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Thermodynamical modeling of viscous dissipation in magnetohydrodynamic flow

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Abstract

A genuine variational principle developed by Gyarmati, in the field of thermodynamics of irreversible processes unifying the theoretical requirements of technical, environmental and biological sciences is employed to study the effects of viscous dissipation and stress work on MHD forced convection flow adjacent to a non-isothermal wedge. The velocity and temperature distributions inside the boundary layer are considered as simple polynomial functions and the variational principle is formulated. The Euler-Lagrange equations are reduced to simple polynomial equations in terms of boundary layer thicknesses. The values of skin friction coefficient and the Nusselt number are presented for various values of wedge angle parameter m, wall temperature exponent $2m$, magnetic parameter ξ, Prandtl number (Pr) and Eckert number (Ec). The present results are compared with known available results and the comparison is found to be satisfactory and the present study establishes the fact that the accuracy is remarkable.

Keywords: Magnetohydrodynamics, stress work, viscous dissipation, skin friction, heat transfer, variational principle, non-isothermal wedge.

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Nomenclature

x	coordinate measuring distance along the plate.
y	coordinate measuring distance normal to plate.
u	velocity component in the x-direction.
v	velocity component in the y-direction.
T	temperature of fluid.
T_0	temperature of plate.
T_∞	temperature of ambient fluid.
U_∞	free stream velocity.
d_1	hydrodynamical boundary layer thickness.
d_2	thermal boundary layer thickness.
P_{12}	momentum flux.
J_q	thermal flux.
L	Lagrangian function.
L_s, L_λ	conductivities.
ν	kinematic viscosity.
d_1^*, d_2^*	non dimensional boundary layer thicknesses.
α	thermal diffusivity.
Pr	Prandtl number.
δ	symbol for variation.
σ	entropy production.
Ψ^*, Φ^*	local dissipation potentials in energy picture.
τ_w^*	Non-dimensional skin friction.
Nu_l	Nusselt number.
ρ	density of the fluid.
κ	electrical conductivity.
Ec	Eckert number.
m	pressure gradient parameter.
Ω	total angle of the wedge.

1 Introduction

The problem of magnetohydrodynamic (MHD) incompressible, steady viscous flow has many important practical engineering applications in areas such as MHD power generator designs, design for cooling of nuclear reactors, construction of heat exchangers, installation of nuclear accelerators and blood flow measurement techniques. Magnetohydrodynamic forced convection flow over a wedge is of considerable interest to the technical field due to its frequent role in industrial and technological applications.

According to the boundary layer theory, the velocity increases from 0 at the wall surface to the free stream velocity at the edge of the boundary layer and thus velocity gradient may be appreciable even if the viscosity is very small. Analyzing the shear stress and heat transfer is one of the most important objectives in the solution of the boundary layer equations. The governing equations of boundary layer flow become nonsimilar due to the presence of a magnetic field or variable fluid properties. The boundary layer flow of a laminar incompressible electrically conducting fluid over a wedge in the presence of transverse magnetic field has been investigated by many researchers.

Watanabe[14, 15] reduced the momentum partial differential equation to ordinary differential equation by employing difference-differential method and obtained solution in a form of integral equation. The solution for the heat transfer of an electrically conducting fluid over a semi-infinite flat plate in the presence of a transverse magnetic field was studied by Watanabe and Pop[18] by means of difference-differential method. The problems of stagnation point and asymmetric flow were investigated by Raptis[11] and Chamkha[1]. Watanabe[15] analyzed the magnetohydrodynamic boundary layer flow along a wedge and he has not considered the energy equation. Hossain [8] treated the viscous and joule heating effects on MHD free convection flow with variable plate temperature. Watanabe and Pop[17] solved the MHD free convection flow over a wedge in the presence of a transverse magnetic field. Yih[19] presented an analysis of forced convection boundary layer flow over a wedge with uniform suction and blowing whereas Watanabe[16] investigated the behaviour of the boundary layer over a wedge with suction/injection in forced flow.

