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ENTROPY GENERATION IN POISEUILLE FLOW THROUGH A CHANNEL PARTIALLY FILLED WITH A POROUS MATERIAL

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Abstract. In the present paper, a theoretical analysis of entropy generation due to fully developed flow and heat transfer through a parallel plate channel partially filled with a porous medium under the effect of transverse magnetic field and radiation is presented. Both horizontal plates of the channel are kept at constant and equal temperature. An exact solution of governing equation for both porous and clear fluid regions has been obtained in closed form. The entropy generation number and the Bejan number are also calculated. The effects of various parameters such as magnetic field parameter, radiation parameter, Brinkman number, permeability parameter, ratios of viscosities and thermal conductivities are examined on velocity, temperature, entropy generation rate.

1. Introduction

Entropy generation minimization studies are vital for ensuring optimal thermal systems in contemporary industrial and technological fields like geothermal systems, electric cooling, heat exchangers, MHD power generators and energy storage systems etc. Analysis of thermodynamic irreversibility in a channel partially filled with a porous medium and partially with a clear fluid, in the presence of transverse magnetic field and thermal radiation appears to be increasingly important due to its various applications in engineering and industries such as in the field of solar energy collection, enhanced oil recovery, nuclear reactor cooling, electronic packages, thermal insulation and petroleum reservoirs. Moreover, many engineering processes occur at high temperature and acknowledge radiation heat transfer become very important for the design of pertinent equipment (Sparrow and Cess [**1**], Raptis et al. [**2**], Singh [**3**]). Several researchers such as Chauhan and Rastogi [**4**], Vyas and Srivastava [**5**] and Manglesh and Gorla [**6**] have discussed the MHD flow and

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heat transfer through porous medium in the presence of radiation under various configurations.

The foregoing discussions that deals with the radiative MHD flow in porous channels are very much restricted to the aspect of the first law of thermodynamics, and none of them carried out a second law based analysis to discuss the nature of the irreversibility in terms of entropy generation. The concept of entropy generation due to viscous fluid flow and heat transfer is introduced by Bejan [**7**–**9**] to analyze the efficiency of various system designs. Entropy generation is associated with thermodynamic irreversibility, which is common in all types of heat transfer processes. Moreover, in thermodynamic analysis of flow and heat transfer processes, one thing of core interest is to improve the thermal systems to avoid the energy losses and fully utilize the energy resources. Since entropy generation is the measure of the destruction of the available work of the system, the determination of the factors responsible for the entropy generation is also important in upgrading the system performances. Efficient utilization of energy is the primary objective in the design of any thermodynamic system. This can be achieved by minimizing entropy generation in processes. Many investigators examined the effects of entropy generation in ducts and channels of different configurations filled with a porous material, such as Al-Odat et al. [**10**], Damesh et al. [**11**], Makinde [**12**, **13**], Chauhan and Kumar [**14**], Das and Jana [**15**, **16**], Vyas and Rai [**17**], Eegunjobi and Makinde [**18**], Rajvanshi et al. [**19**]. Entropy generation in channels partially filled with porous materials is not investigated much. Morosuk [**20**], Komurgoz et al. [**21**], Chauhan and Kumar [**22**,**23**] studied entropy generation for incompressible and compressible fluid flow in conduits and channel partially filled with a porous medium.

In the present study, the entropy generation in MHD Poiseuille flow in a horizontal channel partially filled with a porous medium and partially with a clear fluid under the effect of radiation is considered. Both the channel plates are stationary and maintained at constant and equal temperatures. It has been reported by Levy [**24**] that the fluid flow through porous media is governed by the Brinkman equation for smaller particles while the Darcy model is valid for coarse particles. However there is some uncertainty about validity of the Forchheimer model for fine particles, and therefore the Darcy-Brinkman model has been taken for moderate flow in this study. Closed form solution has been obtained for the fluid velocities, temperatures and entropy generation rates in both the clear fluid region and porous region. The variations of the velocity and temperature fields, entropy generation rate, and Bejan number are investigated for various values of the magnetic field parameter, Brinkman number, Radiation parameter, permeability parameter, viscosity and conductivity ratio parameters.

2. Mathematical formulation and solution

We consider fully developed, steady, laminar Poiseuille flow of an electrically conducting, incompressible fluid in channel bounded below by a fluid saturated porous medium under the effect of a transverse magnetic field B_0 applied normal to the flow direction. Both the plates are impermeable and maintained at a uniform temperature T_w. The x-axis is taken in the flow direction and the y-axis is taken normal to the porous layer interface. Let u_1, v_1, w_1 and u_2, v_2, w_2 be the velocity components and t_1, t_2 be the temperatures for the clear-fluid region and porous region respectively. The channel is infinitely long, and the Navier–Stokes equations govern the clear-fluid region flow while the Brinkman model has been used to model the flow through the porous medium.

The governing momentum and energy equations for steady state fully developed Poiseuille flow are given as follows:

For clear-fluid region-I ($h \leqslant y \leqslant d$)

$$\mu \frac{d^2u_1}{dy^2} - \frac{dp}{dx} - \sigma B_0^2 u_1 = 0,$$
$$k\frac{d^2t_1}{dy^2} + \mu\Big(\frac{du_1}{dy}\Big)^2 + \sigma B_0^2 u_1^2 - \frac{\partial q_{r_1}}{\partial y} = 0.$$

For porous region-II ($0 \leqslant y \leqslant h$)

$$\bar{\mu}\frac{d^2u_2}{dy^2} - \frac{dp}{dx} - \frac{\mu}{K_0}u_2 - \sigma B_0^2 u_2 = 0,$$
$$\bar{k}\frac{d^2t_2}{dy^2} + \frac{\mu u_2^2}{K_0} + \bar{\mu}\Big(\frac{du_2}{dy}\Big)^2 + \sigma B_0^2 u_2^2 - \frac{\partial q_{r_2}}{\partial y} = 0.$$

