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An approach to the optimization of thin-walled cantilever open section beams

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Abstract

An approach to the optimization of the thin-walled cantilever open section beams subjected to th bending and to the constrained torsion is considered. The problem is reduced to the determination of minimum mass, i.e. minimum cross-sectional area of structural thin-walled I-beam and channel-section beam elements for given loads, material and geometrical characteristics. The area of the cross-section is assumed to be the objective function. The stress constraints are introduced. Applying the Lagrange multiplier method, the equations, whose solutions represent the optimal values of the ratios of the parts of the chosen cross-section, are formed. The obtained results are used for numerical calculation.
Keywords: optimization, thin-walled beams, optimal dimensions, objective function, stress constraints.

1 Introduction

In most structures it is possible to find the elements in which, depending on loading cases and the way of their introductions, the effect of

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constrained torsion is present and its consequences are particularly evident in the case of thin-walled profiles. The earliest development of the theory of thin-walled structures is assosiated whith the beginning of the 20^{th}c. The most prominent contributor to the development of this theory was S.P.Timoshenko [1], who was among the first to publish a number of books on materials strength, theory of elasticity and theory of stability and developed the theory of beams and plates bending. Kollbruner and Hajdin [2, 3] expanded the field of thin-walled structures by a range of their works. V.Z.Vlasov [4, 5] also contributed largely to the theory of thin-walled structures by developing the theory of thin-walled open section beams. Thin-walled open section beams are widely applied due to their low weight in many structures. Thin-walled beams have a specific behavior and that is the reason why their optimization represents a particular problem.

Optimization is a mathematical process through which a set of conditions is obtained giving as a result the maximum or minimum value of a specified function. Among the authors who developed theoretical fundamentals of the optimization method, Fox [6], Brousse [7] and Prager [8] should be given the most prominent place. Also Fletcher [9] and Bertsekas [10] should be mentioned as the authors of some recent developments in optimization approaches. Analyzing the process of designing various types of the structures, it can be noticed that the classical procedure of defining dimensions of a structure based on the theory of the strength of materials provides sufficient, however, not the optimum geometric parameters Many studies have been conducted on the optimization problems, treating the cases where geometric configurations of structures are specified and only the dimensions of members, such as areas of members cross-sections, are determined in order to attain the minimum structural weight or cost. Many authors, Farkas [11] being among them too, applied mathematical problems of the conditional extreme of the function with more variables onto the cross-sectional area of the structure and defined optimum cross-section from the aspect of load and consumption of the material. Then, a series of works appear where the problem of optimization of various cross-sections, such as triangular cross-section [12], I- section [13, 14], channel-section beams [15] are solved by using the Lagrange multiplier method.

The main purpose of this paper is to present one approach to the optimization of a thin-walled I-section and channelsection beams

2 Definition of the problem

During the process of dimensioning of a structure, apart from defining the requested dimensions that are necessary to permit the particular part of the structure to support the applied loads, it is often very important to find the optimal values of the dimensions. The starting points during the formulation of the basic mathematical model are the assumptions of the thin-walled-beam theory, on one side, and the basic assumptions of the optimum design, on the other.

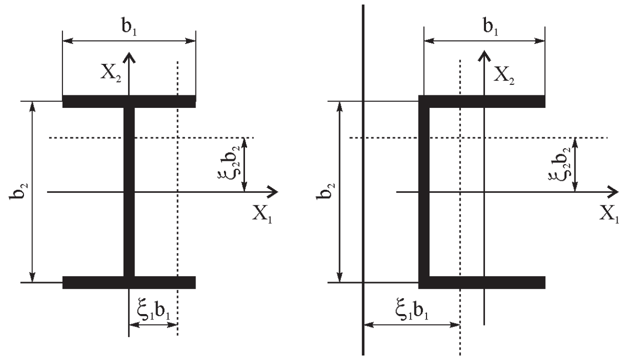
The I-section and the channelsection beams as very often used thin-walled profiles in steel structures are considered in the present paper as the objects of the optimization. The determination of their optimal dimensions is a very important process but not always the simplest one. The aim of the paper is to determine the minimum mass of the whole beam, i.e. the minimum area A of the cross-section of the considered beam for the given loads and material properties.

