JENKINS MODEL BASED FERROFLUID LUBRICATION OF A CURVED ROUGH ANNULAR SQUEEZE FILM: EFFECT OF SLIP VELOCITY

Jimit R. Patel and G. M. Deheri

ABSTRACT. This paper analyzes the combined effect of slip velocity and transverse roughness on the performance of a Jenkins model based ferrofluid lubrication of a squeeze film in curved rough annular plates. The slip model of Beavers and Joseph has been invoked to evaluate the effect of slip velocity. In order to find the effect of surface roughness the stochastic averaging model of Christensen and Tonder has been used. The pressure distribution is obtained by solving the concerned stochastically averaged Reynolds type equation. The load carrying capacity is calculated. The graphical representations of the results indicate that the effect of transverse surface roughness is adverse in general, however, the situation is relatively better in the case of negatively skewed roughness. Further, Jenkins model based ferrofluid lubrication offers some measures in reducing the adverse effect of roughness when slip parameter is kept at reduced level with a suitable ratio of curvature parameters. Lastly, the positive effect of magnetization gets a boost due to the combined effect of variance (-ve) and negatively skewed roughness suitably choosing the aspect ratio.

1. Introduction

The study of a squeeze film between annular plates is a traditional one. Vibrations in jet engines can be captivated using annular squeeze films between engine's bearings and their support. Annular plates are also used in clutch plates. The turbine, brake disc and diaphragm clutch spring are the well-known examples for the industrial application of the annular plate with radial cracks. Due to thermal, elastic and uneven wear effects the plates encountered in practice are not actually flat.

Ferrofluids are stable suspensions of colloidal ferromagnetic particles of the order of 10 nm in suitable non-magnetic carrier liquids. These colloidal particles are coated with surfactants to avoid their agglomeration. Because of the industrial applications of ferrofluids, the investigation on them fascinated the researchers

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and engineers since last five decades. One of the many attractive features of the ferrofluids is the prospect of influencing flow by a magnetic field and vice versa. Owing to the application of the magnetic fluid, quite a good number of authors have dealt with magnetic fluids in different geometry of bearings [1-11]. These investigations have established that the performance of the bearing system gets enhanced due to magnetization.

Less attention has been paid to study the effect of velocity slip at the surface, although, it may be of importance in the flow behaviour of liquids, particularly, when the film is thin, the surface is smooth at the porous boundary. For the effective performance of a bearing system, reduction of friction is relatively essential. It is observed that slip velocity supports in reducing the friction. Flow with slip becomes useful for problems in chemical engineering, for example, flows through pipes in which chemical reactions occur at the walls. [12] investigated the interface between a porous medium and fluid layer in an experimental study and proposed a slip boundary condition at the interface. Many researches concerning with slip velocity have been presented: for instance the circular disks by [13], the slider bearing by [8, 14–16], the radial sleeve bearing by [17] and infinitely long bearing by [18]. In all of above studies, it was found that effect of slip was significant for the bearing performance.

Surface roughness of machined mechanical parts and components has significant influence on various performance characteristics of products such as wear resistance, corrosion resistance and fatigue strength. In all the above studies, smooth bearing surfaces were assumed. But it is not true because, the bearing surfaces develop roughness after having some run-in and wear. Many methods have been developed to deal with the effect of surface roughness on the performance characteristics of squeeze film bearings. [19–21] modified the stochastic theory of [22] to study the effect of surface roughness in general. On the ground of the stochastic model of [19–21], many investigations have been conducted; [23–35]. [36] analyzed the effects of various porous structures on the performance of a Shliomis model based ferrofluid lubrication of a squeeze film in rotating rough porous curved circular plates. It was manifest that the adverse effect of transverse roughness could be overcome by the positive effect of ferrofluid lubrication in the case of negatively skewed roughness by suitably choosing curvature parameters and rotational inertia when Kozeny–Carman's model was deployed for porous structure. [37] theoretically investigated the effect of Shliomis model based ferrofluid lubrication on the squeeze film between curved rough annular plates with comparison between two different porous structures. It was noticed that the effect of morphology parameter and volume concentration parameter increased the load carrying capacity. Recently, [38] studied the effect of slip velocity and surface roughness on the performance of Jenkins model based magnetic squeeze film in curved rough circular plates. It was found that for enhancing the performance characteristics of the bearing system the slip parameter was required to be reduced even if variance (-ve) occurs and suitable magnetic strength was in force.