The corresponding boundary and matching conditions are:

$$\begin{aligned}
&\text{at } y = 0;\ u_2 = 0,\ t_2 = T_w,\\
&\text{at } y = h;\ u_1 = u_2,\ t_1 = t_2,\ \bar{\mu}\frac{du_2}{dy} = \mu\frac{du_1}{dy},\ \bar{k}\frac{dt_2}{dy} = k\frac{dt_1}{dy},\\
&\text{at } y = d;\ u_1 = 0,\ t_1 = T_w.
\end{aligned}$$

Here $\bar{\mu}$ is the effective viscosity; μ the fluid viscosity; K_0 the permeability; $\bar{k}$ the effective thermal conductivity; k the thermal conductivity; σ the electric conductivity; B_0 the magnetic field intensity; q_r the radiation flux; h the width of the porous medium; d the width of the channel; and $\frac{dp}{dx}$ the applied pressure gradient.

It is assumed that the medium is optically thin and with relatively low density. Following Cogley et al. [**25**] equilibrium model, we therefore take expression of the radiative heat flux as follows:

$$\frac{\partial q_r}{\partial y} = 4(t - T_w)\int_0^\infty K_{\lambda\omega}\Big(\frac{\partial e_{b\lambda}}{\partial T}\Big)_\omega d\lambda = 4I^*(t - T_w),$$

where $K_{\lambda\omega}$ is the absorption coefficient at the plate and $e_{b\lambda}$ is the plank constant.

We introduce the following dimensionless quantities:

$$\begin{aligned}
&u_1^* = \frac{u_1}{u_0},\ u_2^* = \frac{u_2}{u_0},\ y^* = \frac{y}{d},\ a = \frac{h}{d},\ \theta_1 = \frac{t_1 - T_w}{T_w},\ \theta_2 = \frac{t_2 - T_w}{T_w},\\
&\phi_1 = \frac{\bar{\mu}}{\mu},\ \phi_2 = \frac{\bar{k}}{k},\ K = \frac{K_0}{d^2},\ M^2 = \frac{\sigma B_0^2 d^2}{\mu},\ \mathrm{Br} = \frac{\mu u_0^2}{kT_w},\ F = \frac{4I^* d^2}{k},
\end{aligned}$$

where $u_0 = -\frac{d^2}{\mu}\frac{dp}{dx}$ is the reference velocity; ϕ_1 the viscosity ratio; ϕ_2 the conductivity ratio; K the permeability parameter; M^2 the magnetic field parameter; Br the Brinkman number and F is the radiation parameter.

Using non-dimensional quantities, the governing dimensionless equations of the present problem, for flow and temperature distribution after dropping asterisks for convenience, are given by:

For clear-fluid region-I ($a \leqslant y \leqslant 1$)

$$\frac{d^2u_1}{dy^2} - M^2u_1 + 1 = 0, \tag{2.1}$$

$$\frac{d^2\theta_1}{dy^2} - F\theta_1 + \mathrm{Br}\left(\frac{du_1}{dy}\right)^2 + M^2\,\mathrm{Br}\,u_1^2 = 0. \tag{2.2}$$

For porous region-II ($0 \leqslant y \leqslant a$)

$$\frac{d^2u_2}{dy^2} - \frac{1}{\phi_1}\Big(M^2 + \frac{1}{K}\Big)u_2 + \frac{1}{\phi_1} = 0, \tag{2.3}$$

$$\frac{d^2\theta_2}{dy^2} - \frac{F}{\phi_2}\theta_2 + \frac{\phi_1}{\phi_2}\,\mathrm{Br}\left(\frac{du_2}{dy}\right)^2 + \frac{\mathrm{Br}}{\phi_2}\Big(M^2 + \frac{1}{K}\Big)u_2^2 = 0. \tag{2.4}$$

The corresponding dimensionless boundary and matching conditions of the present problem are given by:

$$\begin{aligned} &\text{at } y = 0;\ u_2 = 0,\ \theta_2 = 0,\\ &\text{at } y = a;\ u_1 = u_2,\ \theta_1 = \theta_2,\ \phi_1\frac{du_2}{dy} = \frac{du_1}{dy},\ \phi_2\frac{d\theta_2}{dy} = \frac{d\theta_1}{dy},\\ &\text{at } y = 1;\ u_1 = 0,\ \theta_1 = 0. \end{aligned} \tag{2.5}$$

Equations (2.1)–(2.4) are solved under the corresponding boundary/matching conditions (2.5), and we obtain the expressions for velocity and temperature profiles as follows:

$$u_1 = c_1\cos hMy + c_2\sin hMy + \frac{1}{M^2},$$

$$u_2 = c_3\cos h(M_1y) + c_4\sin h(M_1y) + \frac{1}{\phi_1M_1^2},$$

$$\begin{aligned}\theta_1 = {}& c_5\cos h\big(\sqrt{F}y\big) + c_6\sin h\big(\sqrt{F}y\big) + a_1\cos h2My\\ &+ a_2\sin h2My + a_3\cos hMy + a_4\sin hMy + a_5,\end{aligned} \tag{2.6}$$

$$\begin{aligned}\theta_2 = {}& c_7\cos h\left(\sqrt{\frac{F}{\phi_2}}y\right) + c_8\sin h\left(\sqrt{\frac{F}{\phi_2}}y\right) + b_1\cos h(2M_1y)\\ &+ b_2\sin h(2M_1y) + b_3\cos h(M_1y) + b_4\sin h(M_1y) + b_5,\end{aligned} \tag{2.7}$$

where,

$$M_1^2 = \frac{1}{\phi_1}\Big(M^2 + \frac{1}{K}\Big),\ c_1 = -\frac{(c_2M^2\sin hM + 1)}{M^2\cos hM},$$

