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# A TWO DIMENSIONAL FIBRE REINFORCED MICROPOLAR THERMOELASTIC PROBLEM FOR A HALF-SPACE SUBJECTED TO MECHANICAL FORCE

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Abstract. The purpose of this paper is to study the two dimensional deformation of fibre reinforced micropolar thermoelastic medium in the context of Green–Lindsay theory of thermoelasticity. A mechanical force is applied along the interface of fluid half space and fibre reinforced micropolar thermoelastic half space. The normal mode analysis has been applied to obtain the exact expressions for displacement component, force stress, temperature distribution and tangential couple stress. The effect of anisotropy and micropolarity on the displacement component, force stress, temperature distribution and tangential couple stress has been depicted graphically.

## 1. Introduction

The dynamical interaction between the thermal and mechanical has great practical applications in modern aeronautics, astronatics, nuclear reactors, and high-energy particle accelerators. Classical elasticity is not adequate to model the behavior of materials possessing internal structure. Furthermore, the micropolar elastic model is more realistic than the purely elastic theory for studying the response of materials to external stimuli. Eringen and Suhubi [**1**] and Suhubi and Eringen [**2**] developed a nonlinear theory of micro-elastic solids. Later Eringen [**3**–**5**] developed a theory for the special class of micro-elastic materials and called it the "linear theory of micropolar elasticity". Under this theory, solids can undergo macro-deformations and micro-rotations. Eringen [**6**] extended his work to include the axial stretch during the rotation of molecules and developed the theory of micropolar elastic solid with stretch. The micropolar theory was extended to include thermal effects by Nowacki [**7**], Eringen [**8**], Tauchert et al. [**9**], Tauchert [**10**], Nowacki and Olszak [**11**]. One can refer to Dhaliwal and Singh [**12**] for a review

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on the micropolar thermoelasticity and a historical survey of the subject, as well as to Eringen and Kafadar [**13**] in "Continuum Physics" series in which the general theory of micromorphic media has been summed up.

There are two important generalized theories of thermoelasticity. The first is due to Lord and Shulman [**14**]. The second generalization of the coupled theory of elasticity is known as the theory of thermoelasticity with two relaxation time or the theory of temperature-rate-dependent thermoelasticity. Muller [**15**], in the review of thermodynamics of thermoelastic solids, proposed an entropy production inequality, with the help of which he considered restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green and Laws [**16**]. Green and Lindsay [**17**] obtained another version of the constitutive equations. These equations were also obtained independently and more explicitly by Suhubi [**18**]. This theory contains two constants that act as relaxation times and modify all the equations of coupled theory, not only the heat equations. The classical Fourier law of heat conduction is not violated if the medium under consideration has a centre of symmetry.

Barber [**19**] studied thermoelastic displacements and stresses due to a heat source moving over the surface of a half plane. Sherief [**20**] obtained components of stress and temperature distributions in a thermoelastic medium due to a continuous source. Dhaliwal et al. [**21**] investigated thermoelastic interactions caused by a continuous line heat source in a homogeneous isotropic unbounded solid. Chandrasekharaiah and Srinath [**22**] studied thermoelastic interactions due to a continuous point heat source in a homogeneous and isotropic unbounded body. Sharma et al. [**23**] investigated the disturbance due to a time-harmonic normal point load in a homogeneous isotropic thermoelastic half-space. Sharma and Chauhan [**24**] discussed mechanical and thermal sources in a generalized thermoelastic half-space. Sharma et al. [**25**] investigated the steady-state response of an applied load moving with constant speed for infinite long time over the top surface of a homogeneous thermoelastic layer lying over an infinite half-space. Sarbani and Amitava [**26**] studied the transient disturbance in half-space due to moving internal heat source under L-S model and obtained the solution for displacements in the transform domain. Aouadi [**27**] studied thermo-mechanical interaction in a generalized thermo-microstretch elastic half space. Deswal and Choudhary [**28**] studied a two-dimensional problem due to moving loads in generalized thermoelastic solid with diffusion. El. Maghraby [**29**] considered two dimensional problem of generalized thermoelastic half space under the action of body forces and subjected to thermal shock.

A reinforced structural component is designed for all conditions of stresses that may occur and in accordance with the principles of applied mechanics. Fibre-reinforced composites are used in a variety of structures due to their low weight and high strength. The characteristic property of a reinforced composite is that its components act together as single anisotropic units as long as they remain in the elastic condition. A reinforced medium plays a very interesting role in civil engineering and geophysics, as well as aerospace structural dynamics (wings, fuselage etc). The idea of introducing a continuous self-reinforcement at every point of an elastic solid was given by Belfied et al. [**30**]. The model was later applied to the rotation of a tube by Verma and Rana [**31**]. Sengupta and Nath [**32**] discussed the problem of the surface waves in fibre-reinforced anisotropic elastic media. Hashin and Rosen [**33**] gave the elastic moduli for fibre-reinforced materials. The problem of reflection of plane waves at the free surface of a fibre-reinforced elastic half-space was discussed by Singh et al. [**34**]. Chattopadhyay and Choudhury [**35**] discussed the problem of propagation, reflection and transmission of magneto-elastic shear waves in a self-reinforced medium. The reflection and transmission of plane SH wave through a self-reinforced elastic layer sandwiched between two homogeneous visco-elastic solid half-spaces have been studied by Chaudhary et al. [**36**]. Pradhan et al. [**37**] studied the dispersion of Love waves in a self-reinforced layer over an elastic non-homogenous half space. The propagation of plane waves in fibre-reinforced media was discussed by Chattopadhyay et al. [**38**]. The problem of wave propagation in thermally-conducting linear fibre-reinforced composite materials was analyzed by Singh [**39**]. Abbas and Othman [**40**] and Othman and co-workers [**41**–**44**] discussed some problems in fibre-reinforced thermoelastic medium.