In the present study, the effect of Jenkins model based ferrofluid lubrication of a curved rough annular squeeze film considering slip velocity, has been discussed.

2. Analysis

Figure 1 represents the bearing configuration [39] of present study, which consists of two annular plates each of inside radius b and outside radius a, the upper plate and lower plate are curved. Here r denotes the radial coordinates and h_0 is the central film thickness.



FIGURE 1. Configuration of the bearing system

The well known model of magnetic fluid flow due to Neuringer–Rosensweig consists of the following equations [39],

(2.1)
$$\rho(\vec{q}\cdot\nabla)\vec{q} = -\nabla p + \eta\nabla^2\vec{q} + \mu_0(\vec{M}\cdot\nabla)\vec{H}$$

and

(2.2)
$$\nabla \cdot \vec{q} = 0, \nabla \times \vec{H} = 0, \vec{M} = \bar{\mu}\vec{H}, \nabla \cdot (\vec{H} + \vec{M}) = 0$$

In order to improve this model, Jenkins laid a simple model to express the flow of a magnetic fluid in 1972. Using Maugin's modification, the governing equations of the model for steady flow become [4, 40]

(2.3)
$$\rho(\vec{q}\cdot\nabla)\vec{q} = -\nabla p + \eta\nabla^2\vec{q} + \mu_0(\vec{M}\nabla)\vec{H} + \frac{\rho A^2}{2}\nabla\times\left[\frac{\vec{M}}{M}\times\{(\nabla\times\vec{q})\times\vec{M}\}\right]$$

together with (2.1) and (2.2) where ρ is the fluid density, \vec{q} denotes the fluid velocity in the film region, \vec{H} represents external magnetic field, $\bar{\mu}$ represents magnetic susceptibility of the magnetic fluid, p is the film pressure, η denotes the fluid viscosity, μ_0 represents the permeability of the free space, A being a material constant and \vec{M} represents magnetization vector. (2.1) and (2.3) suggest that Jenkins model is a generalization of Neuringer–Rosensweig model with an additional term

$$\frac{\rho A^2}{2} \nabla \times \left[\frac{\vec{M}}{M} \times \left\{ (\nabla \times \vec{q}) \times \vec{M} \right\} \right] = \frac{\rho A^2 \bar{\mu}}{2} \nabla \times \left[\frac{\vec{H}}{H} \times \left\{ (\nabla \times \vec{q}) \times \vec{H} \right\} \right]$$

which affects the velocity of the fluid. At this point one needs to remember that Neuringer–Rosensweig model modifies the pressure while Jenkins model modifies both the pressure and velocity of the ferrofluid. Let (u, v, w) be the velocity of the fluid at any point (r, θ, z) between two solid surfaces, with OZ as axis. Making use of the assumptions of hydrodynamic lubrication and remembering that the flow is steady and axially symmetric, the equations of motion are

(2.4)
$$\left(1 - \frac{\rho A^2 \bar{\mu} H}{2\eta}\right) \frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta} \frac{d}{dr} \left(p - \frac{\mu_0 \bar{\mu}}{2} H^2\right)$$

(2.5)
$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0$$

Solving the above (2.4) using the boundary conditions, u = 0 when z = 0, h, one gets

$$u = \frac{z(z-h)}{2\eta \left(1 - \frac{\rho A^2 \bar{\mu} H}{2\eta}\right)} \frac{d}{dr} \left(p - \frac{\mu_0 \bar{\mu}}{2} H^2\right)$$

Substituting the value of u in (2.5) and integrating it with respect to z over the interval (0, h) one obtains Reynolds type equation for film pressure as

$$\frac{1}{r}\frac{d}{dr}\left(\frac{h^3}{\left(1-\frac{\rho A^2\bar{\mu}H}{2\eta}\right)}r\frac{d}{dr}\left(p-\frac{\mu_0\bar{\mu}}{2}H^2\right)\right) = 12\eta\dot{h_0}$$