Yih[20] extended the work of Watanabe and Pop[18] to investigate the heat transfer characteristic in MHD forced convection flow adjacent to a non-isothermal wedge in the presence of transverse magnetic field. An approximate numerical solution for thermal stratification on MHD steady laminar boundary layer flow over a wedge with suction or injection was investigated by Anjalidevi[2]. The MHD boundary layer flow over a flat plate for two cases, a uniform free stream velocity and a uniform hydrostatic pressure was investigated by Sam Lawrence and Nageswara Rao[12]. Lin and Lin[7] proposed a similarity solution method that provides accurate solutions for laminar forced convection heat transfer for either an isothermal surface or a uniform flux boundary to fluid of any Prandtl number. Chamkha[1] studied steady two dimensional mixed convection flows of an electrically conducting and heat absorbing fluid near stagnation point on a semi infinite vertical permeable surface at arbitrary surface heat flux variations in the presence of a magnetic field. Motivated by the work of the above mentioned authors, the effect of viscous dissipation and stress work on the MHD boundary layer flow over a wedge are analyzed here.

2 The formulation of governing principle of dissipative processes (GPDP)

With the help of boundary layer approximations for the conservation equations of mass, momentum and energy for steady, two dimensional, laminar flow, with constant physical properties with viscous dissipation effects are

$$u_x + v_y = 0, \qquad \text{(mass)} \quad (1)$$

$$uu_x + vu_y = \nu u_{yy} + U_\infty (U_\infty)_x + (\kappa B_0^2)(U_\infty - u)/\rho, \qquad \text{(momentum)} \quad (2)$$

$$uT_x + vT_y = \alpha T_{yy} + (\nu/C_p)(u_y^2) - (u/C_p)[(U_\infty (U_\infty)_x) + ((\kappa B_0^2 U_\infty)/\rho)] + (\kappa B_0^2 u^2)/(\rho C_p). \qquad \text{(energy)} \quad (3)$$

where subscript indicates partial differentiation, u, v, T, U_∞, T_∞, B_0 and C_p represent the velocity component in x-direction, velocity component in y-direction, temperature inside the boundary layer, free stream velocity and the free stream temperature, externally imposed magnetic field in the y-direction and specific heat at constant pressure respectively. The symbols ν, α, ρ, κ represent kinematic viscosity, thermal diffusivity, density and electrical conductivity of the fluid respectively. The initial and boundary conditions of the system are

$$\left.\begin{array}{llll} y = 0: & u = 0,\ v = 0, & T - T_0(x) = Ax^{2m}, & \text{(uniform)} \\ y \to \infty: & u = U_\infty = Cx^m, & T = T_\infty, & \text{(uniform)} \end{array}\right\}$$

where $m = \beta/(2-\beta)$, is the Hartree pressure gradient parameter which corresponds to $\beta = \Omega/\pi$ for a total angle of the wedge. The surface of the wedge is maintained at a variable wall temperature proportional to x^{2m}. In equations(4),

A and C are positive numbers. In this study, the induced magnetic field and the Hall effect are neglected.

Gyarmati[5, 6] introduced a genuine variational principle called the "Governing Principle of Dissipative Processes" (GPDP) which is given in its universal form

$$\delta \int_V [\sigma - \Psi - \Phi] \, dV \quad = 0. \tag{4}$$

The principle(5) is valid for linear, quasi-linear and certain types of non-linear transport processes at any instant of time under constraints that the balance equations

$$\rho \overset{\bullet}{a}_i + \nabla . \vec{J}_i \quad = \sigma_i \qquad\qquad (i = 1, 2, 3, ...f) \tag{5}$$

are satisfied. In equation(5), σ is the entropy production, Ψ and Φ are dissipation potentials and V is the total volume of the thermodynamic system. In equation(6), $\vec{J}_i$ is the flux and σ_i is the source density of i^{th} extensive transport quantity a_i. σ can always be written in the bilinear form

$$\sigma = \sum_{i=1}^{f} \vec{J}_i \, . \, \vec{X}_i \geq 0, \tag{6}$$

where $\vec{J}_i$ and $\vec{X}_i$ are fluxes and forces respectively. According to Onsager's linear theory[9, 10] the fluxes are linear functions, that is

$$\vec{J}_i = \sum_{k=1}^{f} L_{ik} \, \vec{X}_k, \qquad\qquad (i = 1, 2, 3, ...f) \tag{7}$$

or alternatively

$$\vec{X}_i = \sum_{k=1}^{f} R_{ik} \, \vec{J}_k \, . \qquad\qquad (i = 1, 2, 3, ...f) \tag{8}$$

The constants L_{ik} and R_{ik} are conductivities and resistances and they satisfy the reciprocal relations[9, 10]

$$L_{ik} = L_{ki} \text{ and } \quad R_{ik} = R_{ki}, \qquad\qquad (\, i, k = 1, 2, 3...f \,). \tag{9}$$