The formulation is restricted to the stress analysis of thin-walled beams with open sections.

The cross-sections of the considered cantilever beam (Fig. 1) with principal centroidal axes X_i (i= 1, 2) have the axis of symmetry. It is assumed that its flanges have equal widths $b_1 = b_3$, and thicknesses $t_1 = t_3$, and that its web has the width b_2 and thickness t_2. The ratios of thicknesses and widths of flanges and web are treated as not constant quantities.

It is also assumed that the loads are applied in two longitudinal planes, parallel to the principal centroidal axes X_i (i= 1, 2) at the distances $\xi_i b_i$ (i = 1,2) (Fig. 1).If applied in such a way, the loads will cause the bending moments acting in the above mentioned two planes parallel to the longitudinal axis of the beam, and as their consequence the effects of the constrained torsion will appear in the form of the bimoment causing the stresses that depend on the boundary conditions [2, 16].

The aim of the paper is to determine the minimal mass of the beam



(a) I-section (b) channel-section

Figure 1:

or, in other words, to find the minimal cross-sectional area

$$A = A_{\min} \tag{1}$$

for the given loads and material and geometrical properties of the considered beam, while satisfying the constraints.

Formulation of the structural design optimization problem plays an important role in the numerical solution process [6]. A particular choice of the cost function and constraints affect the final solution, and efficiency and robustness of the solution process.

2.1 Objective function

The process of selecting the best solution from various possible solutions must be based on a prescribed criterion known as the objective function. In the considered problem the cross-sectional area will be treated as an objective function and it is obvious from the Fig. 1 that

$$A = \sum b_i t_i, \text{ i = 1,2,3,} \tag{2}$$

or (because $b_1 = b_3$)

$$A = A\,(b_1,\ b_2) = 2\ b_1\ t_1 + b_2\ t_2$$

The ratios of thickness and length of the cross-sectional walls are assumed to be non-constant variables (Fig.1)

$$\mu_i = \frac{t_i}{b_i} \neq const,\ \text{i} = 1{,}2{,}3, \tag{3}$$

where b_i andt_i are widths and thicknesses of the parts of the considered cross-sections.

2.2 Constraints

Only normal stresses will be taken into account in the consideration that follows and the constraints treated in the paper are the stress constraints.

The normal stresses are caused by the bending moments M_{X1} and M_{X2} and by the bimoment B that appears in the case of constrained torsion and they will be denoted as σ_{X1} and σ_{X2} and σ_ω respectively [2, 16].

Bimoment is not a static value, and can not be defined by static equilibrium conditions. In the case when the bending moments are acting in planes parallel to the longitudinal axis (Fig. 1) at the distances $\xi_i b_i (i = 1, 2)$ the bimoment as their consequence will appear and it can be expressed as the function of the bending moments and the eccentricities of their planes $\xi_i b_i (i = 1, 2)$ in the following way [2, 16]

$$B = \xi_1 b_1 M_{X1} + \xi_2 b_2 M_{X2}. \tag{4}$$

For the allowable stress σ_0 the constraint function can be written as

$$\varphi_1 = \varphi\ (\sigma) = \sigma_{X1\max} + \sigma_{X2\max} + \sigma_{\omega\max} \leq \sigma_0. \tag{5}$$

The maximal normal stresses [2, 16] are defined in the form

$$\sigma_{Xi\max} = \frac{M_{Xi}}{W_{Xi}}\ (\text{i} = 1{,}2)\,, \tag{6}$$

$$\sigma_{\omega\max} = \frac{B}{W_\omega}, \tag{7}$$

where $W_{Xi,}$ (i = 1,2) are the section moduli for the principal axes, and W_ω is the sectorial section modulus for the considered cross-section.