$$c_2 = \frac{\frac{1}{M}\sin h(M_1 a)\sin h(Ma)}{\phi_1 M_1 \cos h(M_1 a)\sin hM(1-a) + M\sin h(M_1 a)\cos hM(1-a)} + \frac{\frac{1}{M_1}\big[\cos hM - \cos h(M_1 a)\big\{\cos h(Ma) + \frac{1}{KM^2}(\cos h(Ma) - \cos hM)\big\}\big]}{\phi_1 M_1 \cos h(M_1 a)\sin hM(1-a) + M\sin h(M_1 a)\cos hM(1-a)},$$

$$c_3 = -\frac{1}{\phi_1 M_1^2},$$

$$c_4 = \frac{1}{\sin h(M_1 a)}\Big[c_1\cos h(Ma) + c_2 \sin h(Ma) + \frac{1}{M^2} - \frac{1}{\phi_1 M_1^2}\big\{1-\cos h(M_1 a)\big\}\Big],$$

$$a_1 = -\frac{\mathrm{Br}\,(c_1^2+c_2^2)M^2}{4M^2-F},\ a_2 = -\frac{2\,\mathrm{Br}\,c_1 c_2 M^2}{4M^2-F},\ a_3 = -\frac{2\,\mathrm{Br}\,c_1}{M^2-F},\ a_4 = -\frac{2\,\mathrm{Br}\,c_2}{M^2-F},$$

$$a_5 = \frac{\mathrm{Br}}{FM^2},\ b_1 = -\frac{\mathrm{Br}\,\phi_1(c_3^2+c_4^2)M_1^2}{4\phi_2 M_1^2 - F},\ b_2 = -\frac{2\,\mathrm{Br}\,\phi_1 c_3 c_4 M_1^2}{4\phi_2 M_1^2 - F},$$

$$b_3 = -\frac{2\,\mathrm{Br}\,c_3}{\phi_2 M_1^2 - F},$$

$$b_4 = -\frac{2\,\mathrm{Br}\,c_4}{\phi_2 M_1^2 - F},\ b_5 = \frac{\mathrm{Br}}{\phi_1 F M_1^2},$$

$$c_5 = -c_6 \tan h\sqrt{F} - \frac{1}{\cos h\sqrt{F}}\big[a_1\cos h2M + a_2 \sin h2M + a_3\cos hM + a_4 \sin hM + a_5\big],$$

$$\begin{aligned} c_6 = \Bigg[& \sqrt{F\phi_2}\cos h\sqrt{F}c_7 + \sqrt{F\phi_2}\cos h\sqrt{F}\cos h\sqrt{\frac{F}{\phi_2}}a \\ & \times \begin{Bmatrix} b_1\cos h2M_1a + b_2\sin h2M_1 a + b_3\cos hM_1 a + b_4 \sin hM_1 a + b_5 \\ -a_1\cos h2Ma - a_2\sin h2Ma - a_3\cos hMa - a_4\sin hMa - a_5 \end{Bmatrix} \\ & -\phi_2 M_1 \cos h\sqrt{F}\sin h\sqrt{\frac{F}{\phi_2}}a\big\{2b_1\sin h2M_1 a + 2b_2\cos h2M_1 a \\ & \qquad + b_3 \sin hM_1 a + b_4\cos hM_1 a\big\} \\ & +M\cos h\sqrt{F}\sin h\sqrt{\frac{F}{\phi_2}}a\big\{2a_1\sin h2Ma + 2a_2\cos h2Ma \\ & \qquad + a_3\sin hMa + a_4\cos hMa\big\} \\ & + (a_1\cos h2M + a_2\sin h2M + a_3\cos hM + a_4\sin hM + a_5) \\ & \times\left(\sqrt{F}\phi_2\cos h\sqrt{\frac{F}{\phi_2}}a\cos h\sqrt{F}a - \sqrt{F}\sin h\sqrt{\frac{F}{\phi_2}}a\sin h\sqrt{F}a\right)\Bigg] \Bigg/ \\ & \left[\sqrt{F}\phi_2\cos h\sqrt{\frac{F}{\phi_2}}a\sin h\sqrt{F}(a-1) - \sqrt{F}\sin h\sqrt{\frac{F}{\phi_2}}a\cos h\sqrt{F}(a-1)\right], \end{aligned}$$

$$c_7 = -(b_1+b_3+b_5),$$

$$c_8 = \Bigg[c_5 \cos h\sqrt{F}a + c_6 \sin h\sqrt{F}a + a_1 \cos h2Ma + a_2 \sin h2Ma$$
$$+ a_3 \cos hMa + a_4 \sin hMa + a_5 - c_7 \cos h\sqrt{\frac{F}{\phi_2}}a - b_1 \cos h2M_1a$$
$$-b_2 \sin h2M_1a - b_3 \cos hM_1a - b_4 \sin hM_1a - b_5 \Bigg] \Bigg/ \sin h\sqrt{\frac{F}{\phi_2}}a.$$

The rate of heat transfer at the plates $y = 1$ and $y = 0$ can be obtained from (2.6) and (2.7) respectively as

$$\theta_1'(1) = \sqrt{F}\big(c_5 \sin h\sqrt{F} + c_6 \cos h\sqrt{F}\big)$$
$$+ 2M\big(a_1 \sin h2M + a_2 \cos h2M\big) + M(a_3 \sin hM + a_4 \cos hM),$$
$$\theta_2'(0) = \sqrt{\frac{F}{\phi_2}}c_8 + M_1(2b_2 + b_4).$$

3. Entropy generation

Second law analysis in terms of entropy generation rate is a useful tool for predicting the performance of the engineering processes by investigating the irreversibility arising during the processes. Following Woods [**26**], the local volumetric rate of entropy generation in the presence of a magnetic field for both clear fluid and porous regions can be written as
For clear-fluid region-I ($h \leqslant y \leqslant d$)

$$(S''')_1 = \frac{k}{T_w^2}\Big(\frac{dt_1}{dy}\Big)^2 + \frac{1}{T_w}\Big\{\mu\Big(\frac{du_1}{dy}\Big)^2 + \sigma B_0^2 u_1^2\Big\},$$