The matrices of L_{ik} and R_{ik} are mutual reciprocals. That is

$$\sum_{m=1}^{f} L_{im} R_{mk} = \sum_{m=1}^{f} L_{mk} R_{im} = \delta_{ik}, \qquad (i, k = 1, 2, 3, ...f) \tag{10}$$

where δ_{ik} is the Kronecker delta. The local dissipation potentials Ψ and Φ are defined as[9, 10]

$$\Psi(\vec{X}, \vec{X}) = (1/2)(\sum_{i,k=1}^{f} L_{ik} \vec{X_i} \, . \, \vec{X_k}) \geq 0, \tag{11}$$

$$\Phi(\vec{J}, \vec{J}) = (1/2)(\sum_{i,k=1}^{f} R_{ik} \vec{J_i} \, . \, \vec{J_k}) \geq 0. \tag{12}$$

Since in the case of transport processes $\vec{X_i}$ can be generated as gradients of certain "Γ" variables, it is written as

$$\vec{X_i} = \nabla \Gamma_i. \tag{13}$$

The principle(5) with the help of equations(7),(10),(12),(13) and (14), takes the form

$$\delta \int_V \left[\sum_{i=1}^{f} \vec{J_i} \, .\nabla \Gamma_i - (1/2) \sum_{i,k=1}^{f} L_{ik} \nabla \Gamma_i . \nabla \Gamma_k - (1/2) \sum_{i,k=1}^{f} R_{ik} \vec{J_i} \, . \, \vec{J_k} \right] dV = 0. \tag{14}$$

The principle (5) is also given in energy picture as[6]

$$\delta \int_V [T\sigma \; - \; \Psi^* \; - \Phi^*] dV = 0. \tag{15}$$

Here Tσ is the energy dissipation and the dissipation potentials Ψ^* and Φ^* are given by

$$\Psi^* \; = \; T\Psi \qquad \text{and} \qquad \Phi^* \; = \; T\Phi. \tag{16}$$

It is found that GPDP in energy picture given by equation(16) is always advantageous for dealing with thermohydrodynamic systems. This variational principle has already been applied for various dissipative systems and was established as the most general and exact principle of macroscopic continuum physics. For the description of viscous flow systems, Vincze[13] used the GPDP to derive the equation of thermohydrodynamics. Many other variational principles have already been shown as partial forms of Gyarmati's principle.

The balance equations of the system play a central role in the formulation of Gyarmati's variational principle and hence the governing equations (1)-(3) are written in the balance form as

$$\nabla\cdot \vec{V} = 0, \qquad (\vec{V}= iu + jv). \tag{17}$$

$$\rho(\vec{V}\cdot\nabla)\ \vec{V} +\nabla\cdot \overline{\overline{P}} = (\kappa B_0^2)[U_\infty - (\vec{i}\cdot\vec{V})], \tag{18}$$

$$\rho C_p(\vec{V}\cdot\nabla)T + \nabla\cdot \vec{J_q} = \mu(u_y^2) - (\vec{i}\cdot\vec{V})[\rho U_\infty(U_\infty)_x + \kappa B_0^2 U_\infty] + \kappa B_0^2(\vec{i}\cdot\vec{V})^2. \tag{19}$$

These equations represent the mass, momentum and energy balances respectively. In equation(19) $\overline{\overline{P}}$ denotes the pressure tensor which can be decomposed as [5]

$$\overline{\overline{P}} = p\ \overline{\overline{\delta}} + \overset{\circ}{\overline{\overline{P}}}{}^{vs}, \tag{20}$$

where p is the hydrostaticpressure, $\overline{\overline{\delta}}$ is the unit tensor, and $\overset{\circ}{\overline{\overline{P}}}{}^{vs}$ is the symmetrical part of the viscous pressure tensor, whose trace is zero. In the study of heat transfer and fluid problems, the energy picture of Gyarmati's principle is always advantageous over entropy picture. Therefore, we use the energy dissipation $T\sigma$ instead of entropy production σ. In the energy picture (16), the energy dissipation for the present system is given by [5]

$$T\sigma = -J_q(\partial lnT/\partial y) - P_{12}(\partial u/\partial y) \tag{21}$$

the heat flux J_q and P_{12} the only component of momentum flux $\overset{\circ}{\overline{\overline{P}}}{}^{vs}$, satisfy the conservative relations connecting the independent fluxes and forces as

$$J_q = -L_\lambda(\partial lnT/\partial y), \text{ and } \quad P_{12} = -L_s(\partial u/\partial y). \tag{22}$$