After the introduction of (6) and (7) into (5), the constraint function becomes

$$\varphi = \frac{M_{X1}}{W_{X1}} + \frac{M_{X2}}{W_{X2}} + \frac{B}{W_\omega} - \sigma_0 \leq 0. \tag{8}$$

The constraint function (8) is reduced separately to:

- for the I-section:

$$\varphi = \varphi\ (b_1,\ b_2) = 6M_{X1}\frac{1}{t_1 b_1 b_2\left(6 + \dfrac{t_2}{t_1}\dfrac{b_2}{b_1}\right)} +$$

$$3M_{X2}\frac{1}{t_1 b_1^2} + 6B\frac{1}{t_1 b_1^2 b_2} - \sigma_0 \leq 0, \tag{9}$$

  and

- for the channel-section:

$$\varphi = \varphi(b_1,\ b_2) =$$

$$= 6M_{X1}\frac{1}{t_1 b_1 b_2\left(6 + \dfrac{t_2}{t_1}\dfrac{b_2}{b_1}\right)} + 3M_{X2}\frac{1 + \dfrac{t_2}{t_1}\dfrac{b_2}{b_1}}{t_1 b_1^2\left(1 + 2\dfrac{t_2}{t_1}\dfrac{b_2}{b_1}\right)} + \tag{10}$$

$$6B\frac{3 + \dfrac{t_2}{t_1}\dfrac{b_2}{b_1}}{t_1 b_1^2 b_2\left(3 + 2\dfrac{t_2}{t_1}\dfrac{b_2}{b_1}\right)} - \sigma_0 \leq 0.$$

The expressions (11) and (12) represent the constraint functions corresponding to the given stress constraints.

3 Results and discussion

3.1 Analytic solution

One of the most common problems is that of finding maxima or minima (in general, "extrema") of a function. The Lagrange multiplier method [6, 7, 10, 17-19] is a powerful tool for solving this class of problems and represents the classical approach to the constraint optimization. Lagrange multiplier, labeled as λ, measures the change of the objective function with respect to the constraint. The Lagrange multiplier method is a powerful tool for solving this class of problems without the need to explicitly solve the conditions and use them to eliminate extra variables. It is a method for finding the extremum of the function of several variables when the solution must satisfy a set of constraints, and for the analogous problem in the calculus of variations.

Applying this method to the vector depending on two parameters b_i, (i = 1, 2), the system of equations (13) of the form

$$\frac{\partial}{\partial b_i}\left[A\,(b_1, b_2) + \lambda\,\varphi\,(b_1, b_2)\right] = 0,\ \ (\text{ i} = 1{,}2) \tag{11}$$

will be obtained and after the elimination of the multiplier λ, it will become

$$\frac{\partial\, A\,(b_1, b_2)}{\partial b_1}\cdot\frac{\partial\,\varphi\,(b_1, b_2)}{\partial b_2} = \frac{\partial\, A\,(b_1, b_2)}{\partial b_2}\cdot\frac{\partial\,\varphi\,(b_1, b_2)}{\partial b_1}. \tag{12}$$

The beams of the given cross-sections (Fig.1) are the objects of the optimization.

Let the ratio

$$z = b_2/b_1 \tag{13}$$

be the optimal ratio of the parts of the considered cross-section and let

$$\psi = t_2/t_1, \tag{14}$$

be the ratio of the flange and web thicknesses.

After the introduction of the expression (4) for the bimoment into the equations (9) and (10), the equation (12) can be reduced to the equation whose solutions give the optimal values of the ratio (13). The solutions are in the form:

- of the fourth order for the considered I-beam

$$\sum_{k=0}^{k=4} c_k z^k = 0, \tag{15}$$

where the coefficients c_k in (15) are defined by (16)

$$c_0 = -12\,(1+6\xi_1)\,,$$
$$c_1 = 2\left[\psi\,(1+24\xi_1) - 36\xi_2\frac{M_{X2}}{M_{X1}}\right],$$
$$c_2 = 2\psi\left[11\psi\xi_1 + 6\,(3+4\xi_2)\,\frac{M_{X2}}{M_{X1}}\right],$$
$$c_3 = 2\psi^2\left[\psi\xi_1 + (6+11\xi_2)\,\frac{M_{X2}}{M_{X1}}\right],$$
$$c_4 = \psi^3\,(1+2\xi_2)\,\frac{M_{X2}}{M_{X1}}, \tag{16}$$

and

- of the eightth order for the considered channel-section beam

$$\sum_{k=1}^{8} c_k z^k = 0, \tag{17}$$

where the coefficients c_k in (17) are defined by (18)

$$c_0 = -216\,(1+6\xi_1)\,,$$
$$c_1 = -36\left[\psi\,(31+168\xi_1) + 36\xi_2\frac{M_{X_2}}{M_{X_1}}\right],$$
$$c_2 = -12\psi\left[\psi\,(160+669\xi_1) + 504\xi_2\frac{M_{X_2}}{M_{X_1}}\right],$$
$$c_3 = -4\psi^2\left[\psi\,(296+117\xi_1) - 9\,(45-223\xi_2)\,\frac{M_{X_2}}{M_{X_1}}\right],$$
$$c_4 = -2\psi^3\left[\psi\,(64-2363\xi_1) - 18\,(111-13\xi_2)\,\frac{M_{X_2}}{M_{X_1}}\right],$$
$$c_5 = \psi^4\left[16\psi\,(4+165\xi_1) + (3645+4726\xi_2)\,\frac{M_{X_2}}{M_{X_1}}\right],$$

$$c_6 = 8\psi^5 \left[63\psi\xi_1 + 6\left(31 + 55\xi_2\right)\frac{M_{X_2}}{M_{X_1}}\right],$$

$$c_7 = 4\psi^6 \left[8\psi\xi_1 + \left(65 + 126\xi_2\right)\frac{M_{X_2}}{M_{X_1}}\right],$$

$$c_8 = 16\psi^7 \left(1 + 2\xi_2\right)\frac{M_{X_2}}{M_{X_1}}. \qquad (18)$$

It is obvious that the coefficients c_k ($k = 1, 2, \ldots, 6$) depend on the ratio of the bending moments M_{X2}/M_{X1} and on the eccentrities $\xi_1$and ξ_2 of their planes.

The results that follow were obtained by the analytical approach.

3.2 Optimal values $z = b_2/b_1$

From the general case, when bending moments about both principal axes appear simultaneously with the bimoment, some particular cases can be considered, depending on the ratio M_{X2}/M_{X1}.

The optimal ratios (13) obtained from the equations (15) and (17) are calculated for M_{X2}/M_{X1} =0, 0.1, 0.5, 1; $\psi = 0.5, 0.75, 1$ and ξ_1, ξ_2 =0, 0.2, 0.4, 0.6, 0.8, 1.0, or in other way for $0 \leq \xi_1 \leq 1$; $0 \leq \xi_2 \leq 1$.

3.2.1 I-beam

The optimal values of z forM_{X2}/M_{X1} =0 and $\psi = 0.5$, 0.75 and 1.0, as the functions of ξ_1 and ξ_2, are shown in Tables 1, 2 and 3 respectively. The columns in Tables 1-3 are given in a shortened form because the ratios z have the same values for each ξ_2. The highest and the lowest optimal

$\downarrow \xi_2$	$\xi_1 \rightarrow$	0	0.2	0.4	0.6	0.8	1
0		12	2.83	2.46	2.32	2.24	2.19
...							
1		12	2.83	2.46	2.32	2.24	2.19

Table 1: I-beam: Optimal z for $M_{X2}/M_{X1} = 0$, $\psi = 0.5$

$\downarrow \xi_2$	$\xi_1 \rightarrow$	0	0.2	0.4	0.6	0.8	1
0		8	1.89	1.64	1.54	1.49	1.46
...							
1		8	1.89	1.64	1.54	1.49	1.46