For porous region-II ($0 \leqslant y \leqslant h$)

$$(S''')_2 = \frac{\bar{k}}{T_w^2}\Big(\frac{dt_2}{dy}\Big)^2 + \frac{1}{T_w}\Big\{\bar{\mu}\Big(\frac{du_2}{dy}\Big)^2 + \sigma B_0^2 u_2^2\Big\}.$$

The non-dimensional form of the entropy generation number in both regions are given by

$$(\mathrm{Ns})_1 = \frac{(S''')_1}{S_0'''} = (\mathrm{HTI})_1 + (\mathrm{FFI})_1 + (\mathrm{MFI})_1,$$
$$(\mathrm{Ns})_2 = \frac{(S''')_2}{S_0'''} = (\mathrm{HTI})_2 + (\mathrm{FFI})_2 + (\mathrm{MFI})_2,$$

where, $S_0''' = \frac{k}{d^2}$ is the reference volumetric entropy generation; $(\mathrm{HTI})_1 = \big(\frac{d\theta_1}{dy}\big)^2$ the heat transfer irreversibility in clear fluid region; $(\mathrm{FFI})_1 = \mathrm{Br}\,\big(\frac{du_1}{dy}\big)^2$ the fluid friction irreversibility in clear fluid region; $(\mathrm{MFI})_1 = M^2\,\mathrm{Br}\,u_1^2$ the magnetic field irreversibility in clear fluid region; $(\mathrm{HTI})_2 = \phi_2\big(\frac{d\theta_2}{dy}\big)^2$ the heat transfer irreversibility in porous region; $(\mathrm{FFI})_2 = \phi_1\,\mathrm{Br}\,\big(\frac{du_2}{dy}\big)^2$ the fluid friction irreversibility in porous region and $(\mathrm{MFI})_2 = M^2\,\mathrm{Br}\,u_2^2$ is the magnetic field irreversibility in porous region.

The Bejan numbers are given by
For clear fluid region:

$$(\mathrm{Be})_1 = \frac{(\mathrm{HTI})_1}{(\mathrm{Ns})_1}.$$

For porous region:

$$(\mathrm{Be})_2 = \frac{(\mathrm{HTI})_2}{(\mathrm{Ns})_2}.$$

4. Results and discussion

In the present paper, the study of fully developed Poiseuille flow in a horizontal channel partially filled by a porous medium is conducted in the presence of transverse magnetic field and radiation effects. A closed form solution is obtained for the velocity profiles, temperature profiles, and rate of heat transfer while the entropy generation due to heat transfer, fluid friction and magnetic field effect is formulated. The effects of various pertinent parameters on the fluid velocity, temperature, rate of heat transfer, entropy generation rate, and Bejan number distributions are reported in Figures 1 to 13. Figure 1 illustrates the effects of different

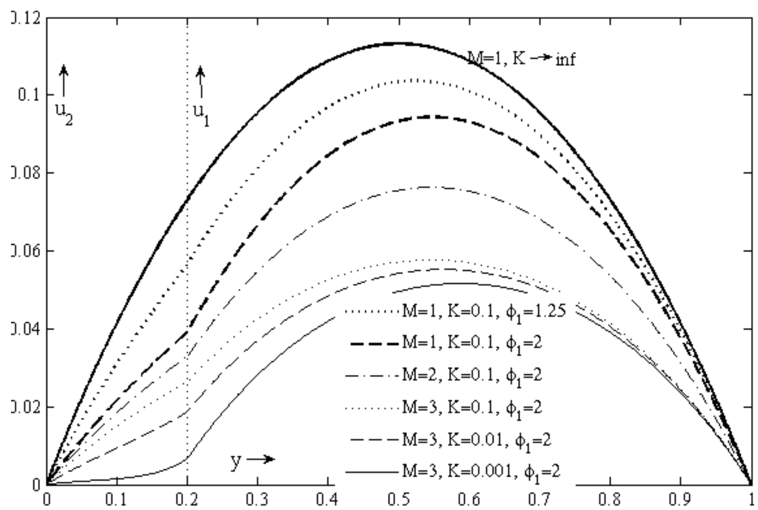

FIGURE 1. Velocity distribution for $a = 0.2$

parameters, e.g., the viscosity ratio parameter (ϕ_1), permeability parameter (K), and magnetic field parameter (M) on the velocity profiles in the composite channel for the dimensionless thickness of the porous medium $a = 0.2$. It is observed that the velocity in the channel decreases with the increase in the value of the magnetic field parameter (M). This is because the presence of Lorentz force acting as a resistance to the flow. Further velocity in the channel decreases by increasing the viscosity ratio (ϕ_1), because porous medium offers resistance to the flow, which increases by increasing ϕ_1. However the flow in the channel increases by

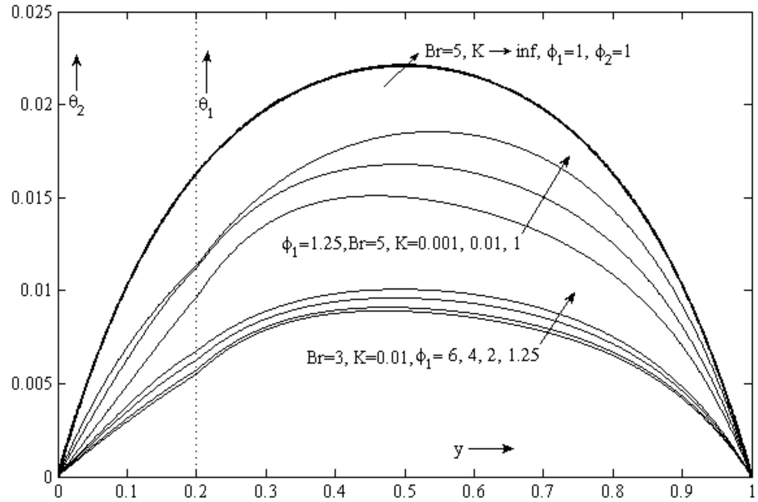

FIGURE 2. Temperature distribution for $M = 2$, $F = 2$, $\phi_2 = 1.5$, $a = 0.2$