The bearing surfaces are considered transversely rough. In view of the stochastic theory of [19-21], the thickness h of the lubricant film is considered as

$$h = \bar{h} + h_s$$

where \bar{h} denotes the mean film thickness and h_s represents the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. h_s is governed by the probability density function

$$f(h_s) = \begin{cases} \frac{35}{32c^7} (c^2 - h_s^2)^3 & \text{for} & -c \leqslant h_s \leqslant c\\ 0 & \text{for} & elsewhere. \end{cases}$$

wherein c represents the maximum deviation from the mean film thickness. The mean α , the standard deviation σ and the parameter ϵ which is the measure of symmetry of the random variable h_s , are adopted according to the theory of [19–21].

It is considered that the upper plate lying along the surface determined by [39,41,42]

$$z_u = h_0 \exp(-\beta r^2); b \leqslant r \leqslant a$$

approaches with normal velocity $\dot{h_0}$ to the lower plate lying along the surface governed by

$$z_l = h_0[\sec(\gamma r^2) - 1]; b \leqslant r \leqslant a$$

where β and γ represent the curvature parameter of the respective plates. The film thickness h(r) then, is defined by [**39**, **41**, **43**]

$$h(r) = h_0 \left[\exp(-\beta r^2) - \sec(\gamma r^2) + 1 \right]; b \leqslant r \leqslant a$$

Modification of the method of [19–21], on certain simplifications yields, under usual assumptions of hydro-magnetic lubrication [28, 39, 44] the Reynolds type equation;

(2.6)
$$\frac{1}{r}\frac{d}{dr}\left(\frac{g(h)}{\left(1-\frac{\rho A^2\bar{\mu}H}{2\eta}\right)}r\frac{d}{dr}\left(p-\frac{\mu_0\bar{\mu}}{2}H^2\right)\right) = 12\eta\dot{h_0}$$

where

$$H^2 = K(r-b)(a-r)$$

and

$$g(h) = (h^{3} + 3h^{2}\alpha + 3(\sigma^{2} + \alpha^{2})h + 3\sigma^{2}\alpha + \alpha^{3} + \epsilon)\left(\frac{4 + sh}{2 + sh}\right)$$

Introducing the following dimensionless quantities

$$\bar{h} = \frac{h}{h_0} = \left[\exp(-BR^2) - \sec(CR^2) + 1\right], R = \frac{r}{b}, P = -\frac{h_0^3 p}{\eta b^2 \dot{h_0}}, B = \beta b^2, C = \gamma b^2, C = \gamma$$

(2.7)
$$\mu^* = -\frac{K\mu_0\bar{\mu}h_0^3}{\eta b^2\dot{h}_0}, k = \frac{a}{b}, \bar{A}^2 = \frac{\rho A^2\bar{\mu}b\sqrt{K}}{2\eta}, \bar{\sigma} = \frac{\sigma}{h_0}, \bar{\alpha} = \frac{\alpha}{h_0}, \bar{\epsilon} = \frac{\epsilon}{h_0^3}, \bar{s} = sh_0$$

and making use of the (2.7), (2.6) paves the way for

(2.8)
$$\frac{1}{R}\frac{d}{dR}\left(\frac{g(\bar{h})}{(1-\bar{A}^2\sqrt{(R-1)(k-R)})}R\frac{d}{dR}\left(p-\frac{1}{2}\mu^*(R-1)(k-R)\right)\right) = -12$$

where

$$g(\bar{h}) = (\bar{h}^3 + 3\bar{h}^2\bar{\alpha} + 3(\bar{\sigma}^2 + \bar{\alpha}^2)\bar{h} + 3\bar{\sigma}^2\bar{\alpha} + \bar{\alpha}^3 + \bar{\epsilon})\left(\frac{4+\bar{s}h}{2+\bar{s}\bar{h}}\right)$$