In the present problem the authors have discussed deformation in micropolar fibre-reinforced thermoelastic medium subjected to a mechanical force. The force is applied along the interface of fluid half-space and micropolar fibre-reinforced thermoelastic half-space. The normal mode analysis is used to obtain the exact expressions for the considered variables. The distributions of the considered variables are represented then graphically.

## 2. Formulation of Problem

The constitutive equation for a fibre-reinforced linearly elastic anisotropic medium with respect to reinforcement direction $\vec{a}$ (Belfied et al. [**30**]) are

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki})$$

$$+\beta a_k a_m e_{km} a_i a_j - \varepsilon_{ijr} k\phi_r - \nu\Big(1 + \nu_0\frac{\partial}{\partial t}\Big)\hat{T}\delta_{ij}, \tag{2.1}$$

where $\sigma_{ij}$ is a force stress tensor, $e_{ij}$ are component of strain, $\lambda$, $\mu_T$ are elastic constants; $\alpha$, $\beta$ and $(\mu_L - \mu_T)$ are reinforcement parameter, $\nu = (3\lambda + 2\mu)\alpha_t$; $\alpha_t$ is thermal expansion coefficient, $\hat{T} = T - T_0$ where T is temperature above reference temperature $T_0$, $\varepsilon_{ijr}$ is alternate tensor, $\delta_{ij}$ is $\vec{a} \equiv (a_1, a_2, a_3), a_1^2 + a_2^2 + a_3^2 = 1$, we choose the fibre direction as $\vec{a} \equiv (1, 0, 0)$. The strain can be expressed in terms of the displacement $u_i$ as, $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$.

We consider a normal force of magnitude $P_1$ acting along the interface of fibre reinforced micropolar thermoelastic medium (medium I) occupying the region $0 \leqslant x \leqslant \infty$ and a non viscous fluid (medium II) in the region $-\infty \leqslant x \leqslant 0$ as shown in Figure 1.

We restrict our analysis to the plane strain parallel to $xy$-plane with displacement vector $\vec{u} = (u, v, 0)$ with micro-rotation vector $\vec{\varphi} = (0, 0, \varphi_3)$ then the constitutive relation for stress components can be written as,

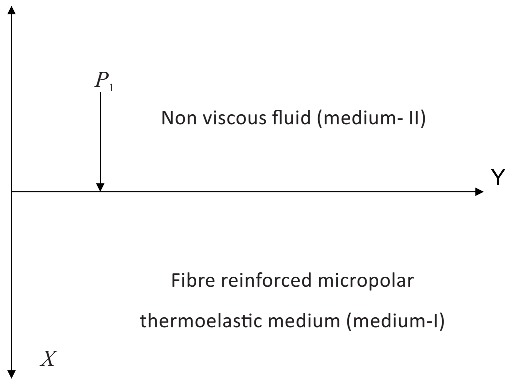
$$\sigma_{xx} = A_{11}u_{,x} + A_{12}v_{,y} - \nu\Big(1 + \nu_0\frac{\partial}{\partial t}\Big)\hat{T}, \tag{2.2}$$

$$\sigma_{yy} = A_{12}u_{,x} + A_{22}v_{,y} - \nu\Big(1+\nu_0\frac{\partial}{\partial t}\Big)\hat{T}, \tag{2.3}$$

$$\sigma_{zz} = A_{12}u_{,x} + \lambda v_{,y} - \nu\Big(1+\nu_0\frac{\partial}{\partial t}\Big)\hat{T}, \tag{2.4}$$

$$\sigma_{xy} = \mu_L(u_{,y}+v_{,x}) - k\varphi_3, \sigma_{yx} = \mu_L(u_{,y}+v_{,x}) + k\varphi_3, \sigma_{zx} = \sigma_{zy} = 0. \tag{2.5}$$

where $A_{11} = \lambda + 2(\alpha+\mu_T) + 4(\mu_L - \mu_T) + \beta, A_{12} = \alpha + \lambda, A_{22} = \lambda + 2\mu_T$.