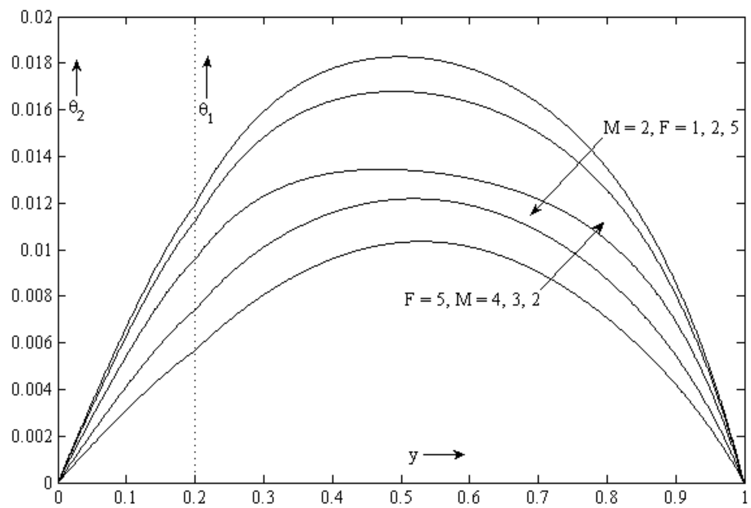

FIGURE 3. Temperature distribution for $K = 0.01$, Br $= 5$, $\phi_1 = 1.25$, $\phi_2 = 1.5$, $a = 0.2$

increasing the permeability of the porous medium. The effects of various parameters on temperature field are shown in Figures 2 and 3. Figure 2 compares the temperature profiles in the composite channel with that when there is no porous medium. The temperature profiles are parabolic for the Poiseuille flow, when the channel is free from porous material. It is seen that with the introduction of the

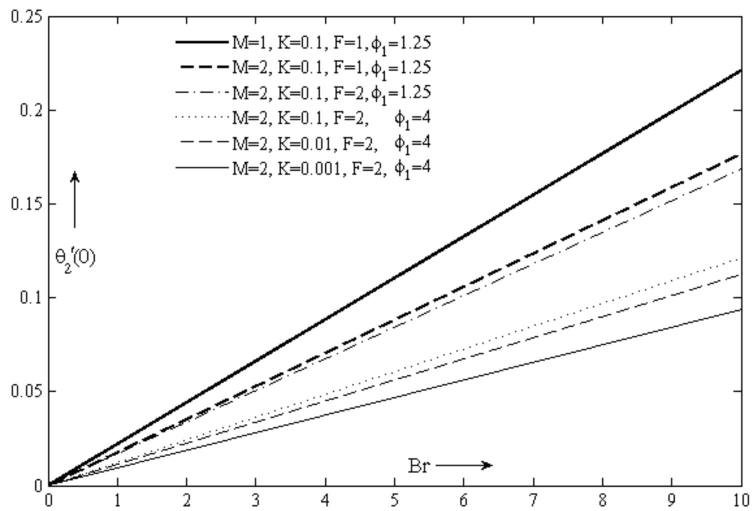

FIGURE 4. Rate of heat transfer at the lower impermeable plate for $\phi_2 = 1.5,\ a = 0.2$

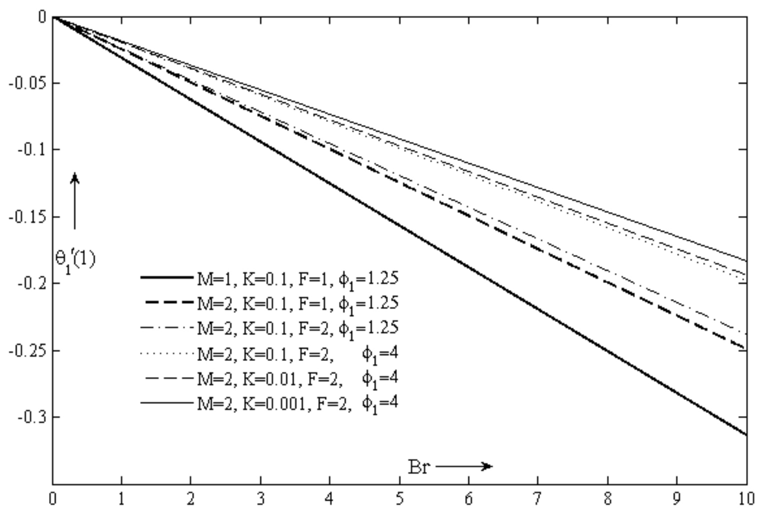

FIGURE 5. Rate of heat transfer at the upper impermeable plate for $\phi_2 = 1.5,\ a = 0.2$

porous medium at the plate the temperature in the channel reduces. Further, we observe from the Figure 2 that the temperature field increases with the increase in the value of Brinkman number (Br) and permeability parameter (K), while it reduces by increasing the viscosity ratio (ϕ_1). Figure 3 shows that the temperature in the channel decreases by increasing the radiation parameter (F) or the magnetic