Table 2: I-beam: Optimal z for $M_{X2}/M_{X1} = 0$, $\psi = 0.75$

$\downarrow \xi_2$	$\xi_1 \rightarrow$	0	0.2	0.4	0.6	0.8	1
0		6	1.42	1.23	1.16	1.12	1.09
...							
1		6	1.42	1.23	1.16	1.12	1.09

Table 3: I-beam: Optimal z for $M_{X2}/M_{X1} = 0$, $\psi = 1$

M_{X2}/M_{X1}	ψ	z
0.1	0.5	$2 \leq z \leq 2.02$
	0.75	$1.37 \leq z \leq 1.58$
	1	$1.04 \leq z \leq 1.32$
0.5	0.5	$1.02 \leq z \leq. 1.72$
	0.75	$0.82 \leq z \leq 1.20$
	1	$0.69 \leq z \leq 0.93$
1	0.5	$0.75 \leq z \leq 1.59$
	0.75	$0.59 \leq z \leq 1.11$
	1	$0.51 \leq z \leq 0.86$

Table 4: I-beam: Optimal $z = b_2/b_1$ for $M_{X2}/M_{X1} = 0.1, 0.5, 1$ and $\psi = 0.5; 0.75; 1$

values $z = b_2/b_1$ for $M_{X2}/M_{X1} = 0.1, 0.5, 1$ are given in a shortened form in Table 4.

From the Tables 1-4 it can be concluded that the values of z are decreasing when the ratio $\psi = t_2/t_1$ is increasing and that they are decreasing when the load ratio is increasing.

3.2.2 Channel-section beam

The relations between z and the eccentricities ξ_1 and ξ_2 for M_{X2}/M_{X1} =0, ψ = 0.5, 0.75 and 1 are shown in Tables 5, 6 and 7, respectively. From

$\downarrow \xi_2$	$\xi_1 \rightarrow$	0	0.2	0.4	0.6	0.8	1
0		12	3.60	3.12	2.94	2.84	2.78
...							
1		12	3.60	3.12	2.94	2.84	2.78

Table 5: Channel-section beam: Optimal z for M_{X2}/M_{X1} =0, ψ = 0.5

$\downarrow \xi_2$	$\xi_1 \rightarrow$	0	0.2	0.4	0.6	0.8	1
0		8	2.34	2.08	1.96	1.89	1.85
...							
1		8	2.34	2.08	1.96	1.89	1.85

Table 6: Channel-section beam: Optimal z for for M_{X2}/M_{X1} =0, ψ = 0.75

$\downarrow \xi_2$	$\xi_1 \rightarrow$	0	0.2	0.4	0.6	0.8	1
0		6	1.80	1.56	1.47	1.42	1.40
...							
1		6	1.80	1.56	1.47	1.42	1.40

Table 7: Channel-section beam: Optimal z for for M_{X2}/M_{X1} =0, ψ = 1

the results presented in Tables 5-7, it is obvious that the quantity z for ratio $M_{X2}/M_{X1} = 0$, does not depend on the eccentricity ξ_2, i.e. the ratios z have the same values for each ξ_2. That is why the relations between z and the eccentrities ξ_1 and ξ_2 for M_{X2}/M_{X1} =0, ψ = 0.75 and ψ = 1 are presented in Tables 1, 2 and 3, respectively, in a shortened form.

Like in the case of the I-section, the optimal values $z = b_2/b_1$ for the channel-section beam for ratios M_{X2}/M_{X1}= 0.1, 0.5, 1 are given in a shortened form in Table 8.