(Geometry of the problem)

FIGURE 1

The field equations and constitutive relations for micropolar generalized thermo-elastic medium in the absence of body forces, body couples and heat sources in the context of generalized thermo-elasticity can be expressed as:

Equation of Motion:

$$\rho\ddot{u}_j = \sigma_{ij,i}, (i,j = 1,2,3), \tag{2.6}$$

Heat conduction Equation:

$$K^*T_{,ii} = \Big(n_1\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\Big)\rho C_E T + \nu T_0\Big(n_1\frac{\partial}{\partial t} + n_0\tau_0\frac{\partial^2}{\partial t^2}\Big)u_{i,i}, \tag{2.7}$$

where $\rho$ denotes the density of micropolar thermoelastic solid, $K^*$ is the thermal conductivity, $C_E$ is the specific heat at constant strain, $n_1$, $n_0$ are parameters and $\tau_0$, $\nu$ are thermal thermal relaxation times. Equation of micropolar material:

$$(\alpha_l + \beta_l + \gamma)\nabla(\nabla.\vec{\varphi}) - \gamma\nabla\times(\nabla\times\vec{\varphi}) + k(\nabla\times\vec{u}) - 2k\vec{\varphi} = J\rho\frac{\partial^2\vec{\varphi}}{\partial t^2}, \tag{2.8}$$

$$m_{il} = \alpha_l\varphi_{r,r}\delta_{il} + \gamma\varphi_{l,i}, \tag{2.9}$$

where $\alpha_l$, $\beta_l$, $\gamma$, k are material constant, J is micro-inertia, $m_{il}$ is couple stress tensor.

The equations of motion and stress components in fluid (Ewing et. al. [**45**]) are,

$$(\lambda^f)\nabla(\nabla.\vec{u^f}) = \rho^f\frac{\partial^2\vec{u^f}}{\partial t^2}, \tag{2.10}$$

$$\sigma_{ij}^f = \lambda^f u_{r,r}^f \delta_{ij}, \tag{2.11}$$

where, $\vec{u^f}$ is displacement vector, $\lambda^f$ is Lame's constant and $\rho^f$ is density of fluid. Using (2.2)–(2.5), we note that the third equation of motion in (2.6) identically satisfied and first two equation become,

$$\rho\frac{\partial^2 u}{\partial t^2} = A_{11}\frac{\partial^2 u}{\partial x^2} + B_2\frac{\partial^2 v}{\partial x \partial y} + B_1\frac{\partial^2 u}{\partial y^2} + k\frac{\partial \varphi_3}{\partial y} - \nu\Big(1+\nu_0\frac{\partial}{\partial t}\Big)\frac{\partial \hat{T}}{\partial x}, \tag{2.12}$$

$$\rho\frac{\partial^2 v}{\partial t^2} = A_{11}\frac{\partial^2 v}{\partial y^2} + B_2\frac{\partial^2 u}{\partial x \partial y} + B_1\frac{\partial^2 v}{\partial x^2} - k\frac{\partial \varphi_3}{\partial x} - \nu\Big(1+\nu_0\frac{\partial}{\partial t}\Big)\frac{\partial \hat{T}}{\partial y}, \tag{2.13}$$

where $B_1 = \mu_L$, $B_2 = A_{12} + B_1$.

Equaton (2.7)–(2.9) becomes

$$K^*\nabla^2 T = \Big(n_1\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\Big)\rho C_E T + \nu T_0\Big(n_1\frac{\partial}{\partial t} + n_0\tau_0\frac{\partial^2}{\partial t^2}\Big)e, \tag{2.14}$$

$$\gamma\nabla^2\varphi_3 + k\Big(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\Big) - 2k\varphi_3 = J\rho\frac{\partial^2 \varphi_3}{\partial t^2}, \tag{2.15}$$

$$m_{yz} = \gamma\frac{\partial \varphi_3}{\partial y}. \tag{2.16}$$

Equations (2.12)–(2.16) are stress components and heat conduction equation of generalized thermoelastic solid applicable to L-S, G-L and C-D theories as follows:

1. The equations of coupled thermoelasticity (CD) theory are retrieved when $n_0 = 0$, $n_1 = 1$, $\tau_0 = 0$, $\nu_0 = 0$
2. Lord–Shulman (L-S) theory is retrieved when $n_0 = 1$, $n_1 = 1$, $\tau_0 > 0$, $\nu_0 = 0$
3. Green–Lindsay (G-L) theory is retrieved when $n_0 = 0$, $n_1 = 1$, $\nu_0 \geqslant \tau_0 > 0$
4. The corrosponding equations for three theories in the absence of fiber reinforcement are retrieved from the above mentioned cases by taking $\alpha = 0$, $\beta = 0$, $(\mu_L - \mu_T) = 0$

For convenience the following non-dimensional variables are used: $x' = c_1\eta x$, $y' = c_1\eta y$, $u' = c_1\eta u$, $v' = c_1\eta v$, $u^{f'} = c_1\eta u^f$, $v^{f'} = c_1\eta v^f$, $t' = c_1^2\eta t$, $\tau_0' = c_1^2\eta\tau_0$,

$\nu_0' = c_1^2\eta\nu_0$, $\theta = \frac{\nu\hat{T}}{\rho c_1^2}$, $\sigma_{ij}' = \frac{\sigma_{ij}}{\rho c_1^2}$, $\varphi_3' = \varphi_3$, $m_{ij}' = \frac{\eta m_{ij}}{\rho c_1}$, $\sigma_{ij}^{f'} = \frac{\sigma_{ij}^f}{\rho^f c_1^2}$, $P_1' = \frac{P_1}{\rho c_1^2}$,
Where, $\eta = \frac{\rho C_E}{K^*}$, $c_1^2 = \frac{\lambda + 2\mu_T}{\rho}$.