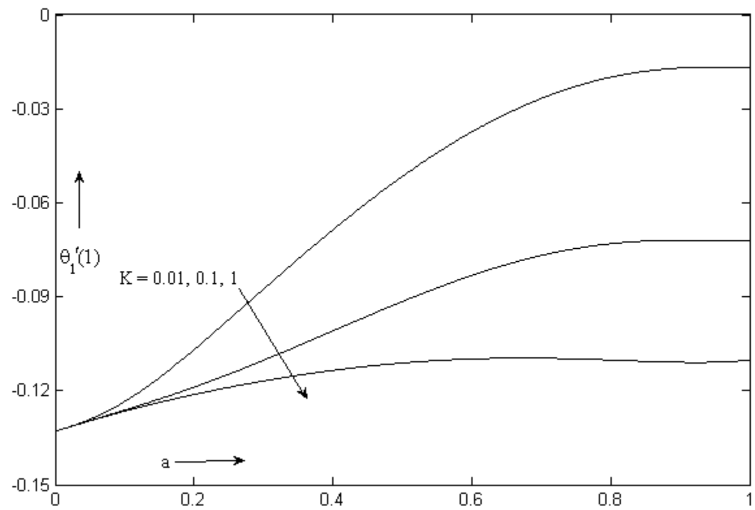

FIGURE 6. Rate of heat transfer at the upper impermeable plate for $M = 2,\ F = 2,\ \phi_1 = 1.25,\ \phi_2 = 1.5,\ \mathrm{Br} = 5$

parameter (M). Figures 4 to 6 illustrate the variations of the dimensionless rate of heat transfer at the lower impermeable plate and at the upper impermeable plate respectively for different values of the parameters M, K, F,Br, a and ϕ_1. It is noticed in Figure 4 that the dimensionless rate of heat transfer, $\theta_2'(0)$ increases at the lower impermeable plate where porous medium is attached with the increase in the permeability parameter K and the Brinkman number Br. However radiation parameter F or the magnetic field parameter M or the viscosity ratio $\phi_1$decreases $\theta_2'(0)$. Figures 5 and 6 shows that $\theta_1'(1)$ increases with the increase in the permeability parameter K and the Brinkman number Br, while radiation parameter F or the magnetic field parameter M or the viscosity ratio $\phi_1$decreases $\theta_1'(1)$. Further by the introduction of porous medium in the channel, the rate of heat transfer at the upper plate decreases numerically and it attains its minimum value when the channel is fully filled with a porous medium. This minimum value also increases rapidly with the increase in the permeability of the porous medium. Figures 7 to 10, depict variations of total entropy generation Ns in both regions for different values of the pertinent parameters, such as Brinkman number Br, magnetic field parameter M, permeability parameter K, width of the porous medium a, and viscosity ratioϕ_1. It is seen that the entropy generation number Ns is very low in the middle part of the channel because of gradually varying small velocity and temperature gradient there, and attains high values in the vicinity of the channel plates and porous interface. It is more pronounced in the region near the upper plate because of high velocity and temperature gradient there. It is seen from Figure 7 that the Brinkman number Br enhances the total entropy generation in both the free fluid and the porous region. Figure 8 reveals the effect of increasing magnetic field parameter M on total entropy generation Ns. As M increases, the total entropy

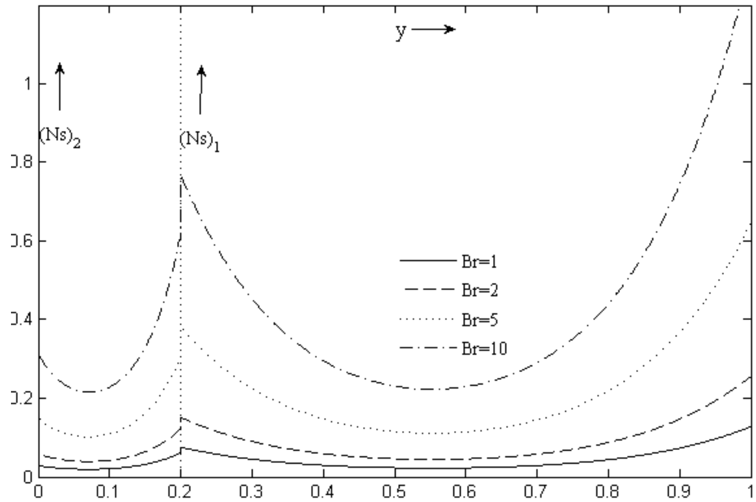

FIGURE 7. Total entropy generation for $M = 2$, $K = 0.01$, $F = 2$, $\phi_1 = 1.25$, $\phi_2 = 1.5$, $a = 0.2$

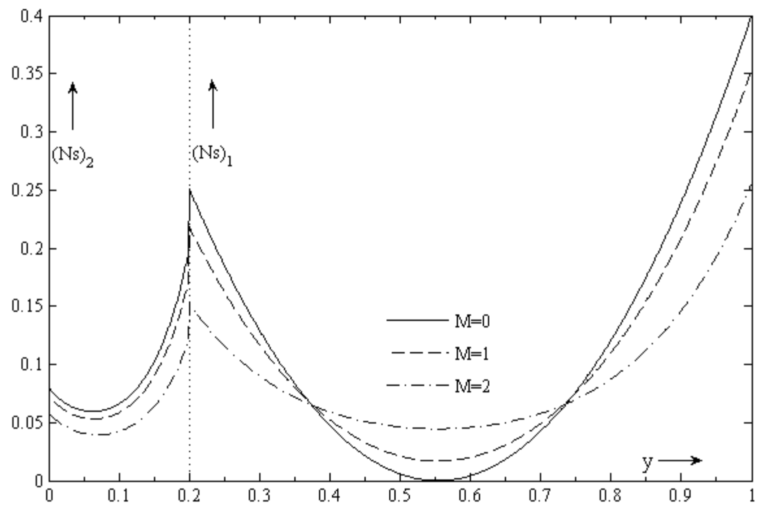

FIGURE 8. Total entropy generation for Br $= 2$, $K = 0.01$, $F = 2$, $\phi_1 = 1.25$, $\phi_2 = 1.5$, $a = 0.2$

generation decreases in the channel except the middle part of the clear fluid region where it increases by increasing M. Meanwhile, it is interesting to note that within the flow field there exists two points where the entropy production is not affected by increasing M. The results on Ns are also compared with that of a channel free from porous material in Figure 9. It is observed that by attaching a porous medium