M_{X2}/M_{X1}	ψ	z
0.1	0.5	$2.54 \leq z \leq 2.62$
	0.75	$2.02 \leq z \leq.\ 1.74$
	1	$1.32 \leq z \leq 1.68$
0.5	0.5	$1.41 \leq z \leq.\ 2.22$
	0.75	$1.10 \leq z \leq 1.54$
	1	$0.93 \leq z \leq 1.18$
1	0.5	$1.07 \leq z \leq 2.10$
	0.75	$0.84 \leq z \leq 1.45$
	1	$0.71 \leq z \leq 1.12$

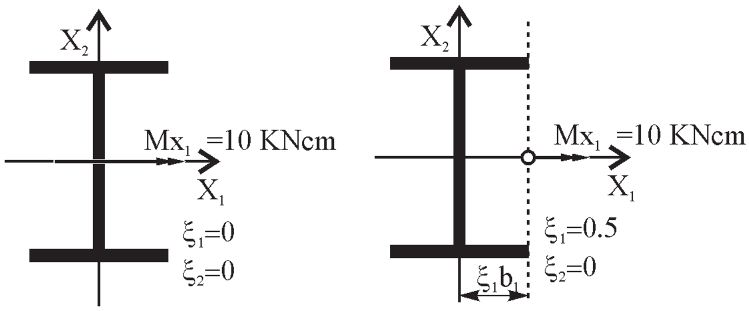
Table 8: Channel-section beam: Optimal $z = b_2/b_1$ for M_{X2}/M_{X1}= 0.1, 0.5, 1 and ψ = 0.5; 0.75; 1

From the Tables 5-8 it can be also concluded that the values of z are decreasing when the ratio $\psi = t_2/\ t_1$ is increasing and that they are decreasing when the load ratio is increasing.

3.3 The loading cases

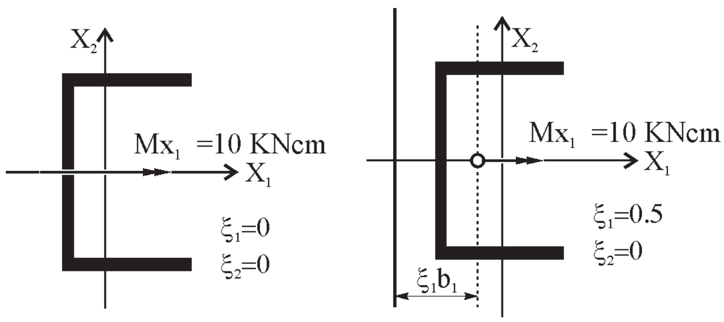
From the general case, when bending moments about both principal axes appear simultaneously with the bimoment, some particular cases can be considered depending on the loading case.

In this section an I-beam and channel-section beam are fixed at one end and subjected to the concentrated bending moment $M_{X1} = 10$ kNcm ($M_{X2} = 0$) at the free end of the beam in two ways (Figs.2 and 3) as: a) Loading case 1: $\xi_1 = \xi_2 = 0$ and b) Loading case 2: $\xi_1 = 0.5$, $\xi_2 = 0$.



(a) Loading case 1 (b) Loading case 2

Figure 2: I-section



(a) Loading case 1 (b) Loading case 2

Figure 3: Channel-section

4 Numerical example

As the numerical example, considered cantilever beams with the lengths l=150 cm, fixed at one end are subjected to the bending moments M_{X1} = 10 kNcm, $M_{X2} = 0$.

The initial cross-sectional geometrical characteristics are calculated taking into account the initial dimensions of the I-section and the channel-section. It is assumed that considered sections have the same initial cross-sectional geometrical characteristics: b_1 =5.175 cm, b_2 =9.2 cm,t_1 = 0.8 cm, t_2 =0.65 cm. For the given loads (Figs. 2 and 3) and the defined geometry of the profile, the initial stresses are calculated.

4.1 Determination of the minimum cross-sectional area

The problem is considered in two ways:

1. The optimal dimensions of the cross-section $b_{1optimal}$ and $b_{2optimal}$ are obtained by equalizing the "Initial" and the "Optimal area" ($_{initial} = _{optimal}$) and by using the calculated optimal relation z. In that case the normal stress lower than the initial one is obtained ($\sigma_{optimal}$¡$\sigma_{initial}$). It represents the model used for the control or **Optimal model no.1**.