Using above non dimensional variables, the equations (2.12)–(2.16) reduces to (after dropping superscripts)

$$\frac{\partial^2 u}{\partial t^2} = h_{11}\frac{\partial^2 u}{\partial x^2} + h_2\frac{\partial^2 v}{\partial x \partial y} + h_1\frac{\partial^2 u}{\partial y^2} + h_3\frac{\partial \varphi_3}{\partial y} - \nu\Big(1+\nu_0\frac{\partial}{\partial t}\Big)\frac{\partial \theta}{\partial x}, \tag{2.17}$$

$$\frac{\partial^2 v}{\partial t^2} = h_{22}\frac{\partial^2 v}{\partial y^2} + h_2\frac{\partial^2 u}{\partial x \partial y} + h_1\frac{\partial^2 v}{\partial x^2} - h_3\frac{\partial \varphi_3}{\partial x} - \nu\Big(1+\nu_0\frac{\partial}{\partial t}\Big)\frac{\partial \theta}{\partial y}, \tag{2.18}$$

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = \Big(n_1\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\Big)\theta + \varepsilon\Big(n_1\frac{\partial}{\partial t} + n_0\tau_0\frac{\partial^2}{\partial t^2}\Big)\Big(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\Big), \tag{2.19}$$

$$\nabla^2\varphi_3 + b_1\Big(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\Big) - b_2\varphi_3 = b_3\frac{\partial^2\varphi_3}{\partial t^2}, \tag{2.20}$$

$$m_{yz} = b_4\frac{\partial\varphi_3}{\partial y}, \tag{2.21}$$

where,

$$(h_{11}, h_{22}, h_1, h_2, h_3) = \frac{(A_{11}, A_{22}, B_1, B_2, k)}{\rho c_1^2}, \quad \varepsilon = \frac{\nu^2 T_0}{K^*\eta\rho c_1^2},$$

$$(b_1, b_2) = \frac{(k, 2k)}{(\gamma\eta^2 c_1^2)}, \quad b_3 = \frac{J\rho c_1^2}{\gamma}, \quad b_4 = \frac{\gamma\eta^2}{\rho}.$$

Equation (2.2)-(2.5) in dimensionless form reduces to

$$\rho c_1^2\sigma_{xx} = A_{11}u_{,x} + A_{12}v_{,y} - \rho c_1^2\Big(1 + \nu_0\frac{\partial}{\partial t}\Big)\theta, \tag{2.22}$$

$$\rho c_1^2\sigma_{yy} = A_{12}u_{,x} + A_{22}v_{,y} - \rho c_1^2\Big(1 + \nu_0\frac{\partial}{\partial t}\Big)\theta, \tag{2.23}$$

$$\rho c_1^2\sigma_{zz} = A_{12}u_{,x} + \lambda v_{,y} - \rho c_1^2\Big(1 + \nu_0\frac{\partial}{\partial t}\Big)\theta, \tag{2.24}$$

$$\rho c_1^2\sigma_{xy} = \mu_L(u_{,y} + v_{,x}) - k\varphi_3, \tag{2.25}$$

$$\rho c_1^2\sigma_{yx} = \mu_L(u_{,y} + v_{,x}) + k\varphi_3, \tag{2.26}$$

$$\sigma_{zx} = \sigma_{zy} = 0. \tag{2.27}$$

## 3. Normal Mode Analysis

The solution of the considered physical variables can be decomposed in terms of normal mode and can be considered in the following form,

$$(u, v, \theta, \varphi_3, \sigma_{ij}, m_{ij}, u^f, v^f, \sigma_{ij}^f)(x, y, t) = (u^*, v^*, \theta^*, \varphi_3^*, \sigma_{ij}^*, m_{ij}^*, u^{f^*}, v^{f^*}, \sigma_{ij}^{f^*})(x)e^{\omega t + iby}$$

where $\omega$ is complex frequency, $b$ is wave number in $y$-direction and $u^*(x), v^*(x), \theta^*(x), \varphi_3^*(x), \sigma_{ij}^*(x), m_{ij}^*(x), u^{f^*}(x), v^{f^*}(x), \sigma_{ij}^{f^*}(x)$ are the amplitudes of field quantities.