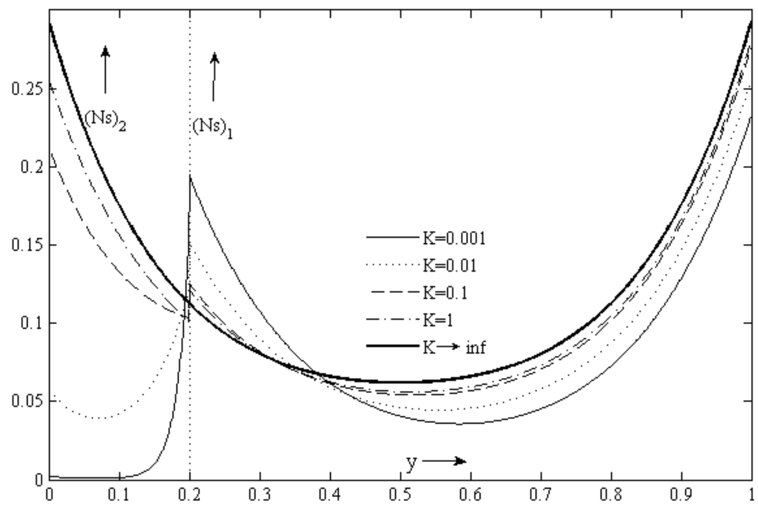

FIGURE 9. Total entropy generation for $M = 2,\ \mathrm{Br} = 2,\ F = 2,\ \phi_1 = 1.25,\ \phi_2 = 1.5,\ a = 0.2$

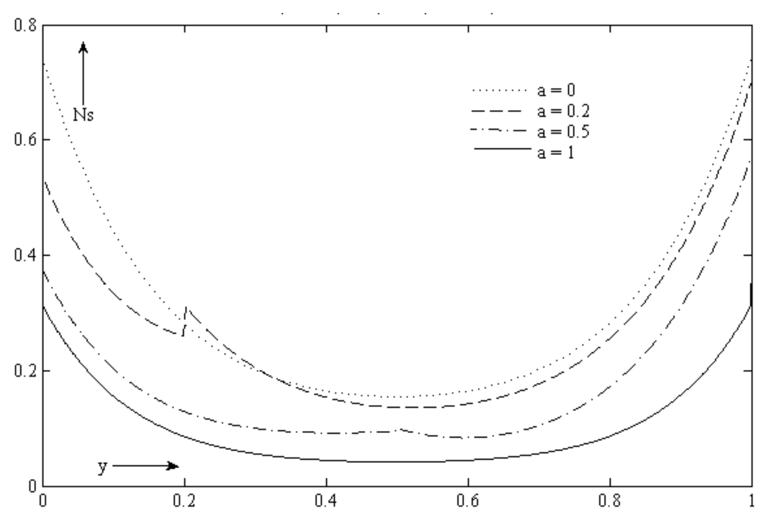

FIGURE 10. Total entropy generation for $M = 2,\ K = 0.1,\ \mathrm{Br} = 5,\ F = 2,\ \phi_1 = 1.25,\ \phi_2 = 1.5$

layer to the lower plate Ns is increased in the clear fluid region near the porous interface while it decreases in the rest part of the channel. Figure 10 shows the effect of porous medium width a on total entropy generation. We see that total entropy decreases throughout the region for large values of a and it is minimum for $a = 1$, i.e. when the channel if completely filled with a porous medium. In Figure 11,

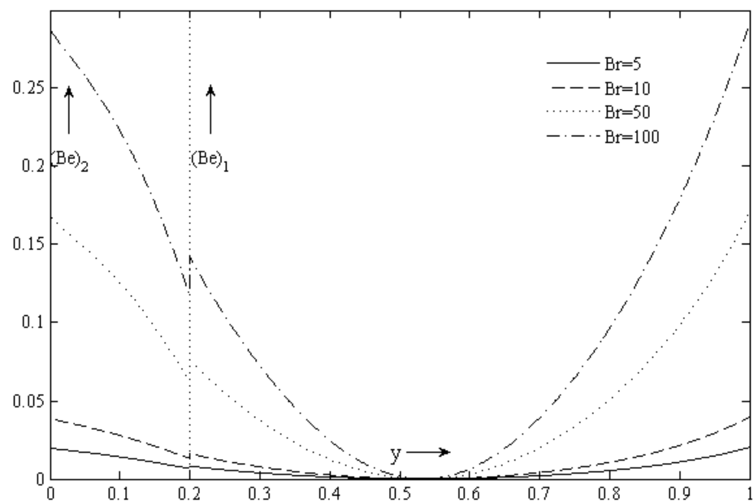

FIGURE 11. Bejan number for $M = 2,\ K = 0.1,\ F = 2,\ \phi_1 = 1.25,\ \phi_2 = 1.5,\ a = 0.2$

