q\theta(y) + L^{-2}Mu(y), \tag{3.26}$$

where $\theta(0) = 0$. The equivalent nonlinear Volterra integral equation with one undetermined constant of integration as an intermediate step is

$$\theta(y) = y\frac{d\theta}{dy}(0) + L^{-2}Q\theta(y) + L^{-2}Mu(y). \tag{3.27}$$

Next, we use the remaining boundary condition and appropriate algebraic manipulations to determine the unknown constant of integration by formula. i.e., using $\theta(1) = 0$, we get

$$\frac{d\theta}{dy}(0) = -L_1^{-2}Q\theta(y) - L_1^{-2}Mu(y). \tag{3.28}$$

Upon substitution, we get

$$\theta(y) = y\{-L_1^{-2}Q\theta(y) - L_1^{-2}Mu(y)\} + L^{-2}Q\theta(y) + L^{-2}Mu(y). \tag{3.29}$$

The equivalent nonlinear Fredholm-Volterra integral equation for $\theta(y)$ without any undetermined constants of integration is

$$\theta(y) = -y\{L_1^{-2}Q\theta(y) + L_1^{-2}Mu(y)\} + L^{-2}Q\theta(y) + L^{-2}Mu(y). \tag{3.30}$$

By Adomian decomposition series method of solution, we let

$$\theta(y) = \sum_{n=0}^{\infty}\theta_n(y), \tag{3.31}$$

such that

$$\begin{aligned}\sum_{n=0}^{\infty}\theta_n(y) = &- y\Big\{L_1^{-2}Q\sum_{n=0}^{\infty}\theta_n(y) + L_1^{-2}\sum_{n=0}^{\infty}B_n(y)\Big\}\\ &+ L^{-2}Q\sum_{n=0}^{\infty}\theta_n(y) + L^{-2}\sum_{n=0}^{\infty}B_n(y),\end{aligned} \tag{3.32}$$

and the Duan–Rach modified recursion scheme can be written as

$$\begin{aligned}&\theta_0(y) = -yL^{-2}Mu(y) + L_1{}^{-2}Mu(y)\\ &\theta_{n+1}(y) = -y\left\{L_1{}^{-2}Q\theta_n(y)\right\} + L^{-2}Q\theta_n(y), \quad n \geq 1.\end{aligned} \tag{3.33}$$

Solution approximant is then given as

$$\psi_{n+1}(y) = \sum_{m=0}^{n}\theta_m(y). \tag{3.34}$$

Error remainder function is

$$\begin{aligned}ER_{2,\,n+1}(y) =& \frac{d^2\psi_{n+1}}{dy^2}(y) - Pe\frac{d\psi_{n+1}}{dy}(y)\\ &+ Br\Big\{\Big(\frac{d\psi_{n+1}}{dy}(y)\Big)^2 + \frac{1}{a^2}\Big(\frac{d^2\psi_{n+1}}{dy^2}(y)\Big)^2 + H^2\psi_{n+1}^2(y)\Big\}\end{aligned} \tag{3.35}$$

and the maximal error remainder parameter

$$MER_{2,n+1} = \max_{0\leq y\leq 1} |ER_{2,n+1}(y)|. \tag{3.36}$$

Obviously, the accuracy of the solution can be improved with increasing the number of components in the partial sums. Equations (3.11) to (3.36) were coded in computer-assisted symbolic package Mathematica, due to the large output of the symbolic solutions only the graphical results are presented as Figures 1 to 8. Other important fluid flow properties include the shear stress at the walls, i.e.,

$$C_f = \frac{\tau_{xy}}{V_0\mu} = \kappa\frac{du}{dy} - \frac{b}{3}\Big(\frac{du}{dy}\Big)^3$$

and the rate of heat transfer at the walls is obtained as

$$Nu = -\frac{d\theta}{dy}.$$

4. Results and Discussion

Table 1 confirms the rapid convergence of the Adomian series solution. Figure 1 shows the effect of the polymer additive on the fluid flow. As observed from the graph, an increase in the couple stress inverse parameter is seen to improve the fluid flow velocity. It is therefore expected that the couple stress parameter will decrease the flow of the non-Newtonian fluid. Moreover, as observed in Figure 2, an increase in the magnetic field intensity reduces the flow velocity due to the retarding effect of Lorentz force present in the magnetic field place in transverse direction to the fluid flow.