Using Normal mode in equation (2.17)–(2.27), we get

$$(h_{11}D^2 - A_1)u^* + ibh_2Dv^* + ibh_3\varphi_3^* - A_4D\theta^* = 0, \tag{3.1}$$

$$ibh_2Du^* + (h_1D^2 - A_2)v^* - h_3D\varphi_3^* - ibA_4\theta^* = 0, \tag{3.2}$$

$$-ibb_1u^* + b_1Dv^* + (D^2 - A_3)\varphi_3^* = 0, \tag{3.3}$$

$$A_6Du^* + ibA_6v^* - (D^2 - A_5)\theta^* = 0, \tag{3.4}$$

$$m_{yz}^* = ibb_4\varphi_3^*, \tag{3.5}$$

$$\rho c_1^2\sigma_{xx}^* = A_{11}Du^* + ibA_{12}v^* - \rho c_1^2A_4\theta^*, \tag{3.6}$$

$$\rho c_1^2\sigma_{yy}^* = A_{12}Du^* + ibA_{22}v^* - \rho c_1^2A_4\theta^*, \tag{3.7}$$

$$\rho c_1^2\sigma_{zz}^* = A_{11}Du^* + \lambda ibv^* - \rho c_1^2A_4\theta^*, \tag{3.8}$$

$$\rho c_1^2 \sigma_{xy}^* = \mu_L (ibu^* + Dv^*) - k\varphi_3^*, \tag{3.9}$$

$$\rho c_1^2 \sigma_{yx}^* = \mu_L (ibu^* + Dv^*) + k\varphi_3^*, \tag{3.10}$$

where, $A_1 = \omega^2 + h_1 b^2$, $A_2 = \omega^2 + h_{22} b^2$, $A_3 = b^2 + b_2 - b_3 b^2$, $A_4 = 1 + \nu_0 \omega$, $A_5 = b^2 + \tau_0 \omega^2 + n_1 \omega$, $A_6 = \varepsilon \omega (n_1 + n_0 \tau_0 \omega)$, $D = \frac{d}{dx}$.
Eliminating $v^*(x)$, $\theta^*(x)$, $\varphi^*(x)$ between equations (3.1)–(3.4), we get the following eight order differential equation for $u^*(x)$ as

$$(D^8 + AD^6 + BD^4 + CD^2 + E)u^*(x) = 0. \tag{3.11}$$

where,

$$A = \frac{-1}{h_{11} h_1}\big(h_1 A_4 A_6 + h_{11} h_1 A_5 + h_{11} h_1 A_3 + h_{11} A_2 + b_1 h_{11} h_3 + h_1 A_1 + ibh_2^2\big),$$

$$\begin{aligned} B = \frac{-1}{h_{11} h_1}(&2b^2 h_2 A_4 A_6 - h_1 A_3 A_4 A_6 + b_1 h_3 A_4 A_6 - A_2 A_4 A_6 + ib^2 h_{11} A_4 A_6 \\ &- h_{11} h_1 A_3 A_5 - h_{11} A_2 A_5 + h_{11} b_1 h_3 A_5 - h_1 A_1 A_5 - ib^2 h_2^2 A_5 - h_{11} A_2 A_3 \\ &- h_1 A_3 A_1 - A_1 A_2 + b_1 h_3 A_1 - ibh_2^2 A_3 + b^2 b_1 h_2 h_3 + b^2 b_1 h_1 h_3), \end{aligned}$$

$$\begin{aligned} C = \frac{-1}{h_{11} h_1}(&-2b^2 h_2 A_3 A_4 A_6 - b^2 b_1 h_3 A_4 A_6 + A_2 A_3 A_4 A_6 - ib^2 A_1 A_4 A_6 \\ &- ib^2 h_{11} A_3 A_4 A_6 + h_{11} A_2 A_3 A_5 + h_1 A_1 A_3 A_5 + A_1 A_2 A_5 - b_1 h_3 A_1 A_5 \\ &+ ib^2 h_2^2 A_3 A_5 - 2b^2 b_1 h_2 h_3 A_5 - b^2 b_1 h_1 h_3 A_5 + A_1 A_2 A_3 - b^2 b_1 h_3 A_2), \end{aligned}$$

$$E = \frac{-1}{h_{11} h_1}(b^4 b_1 h_3 A_4 A_6 + ib^2 A_1 A_3 A_4 A_6 - A_1 A_2 A_3 A_5 + b^2 b_1 h_3 A_2 A_5).$$

In a similar manner we can show that $v^*(x)$, $\theta^*(x)$, $\varphi^*(x)$ satisfies the equation

$$(D^8 + AD^6 + BD^4 + CD^2 + E)(v^*(x), \theta^*(x), \varphi^*(x)) = 0, \tag{3.12}$$

which can be factorized as follows

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)u^*(x) = 0, \tag{3.13}$$

The Series solution of equation (3.12) has the form

$$u^*(x) = \sum_{n=1}^{4} M_n(b, \omega) e^{-k_n x}, \tag{3.14}$$

$$v^*(x) = \sum_{n=1}^{4} M_n'(b, \omega) e^{-k_n x}, \tag{3.15}$$

$$\varphi_3^*(x) = \sum_{n=1}^{4} M_n''(b, \omega) e^{-k_n x}, \tag{3.16}$$

$$\theta^*(x) = \sum_{n=1}^{4} M_n'''(b, \omega) e^{-k_n x}. \tag{3.17}$$

where $M_n(b,\omega), M_n'(b,\omega), M_n''(b,\omega), M_n'''(b,\omega)$ are specific function depending upon $b$, $\omega$ and $k_n^2$ ; $n$=1,2,3,4 are the roots of characteristic equation (3.13).