Solving (2.8) under the boundary conditions

$$P(1) = P(k) = 0$$

one gets the expression for the dimensionless pressure distribution as

$$\begin{split} P = & \frac{\mu^*}{2} (R-1)(k-R) - 6 \int_1^R \frac{R}{g(\bar{h})} \Big(1 - \bar{A}^2 \sqrt{(R-1)(k-R)} \Big) dR \\ &+ 6 \frac{\int_1^k \frac{R}{g(\bar{h})} \Big(1 - \bar{A}^2 \sqrt{(R-1)(k-R)} \Big) dR}{\int_1^k \frac{1}{Rg(\bar{h})} \Big(1 - \bar{A}^2 \sqrt{(R-1)(k-R)} \Big) dR} \\ &\int_1^R \frac{1}{Rg(\bar{h})} \Big(1 - \bar{A}^2 \sqrt{(R-1)(k-R)} \Big) dR \end{split}$$

The non dimensional load carrying capacity of the bearing system then, is obtained as

(2.9)
$$W = -\frac{h_0^3 w}{2\pi \eta b^4 \dot{h}_0} = \frac{\mu^*}{24} (k^2 - 1)(k - 1)^2 + 3\int_1^k \frac{R^3}{g(\bar{h})} \left(1 - \bar{A}^2 \sqrt{(R - 1)(k - R)}\right) dR - 3\frac{\left[\int_1^k \frac{R}{g(\bar{h})} \left(1 - \bar{A}^2 \sqrt{(R - 1)(k - R)}\right) dR\right]^2}{\int_1^k \frac{1}{Rg(\bar{h})} \left(1 - \bar{A}^2 \sqrt{(R - 1)(k - R)}\right) dR}$$

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3. Result and Discussions

The expression for the load carrying capacity is linear with respect to the magnetization parameter which makes it sure that the load carrying capacity will be increased for increasing values of magnetization parameter. Probably, this may be due to the fact that the magnetization induces an increase in the viscosity of the lubricant. Further, it is noticed from (2.9) that the load carrying capacity enhances by $(\mu^*/24)(k^2-1)(k-1)^2$ as compared to the case of conventional lubricant based bearing system.



FIGURE 2. Variation of Load carrying capacity with respect to \bar{A} and B.



FIGURE 3. Variation of Load carrying capacity with respect to \bar{A} and C.

The effect of material constant parameter on the load carrying capacity is provided in Figures 2 to 8. It is seen that the load carrying capacity decreases with increasing values of the material constant parameter. This is in conformity with the results obtained experimentally [39, 44].



FIGURE 4. Variation of Load carrying capacity with respect to \bar{A} and k



FIGURE 5. Variation of Load carrying capacity with respect to \bar{A} and $\bar{\sigma}$



FIGURE 6. Variation of Load carrying capacity with respect to \bar{A} and $\bar{\epsilon}$.



FIGURE 7. Variation of Load carrying capacity with respect to \bar{A} and $\bar{\alpha}$.



FIGURE 8. Variation of Load carrying capacity with respect to \bar{A} and $1/\bar{s}$.



FIGURE 9. Variation of Load carrying capacity with respect to ${\cal B}$ and C.



FIGURE 10. Variation of Load carrying capacity with respect to B and k.



FIGURE 11. Variation of Load carrying capacity with respect to B and $\bar{\sigma}$.



FIGURE 12. Variation of Load carrying capacity with respect to B and $\bar{\epsilon}.$



FIGURE 13. Variation of Load carrying capacity with respect to B and $\bar{\alpha}.$



FIGURE 14. Variation of Load carrying capacity with respect to B and $1/\bar{s}.$



FIGURE 15. Variation of Load carrying capacity with respect to $C \mbox{ and } k.$



FIGURE 16. Variation of Load carrying capacity with respect to C and $\bar{\sigma}$.



FIGURE 17. Variation of Load carrying capacity with respect to C and $\bar{\epsilon}$.



FIGURE 18. Variation of Load carrying capacity with respect to C and $\bar{\alpha}.$



FIGURE 19. Variation of Load carrying capacity with respect to C and $1/\bar{s}$.



FIGURE 20. Variation of Load carrying capacity with respect to k and $\bar{\sigma}$.

Further, the effect of standard deviation on the distribution of load carrying capacity with respect to the material constant is almost negligible.

The fact that the upper plate's curvature parameter increases the load can be seen from Figures 9 to 14. Further, the effect of aspect ratio on the distribution of load carrying capacity with respect to upper plate's curvature parameter is almost negligible at the initial stage.