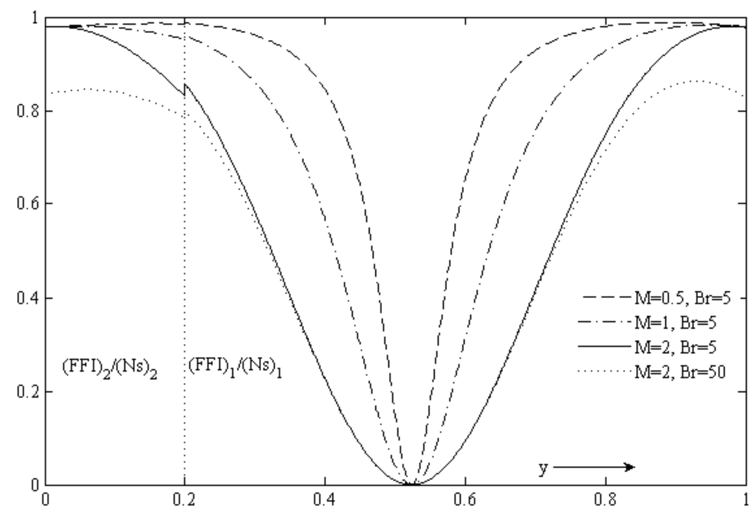

FIGURE 12. Entropy generation due to fluid friction for $K = 0.1,\ F = 2,\ \phi_1 = 1.25,\ \phi_2 = 1.5,\ a = 0.2$

the Bejan number is displayed as a function of the transverse distance y for varying values of the Brinkman number Br. It is observed that the Bejan number increases throughout the channel with the increase in the Brinkman number. We see that in the middle part of the channel, the contribution in the entropy generation due to heat transfer irreversibility is negligible. Figure 11 also shows that Be is always less

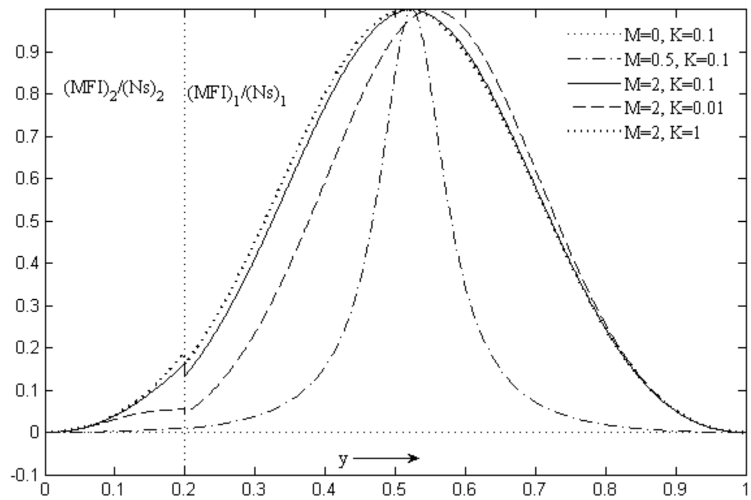

FIGURE 13. Entropy generation due to magnetic field for $F = 2,\ \mathrm{Br} = 5,\ \phi_1 = 1.25,\ \phi_2 = 1.5,\ a = 0.2$

than 0.5 throughout the channel for some moderate values of Brinkman number, and hence in the channel $\mathrm{FFI} + \mathrm{MFI} > \mathrm{HTI}$.

The effects of magnetic field parameter (M), Brinkman number (Br) and permeability parameter (K) on entropy generation due to fluid friction and magnetic field is shown in Figures 12 and 13 respectively. Figure 12 shows that entropy generation due to fluid friction decreases in both regions by increasing the magnetic field parameter (M). This behavior may be explained by the decrease in velocity with the increase in magnetic parameter, see Figure 1. Further the entropy generation due to fluid friction decreases with the increase in Brinkman number (Br), but this increment is significant near the plates only. The influence of magnetic field parameter (M) and permeability parameter (K) on entropy generation due to magnetic field is plotted in Figure 13. We can see that entropy generation due to magnetic field tends to increase in both regions by increasing the value of M. This effect has its maximum value near the centerline of the channel, which shifts towards the upper plate by decreasing the value of K. The effect of increasing the value of K is to increase the entropy generation due to magnetic field in the lower half of the region, while the reverse effects have been obtained in the rest part of the channel.

5. Conclusion

In this paper, the effect of transverse magnetic field on entropy generation of steady fully developed flow in a channel partially filled with a porous material and partially with a clear fluid has been investigated. The velocity and temperature profiles for both regions are obtained and used to compute the entropy generation. The effect of various parameters on velocity, temperature and entropy generation is analyzed. Based on the results presented above, the following conclusions are deduced.

(1) An increase in K increases the velocity profiles, while an increase in M or ϕ_1 decreases the velocity profiles in both regions.
(2) An increase in Br or K, increases the temperature profiles, while an increase in ϕ_1, M or F decreases the temperature profiles in both regions.
(3) Rate of heat transfer at the lower impermeable plate increases by increasing Br or K, while reverse effects are obtained by increasing ϕ_1, M or F.
(4) Rate of heat transfer at the upper impermeable plate increases numerically by increasing Br or K, while reverse effects are obtained by increasing ϕ_1, M, a or F.
(5) As expected, the channel plates and porous interface nearby region act as strong producer of irreversibility in the channel.
(6) Entropy generation due to combined effect of fluid friction and magnetic field dominates over entropy generation due to heat transfer. Moreover, the entropy generation due to heat transfer and fluid friction takes their minimum values near the centerline, while entropy generation due to magnetic field is maximum near the centerline.

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СТВАРАЊЕ ЕНТРОПИЈЕ У ПУАЗЕЉЕВОМ ТОКУ КРОЗ КАНАЛ ДЕЛИМИЧНО ПОПУЊЕН ПОРОЗНИМ МАТЕРИЈАЛОМ

Резиме. У овом раду разматрана је теоријска анализа стварања ентропије услед размене топлоте за ток кроз канал између паралелних плоча делимично попуњен порозним материјалом, под дејством транверзалног магнетног поља и радијације. Обе хоризонталне плоче су на константним и једнаким температурама. Добијено је тачно решење одговарајућих једначина у затвореној форми, како за области испуњене порозним материјалом, тако и за чисте области флуида. Коефицијент стварања ентропије као и *Bejan*-ов број су такође израчунати. Одређен је утицај мењања разних параметара, попут магнетног поља, радијације, односа вискозности и термалне проводљивости, на вредности брзине, температуре, као и брзину стварања ентропије.

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