Using equation (3.14)–(3.17) in equation (3.1)–(3.4), we get the following relations

$$M_n'(b,\omega) = H_{1n} M_n(b,\omega), \tag{3.18}$$

$$M_n''(b,\omega) = H_{2n} M_n(b,\omega), \tag{3.19}$$

$$M_n'''(b,\omega) = H_{3n} M_n(b,\omega). \tag{3.20}$$

Thus we have,

$$v^*(x) = \sum_{n=1}^{4} H_{1n} M_n(b,\omega) e^{-k_n x}, \tag{3.21}$$

$$\varphi_3^*(x) = \sum_{n=1}^{4} H_{2n} M_n(b,\omega) e^{-k_n x}, \tag{3.22}$$

$$\theta^*(x) = \sum_{n=1}^{4} H_{3n} M_n(b,\omega) e^{-k_n x}, \tag{3.23}$$

$$m_{yz}^*(x) = \sum_{n=1}^{4} H_{4n} M_n(b,\omega) e^{-k_n x}, \tag{3.24}$$

$$\sigma_{xx}^*(x) = \sum_{n=1}^{4} H_{5n} M_n(b,\omega) e^{-k_n x}, \tag{3.25}$$

$$\sigma_{yy}^*(x) = \sum_{n=1}^{4} H_{6n} M_n(b,\omega) e^{-k_n x}, \tag{3.26}$$

$$\sigma_{zz}^*(x) = \sum_{n=1}^{4} H_{7n} M_n(b,\omega) e^{-k_n x}, \tag{3.27}$$

$$\sigma_{xy}^*(x) = \sum_{n=1}^{4} H_{8n} M_n(b,\omega) e^{-k_n x}, \tag{3.28}$$

$$\sigma_{yx}^*(x) = \sum_{n=1}^{4} H_{9n} M_n(b,\omega) e^{-k_n x}. \tag{3.29}$$

where,

$$H_{1n} = \frac{[h_{11}k_n^5 - (A_1 + b^2 h_2 + A_5 h_{11} + A_4 A_6)k_n^3 + (A_1 A_5 + b^2 h_2 A_5 + A_4 A_6 b^2)k_n]}{ib\big[(h_1 h_2)k_n^4 - (A_2 + h_1 A_5 + h_2 A_5 + A_4 A_6)k_n^2 + (A_2 A_5 + b^4 A_4 A_6)\big]},$$

$$H_{2n} = \frac{(ibb_1 + b_1 k_n H_{1n})}{(k_n^2 - A_3)},$$

$$H_{3n} = \frac{A_6(-k_n + ibH_{1n})}{(k_n^2 - A_5)},$$

$$H_{4n} = ibb_4 H_{2n},$$

$$H_{5n} = \frac{[-A_{11}k_n + ibA_{12}H_{1n} - \rho c_1^2 A_4 H_{3n}]}{\rho c_1^2},$$
$$H_{6n} = \frac{[-A_{12}k_n + ibA_{22}H_{1n} - \rho c_1^2 A_4 H_{3n}]}{\rho c_1^2},$$
$$H_{7n} = \frac{[-A_{12}k_n + ib\lambda H_{1n} - \rho c_1^2 A_4 H_{3n}]}{\rho c_1^2},$$
$$H_{8n} = \frac{[ib\mu_L - k_n\mu_L H_{1n} - kH_{2n}]}{\rho c_1^2},$$
$$H_{9n} = \frac{[ib\mu_L - k_n\mu_L H_{1n} + kH_{2n}]}{\rho c_1^2}.$$

Similarly for medium II (i.e., fluid half space), the solutions are expressed in the form

$$u^{f^*}(x) = M_5(b,\omega)e^{-k_5 x}, \tag{3.30}$$

$$v^{f^*}(x) = M_5'(b,\omega)e^{-k_5 x}, \tag{3.31}$$

where $M_5(b,\omega)$ and $M_5'(b,\omega)$ are specific function depending upon $b$ and $\omega$ and $k_5$ is root of characteristic equation,

$$(D^2 - b^2 + l\omega^2)u^{f^*}(x) = 0, \tag{3.32}$$

where $l = \frac{\rho^f c_1^2}{\lambda^f}$.

From (3.32), $k_5^2 = b^2 - l\omega^2$.

Thus we have,

$$v^{f^*}(x) = QM_5(b,\omega)e^{-k_5 x}, \tag{3.33}$$

$$\sigma_{xx}^{f^*}(x) = LM_5(b,\omega)e^{-k_5 x}, \tag{3.34}$$

$$\sigma_{xy}^{f^*}(x) = 0, \tag{3.35}$$

where, $Q = \frac{k_5^2 - l\omega^2}{ibk_5}$ and $L = \frac{(\lambda^f)(ibQ - k_5)}{\rho c_1^2}$.