Table 1. Computation showing rapid convergence of the series solution

n	u_n	$\sum_{n=0}^{m} u_n$	$ER_{1,\,n+1}$
0	0.00208333	0.00208333	-2.45624E-2
1	-0.00416667	-0.00208333	-4.16635E-4
2	0.00208333	-6.76635E+20	1.25067E-4
3	1.04199E-21	5.61838E-19	2.37934E-7
4	1.54026E-24	-2.81996E-19	4.26406E-10
5	-2.85658E-28	-2.91379E-19	-5.201E-11

Figure 3 shows the variation of velocity with viscosity in the case of the first approximation of the Eyring–Powell parameter. As observed the first approximation contributes to the fluid viscosity, this lead to the fluid thickening. Moreover, Figure 4 shows the effect of the suction/injection Reynolds number on the velocity profile. It is observed that an increase in the suction Reynolds number enhances the flow velocity. This is physically true since reduction in fluid viscosity means rise in the flow Reynolds number. This translates to an increase in the flow velocity provided the critical Reynolds value is not exceeded.

Figures 5 to 9, shows the temperature profile for different flow parameters. As observed the flow is skewed towards the heated plate with suction due to the

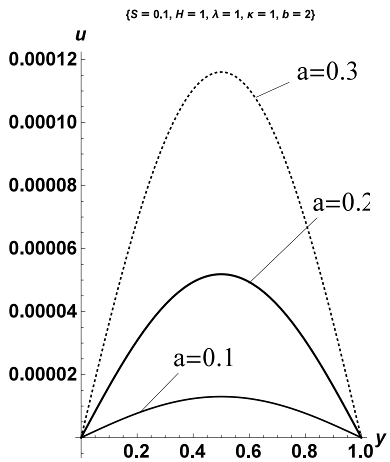

Figure 1. Effect of couple stress inverse parameter on velocity profile

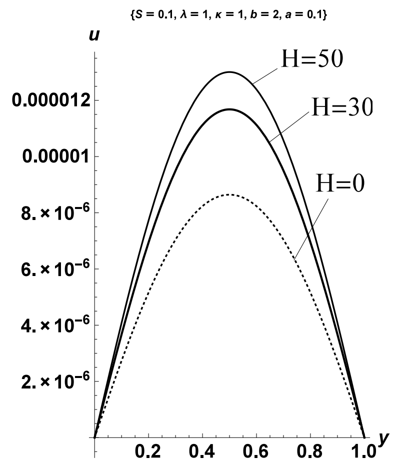

Figure 2. Effect of Hartmann number on velocity profile

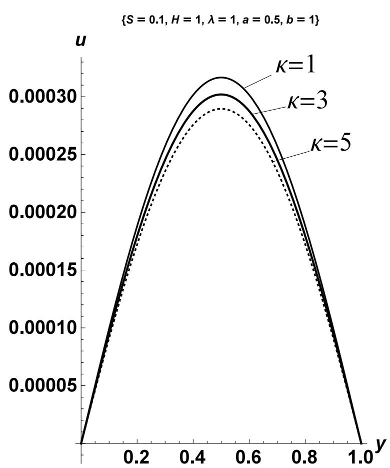

Figure 3. Effect of Viscosity parameter on velocity profile

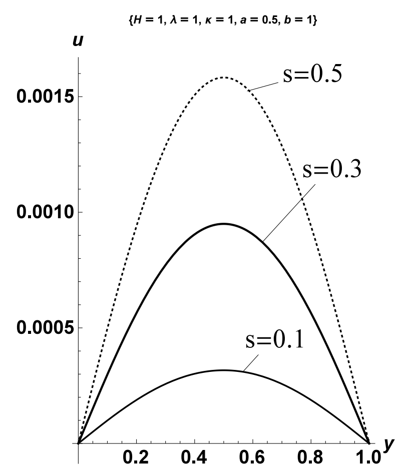

Figure 4. Effect of suction /injection Reynolds' parameter on velocity profile

influence of the porosity of the walls. Figure 5 shows the variation in the fluid temperature distribution with variations in the magnetic field intensity. Interestingly an increase in the Hartmann's number decreases the fluid temperature. This is due to the micro-rotation of the couple stress fluid particles. Similarly, Figure 6 depicts the temperature profile for variations in the flow suction/injection Reynolds number. As observed an increase in the suction/injection Reynolds number lead