The trends of the load carrying capacity with respect to lower plate's curvature parameter are almost opposite to that of upper plate's curvature parameter (Figures 15 to 19).

In this type of bearing system the aspect ratio plays a central role in enhancing the performance of the bearing system (Figures 20 to 23). It is seen that the trends of the combined effect of aspect ratio and standard deviation are identical with those of aspect ratio and skewness.



FIGURE 21. Variation of Load carrying capacity with respect to k and $\bar{\epsilon}.$



FIGURE 22. Variation of Load carrying capacity with respect to k and $\bar{\alpha}.$



FIGURE 23. Variation of Load carrying capacity with respect to k and $1/\bar{s}.$



FIGURE 24. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\epsilon}$.



FIGURE 25. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\alpha}.$



FIGURE 26. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $1/\bar{s}$.



FIGURE 27. Variation of Load carrying capacity with respect to $\bar{\epsilon}$ and $\bar{\alpha}.$



FIGURE 28. Variation of Load carrying capacity with respect to $\bar{\epsilon}$ and $1/\bar{s}.$



FIGURE 29. Variation of Load carrying capacity with respect to $\bar{\alpha}$ and $1/\bar{s}.$

Figures 24, 25 and 26 establish that the standard deviation causes a significant load reduction. This is because the motion of the lubricant gets retarded by the composite roughness of the surfaces. Further, Figure 26 underlines that the combined effect of slip and standard deviation is relatively adverse.

The effect of skewness presented in Figures 27 and 28 makes it clear that the load carrying capacity decreases with positive skewness while the negative skewed roughness induces an increase in the load carrying capacity. The trends of load carrying capacity with respect to variance are almost similar to that of skewness (Figure 29).

The graphical representations suggest the following:

- The adverse effect of roughness can be minimized by the positive effect of magnetization choosing a suitable ratio of curvature parameters, when the slip is at reduced level.
- With a suitable choice of aspect ratio, the combined effect of variance (-ve) and negatively skewed roughness goes a long way in reducing the adverse effect of material constant and slip parameter.
- For an effective performance the slip requires to be kept at minimum even if suitable magnetic strength is in place.

Conclusion

- This investigation reveals that the Jenkins model goes ahead of the Neuringer–Rosensweig model for augmenting the performance characteristics of the bearing system.
- This article asserts that the roughness aspect must be addressed judiciously when designing the bearing system.
- This type of bearing system supports a good amount of load even in the absence of flow in spite of the fact that there are several parameters bringing down the load carrying capacity.

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JENKINS-ОВ МОДЕЛ ПОДМАЗИВАЧА НА БАЗИ ФЕРОФЛУИДА ЗАКРИВЉЕНОГ ХРАПАВОГ ПРСТЕНАСТОГ СТИСНУТОГ ФИЛМА: УТИЦАЈ БРЗИНЕ ПРОКЛИЗАВАЊА

РЕЗИМЕ. У раду се анализира комбиновани ефекат брзине проклизавања и трансверзалне храпавости на особине *Jenkins*-овог модела подмазивача на бази ферофлуида стиснутог филма код храпавих закривљених прстенастих плоча. Искоришћен је модел проклизавања *Beavers*-а и *Joseph*-а да би се одредили ефекти брзине проклизавања. Да би се добио ефекат површинске храпавости, користи се стохастички модел *Christensen*-а и *Tonder*-а. Расподела притиска је добијена решавањем усредњене стохастичке једначине Рејнолдсовог типа. Графичко представљање резултата упућује да су ефекти трансверзалне храпавости површине у општем случју негативни. Даље, *Jenkins*-ов модел нуди одредђене мере за смањивање негативних ефекта храпавости, када је параметар проклизавања одржан на смањеном нивоу са одређеним размером параметара кривина.

Department of Mathematics Sardar Patel University Vallabh Vidyanagar, Anand, Gujarat India patel.jimitphdmarch2013@gmail.com

Department of Mathematics Sardar Patel University Vallabh Vidyanagar, Anand, Gujarat India gm.deheri@rediffmail.com (Received 30.12.2014) (Revised 02.07.2015)