## 4. Applications

In this section we determine the parameters $M_n$; ($n$=1,2,3,4,5). In the physical problem, we should suppress the positive exponentials that are unbounded at infinity. Constants $M_1, M_2, M_3, M_4$ and $M_5$ have to be selected such that boundary condition (assuming closed pore condition) at the surface $x = 0$ takes the form, $\sigma_{xx} = \sigma_{xx}^f - P_1 e^{\omega t + iby}$; $\sigma_{xy} = \sigma_{xy}^f$; $\dot{v} = \dot{v}^f$; $\frac{\partial\theta}{\partial x} = 0$; $m_{yz} = 0$ where $P_1$ is the magnitude of mechanical force.

Using the expressions of $\sigma_{xx}$, $\sigma_{xx}^f$, $\sigma_{xy}$, $\sigma_{xy}^f$, $v$, $v^f$, $\theta$, $m_{yz}$ into above boundary conditions, gives the following equations satisfied by the parameters; ($n = 1,2,3,4,5$),

$$\sum_{n=1}^{4} H_{5n}M_n - LM_5 = -P_1 \qquad \sum_{n=1}^{4} H_{8n}M_n = 0$$

$$\sum_{n=1}^{4} H_{1n}M_n - QM_5 = 0 \qquad \sum_{n=1}^{4} H_{3n}k_nM_n = 0$$

$$\sum_{n=1}^{4} H_{4n}M_n = 0$$

After solving the above system of five equations, we get the values of constants $M_1, M_2, M_3, M_4, M_5$ and hence obtain the component of normal displacement, normal force stress, temperature distribution and and tangential couple stress at the interface of fibre reinforced micropolar thermoelastic half space and fluid half space.

## 5. Special Cases

(i) Taking $\alpha = 0$, $\beta = 0$, $(\mu_L - \mu_T) = 0$, we obtain the expressions for micropolar thermoelastic solid (MTS).

(ii) Neglecting micropolarity effect ($\alpha$, $\beta$, $\gamma$, $k$, $J \to 0$), we obtain the expressions for fibre reinforced thermoelastic solid (FRTS).

(iii) Neglecting micropolarity effect ($\alpha$, $\beta$, $\gamma$, $k$, $J \to 0$) and $\mu_L - \mu_T = 0$, we obtain the expressions for generalized thermoelastic solid (TS).

## 6. Numerical Results and Discussions

In order to illustrate the theoretical results obtained in the preceding section, we present some numerical values for the physical constants (Othman et al. [**44**]):

$\lambda$=$9.4 \times 10^9 N/m^2$, $\mu_T$=$1.89 \times 10^9 N/m^2$, $\mu_L$=$2.45 \times 10^9 N/m^2$,
$\alpha$=$-1.28 \times 10^9 N/m^2$, $\beta$=$0.32 \times 10^9 N/m^2$, $C_E = 5J/(kgk)$, $k = 10^{11} N/m^2$,
$J = 0.2 \times 10^{-11} m^2$, $\gamma = 0.779 \times 10^{-1} N$, $K^* = 0.3W/(mk)$, $T_0 = 1.5^0 k$,
$\alpha_t = 7.4033 \times 10^{-7} k^{-1}$, $\rho = 1.7 \times 10^3 kg/m^3$, $\mu = 3.84 \times 10^{11} N/m^2$,

The physical constant for water are given by (Ewing et. al. [**45**]):
$\lambda^f = 2.14 \times 10^9 N/m^2$, $\rho^f = 10^3 kg/m^3$.

The computations are carried out for the value of non-dimensional time $t = 0.4$ in the range $0 \leqslant y \leqslant 10$ and on the surface $x = 1.3$. The numerical values for normal displacement, normal force stress, temperature distribution and tangential couple stress are shown in Figures 2 to 5 for mechanical force with magnitude $P_1 = 1.0$, $\omega = \omega_0 + \iota\xi$, $\omega_0 = -0.3$, $\xi = 0.1$ and $b = 0.7$.

The variations for quantities are shown graphically for,
(a) Fibre-Reinforced Micropolar thermoelastic solid(FRMTS) by solid line with dashed symbol ⧫.
(b) Micropolar thermoelastic solid(MTS) by dashed line with centered symbol ■.
(c) Fibre-Reinforced thermoelastic solid(FRTS) by dashed line with centered symbol ▲.
(d) Thermoelasic solid(TS) with dashed line with centered symbol $\times$.