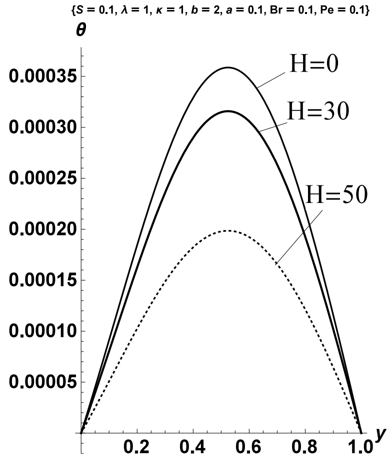

Figure 5. Effect of Hartmann number on temperature distribution

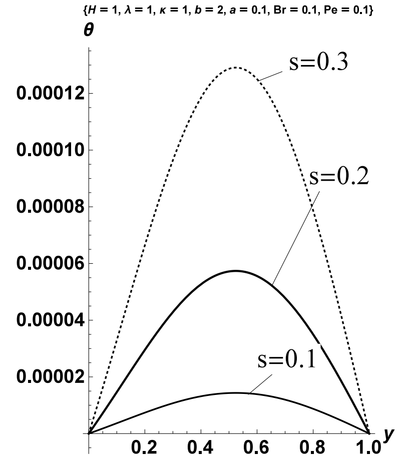

Figure 6. Effect of suction /injection Reynolds' parameter on temperature distribution

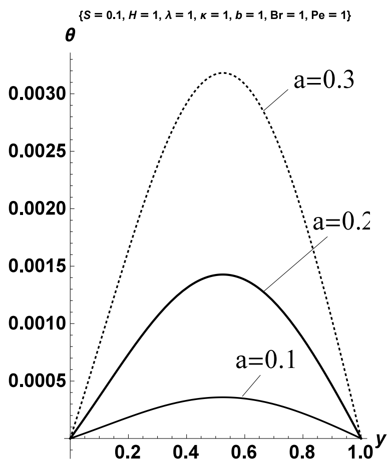

Figure 7. Effect of couple stress inverse parameter on temperature distribution

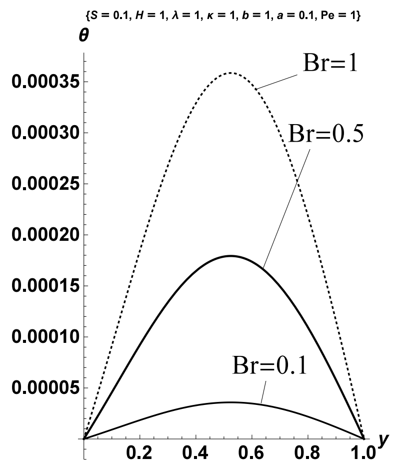

Figure 8. Effect of Brinkman number on temperature distribution

to an increase in the temperature distribution within the channel. In Figure 7, the effect of couple stresses is seen to decrease the fluid temperature within the channel. However, due to the presence of couple stresses an increase in the Peclet number is observed to enhance the fluid temperature as observed in Figure 8. The same phenomenon is experience in Figure 9 in which an increase in the Brinkman number increases the temperature distribution within the channel.

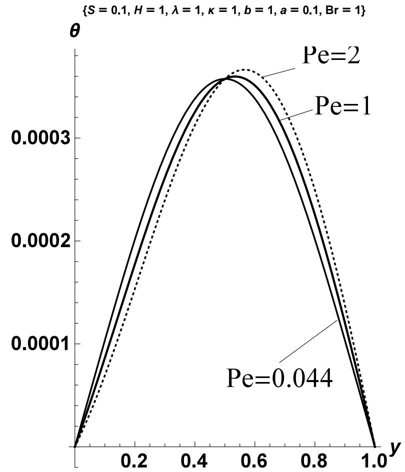

FIGURE 9. Effect of Peclet number on temperature distribution

5. Conclusion

In this paper, the nonlinear momentum and energy equations that model the hydromagnetic flow of the non-Newtonian fluid under the influence of couple stresses and Joule dissipation are investigated. The dimensionless nonlinear governing equations are solved using ADM. In particular, accurate solutions of the velocity and temperature profiles are obtained and shown to be convergent. Again, the strength and the ability of the semi-analytical method has been should read handling of nonlinear coupled-differential equations.

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ЕФЕКАТ ВЕЗАНИХ НАПОНА НА ХИДРОМАГНЕТНИ ЕЈРИНГ–ПАУЕЛОВ ТОК ФЛУИДА КРОЗ ПОРОЗАН КАНАЛ

Резиме. У овом раду истраује се ток хидромагнетног не-Њутновог флуида услед везаних напона кроз порозни канал кориењем Ејринг-Пауеловог модела. Флуид се креe са аксијалним константним градијентом притиска. Приближна решења бездимензионих нелинеарних једначина тока флуида су добијена коришћењем нове модификације Адомиан методе декомпозиције (АДМ). Изведени и дискутовани су ефекти варијације различитих параметара тока флуида, укључујући трење површине канала и флуида и брзину преноса топлоте, на брзину и температурно поље флуида.

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