2. From the condition prescribing that the stresses must be lower than the allowable one, i.e. the "Initial stress", the optimal values $b_{1optimal}$ and $b_{2optimal}$ are obtained by using the calculated optimal relation $z$and by comparing the stress defined by the optimal geometrical characteristics to the "Initial stress". It represents the **Optimal model no.2**. Starting from the optimal cross-sectional dimensions ($b_{1optimal}$ and $b_{2optimal}$), the optimal-minimum cross-sectional area $_{min}$ is calculated for each loading case and the results including the saved mass of the material are given in Tables 9 and 10.

From the Tables 9 and 10 it can be seen that greater saved mass of the material was obtained for I-section than for channel-section. Also, for all loading cases the level of stresses is decreased in the Optimal model no.1

Load. case	z_{init}	z_{optim}	σ_{init} $[kN/cm^2]$	σ_{optim_1} $[kN/cm^2]$	σ_{optim_2} $[kN/cm^2]$	$A_{init}=$ A_{optim_1} $[cm^2]$	$A_{min}=$ A_{optim_2} $[cm^2]$	Saved mass [%]
1	1.67	7.38	0.20	0.19	0.20	14.26	12.60	11.64
2		1.46	0.94	0.94	0.94		14.23	0.22

Table 9: I-beam: Normal stresses and saved mass

Load. case	z_{init}	z_{optim}	σ_{init} $[kN/cm^2]$	σ_{optim_1} $[kN/cm^2]$	σ_{optim_2} $[kN/cm^2]$	$A_{init}=$ A_{optim_1} $[cm^2]$	$A_{min}=$ A_{optim_2} $[cm^2]$	Saved mass [%]
1	1.78	7.38	0.22	0.17	0.22	14.26	12.80	10.25
2		1.84	0.84	0.84	0.84		14.26	0.12

Table 10: Channel-section-beam: Normal stresses and saved mass

with the area of the cross-section having the same value as in the "Initial model" and the saved mass of material is increased with respect to the initial stress limits in the Optimal model no.2 where the area is smaller than the initial one. The calculation showed that the maximum saved material is obtained in the Loading case 1 and the minimum in the Loading case 2 for both shapes of cross-sections. This allows to conclude that if the distance of the loading plane from the shearing plane is increased the optimization of the cross-section is less necessary to be done.

5 Conclusions

The paper presents one approach to the optimization of the thin-walled open section beams, loaded in a complex way, using the Lagrange multiplier method.

Accepting the cross-sectional area as the objective function and the stress constraints as the constrained function, it is possible to find the optimal relation between the dimensions of the web and the flanges of the considered thin-walled profiles in a very simple way.

In addition to the general case, some particular loading cases are considered. As the result of the calculation the modified constrained functions are derived as the polynomials of the fourth and eight order.

Particular attention is directed to the calculation of the saved mass using the proposed analytical approach. It is also possible to calculate the saved mass of the used material for different loading cases.

The aim of the paper is the optimization of thin-walled elements subjected to the complex loads, and it may be concluded that the paper gives the general results permitting the derivation of the expressions that can be recommended for technical applications.

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Jedan pristup optimizaciji tankozidih konzolnih nosaca otvorenih poprecnih preseka

U ovom radu je razmatran jedan pristup optimizaciji tankozidih konzolnih konstrukcionih elemenata otvorenih poprecnih preseka izloženih savijanju i ogranicenoj torziji. Problem je redukovan na odredjivanje minimalne mase, t.j. minimalne povrsine konstrukcionih tankozidih elemenata oblika I i U-profila za zadata opterecenja, materijal i geometrijske karakteristike. Površina poprecnog preseka nosaca je izabrana za funkciju cilja. Uvedeno je naponsko ogranicenje. Primenom metode Lagranžovog množitelja formirane su jednacine cija rešenja predstavljaju optimalne odnose dimenzija poprecnog preseka izabranog oblika. Dobijeni rezultati su iskorišceni pri numerickom proracunu.

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