These graphical results represent the solutions obtained by using the generalized theory with two relaxation times (G-L theory) by taking $\tau_0 = 0.4$, $\upsilon_0 = 0.6$. The variations of all the quantities are similar in nature which decreases with increase in horizontal distance. The values of normal displacement, normal force stress and temperature distribution for fibre reinforced thermoelastic medium are

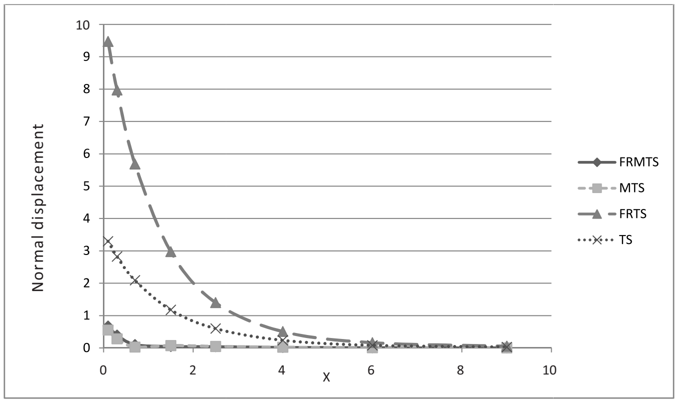



Figure 2

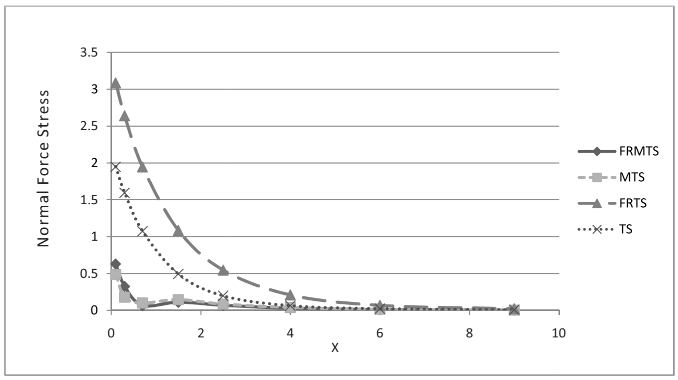



Figure 3

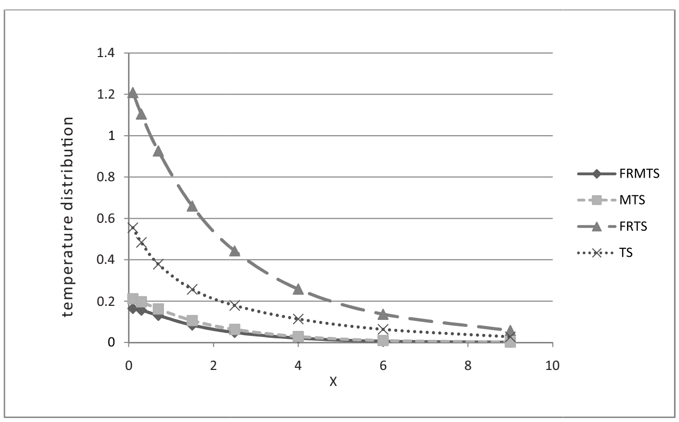



Figure 4

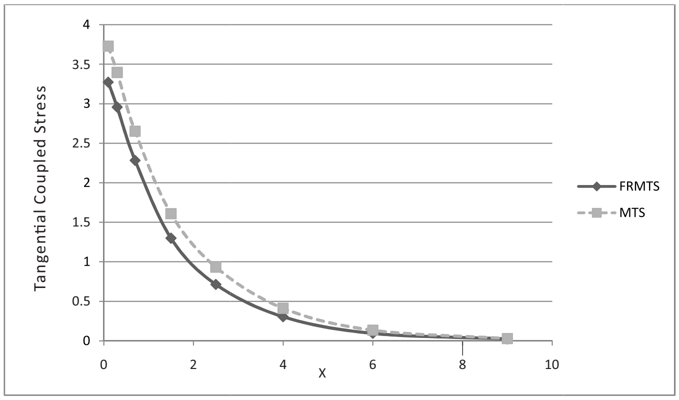



FIGURE 5

more near the point of application of source. For a micropolar thermoelastic medium (FRMTS and MTS), the values of these quantities are less near the application of source. The variations of normal displacement, normal force stress and temperature distribution are shown in Figures 2 to 4 respectively. The variations of tangential couple stress for both medium (FRMTS and MTS) follow similar pattern, which is decreasing in nature. In terms of magnitude there is not much difference in tangential couple stress as shown in Figure 5.

## 7. Conclusion

The values of all the quantities decrease with increase in horizontal distance and approaches to zero. Anisotropy and micropolarity have significant effects on all the quantities. Due to anisotropy and micropolarity, the values of normal displacement, normal force stress and temperature distribution are more near the point of application of mechanical source.

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## ДВОДИМЕНЗИОНИ, ВЛАКНИМА ОЈАЧАН, МИКРОПОЛАРНИ ТЕРМОЕЛАСТИЧНИ ПРОБЛЕМ ЗА ПОЛУПРОСТОР НА КОМЕ ДЕЛУЈУ МЕХАНИЧКЕ СИЛЕ

Резиме. Циљ рада је да се проуче дводимензионе деформације влакнима ојачане микрополарне термоеластичне средине у оквиру *Green–Lindsay*-еве теорије термоеластичности. Механичка сила делује дуж границе полупростора са флуидом и полупростора микрополарне термоеластичне средине ојачане влакнима. Анализа нормалних модова је примењена ради добијања тачних облика компонената померања, напона, температурске расподеле и тангенцијалних спрегнутих напона. Ефекти анизотропије и микрополарности на дате механичке величине су приказане графички.

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