Here $L_\lambda = \lambda T$ and $L_s = \mu$ where λ and μ are the thermal conductivity and viscosity respectively. It is well known that lnT is the proper state variable instead of T when the principle assumes energy picture[5]. With the help of equation(23) the dissipation potentials in the energy picture are found as follows.

$$\Psi^{\star} = (1/2)[L_\lambda(\partial lnT/\partial y)^2 + L_s(\partial u/\partial T)^2], \tag{23}$$

$$\Phi^{\star} = (1/2)[R_\lambda {J_q}^2 + R_s {P_{12}}^2], \tag{24}$$

where $L_\lambda = R_\lambda^{-1}$ and $L_s = R_s^{-1}$. Using the equations(22), (23), (24) and (25) Gyarmati's variational principle in the energy picture(16) is formulated in the following form

$$\delta \int_0^l \int_0^\infty [-J_q(\partial lnT/\partial y) - P_{12}(\partial u/\partial y) - (L_\lambda/2)(\partial lnT/\partial y)^2$$
$$- (L_s/2)(\partial u/\partial y)^2 - (R_\lambda/2){J_q}^2 - (R_s/2)\, P_{12}^2]\, dydx = 0, \tag{25}$$

in which l is the representative length of the surface.

3 Solution procedure

It is considered that the system of two dimensional MHD laminar, inviscid potential flow past an unlimited wedge placed symmetrically in a stream with apex at the origin where x and y are coordinates measured along and normal to the surface respectively. The main stream velocity and the wall temperature are assumed to vary as power functions of distance from the start of the boundary layer respectively as

$$U_\infty = Cx^m, \;\; T_0 - T_\infty = Ax^{2m}, \tag{26}$$

where C and A are constants and the exponent m is connected with the apex angle $\pi\beta$ by the relation

$$m = \beta/(2-\beta) \quad \text{or} \quad \beta = 2m/(m+1). \tag{27}$$

In equation(27), T_0 is the temperature of the surface and T_∞ is the free stream temperature respectively. Here the analysis is carried out for the entire range of realistic flow, that means when $0 \leq m < \infty$ or $0 \leq \beta < 2$. The velocity and temperature fields in their respective boundary layer regions are suitably described by the following functions

$$\left.\begin{aligned} u/U_\infty &= 3y/d_1 - 3y^2/d_1^2 + y^3/d_1^3 \quad (y < d_1), \\ u &= U_\infty \qquad\qquad\qquad\qquad (y \geq d_1), \\ (T - T_\infty)/(T_0 - T_\infty) &= 1 - 3y/2d_2 + y^3/2d_2^3 \qquad (y < d_2), \\ T &= T_\infty. \qquad\qquad\qquad\qquad (y \geq d_2) \end{aligned}\right\} \tag{28}$$

Where d_1 and d_2 are the momentum and thermal boundary layer thicknesses respectively. The velocity and thermal profiles (29) satisfy the following compatibility conditions

$$\left.\begin{aligned} &y = 0, \quad u = 0, \quad v = 0, \quad T = T_0(x), \quad T_y = 0, \\ &y = d_1, \quad u = U_\infty, \quad u_y = 0, \quad u_{yy} = 0, \\ &y = d_2, \quad T = T_\infty, \quad T_y = 0, \end{aligned}\right\} \tag{29}$$

The smooth fit boundary conditions $u_y = 0$ and $T_y = 0$ correspond to $P_{12} = 0$ and $J_q = 0$ at the respective edges of the boundary layers. Here d_1 and d_2 are unknown parameters and they are to be determined by the present thermodynamic analysis.

The transverse velocity component v is obtained from the mass balance equation(1) as

$$\begin{aligned} v &= (U_\infty)[(3y^2/2d_1^2 - 2y^3/d_1^3 + 3y^4/4d_1^4)d_1'] \\ &\quad +(U_\infty)[(-3y^2/2d_1 + y^3/d_1^2 - y^4/4d_1^3)(m/x)]. \end{aligned} \tag{30}$$

The velocity and temperature functions (29) are substituted in the momentum and energy balance equations(1)-(3), and on direct integration with respect to y with the help of smooth fit boundary conditions the fluxes P_{12} and J_q are obtained respectively. The expression for P_{12} remains the same for any Prandtl number Pr. But the energy flux J_q assumes different expression for $Pr \leq 1$ and$Pr \geq 1$ respectively. When $Pr \leq 1$ the expression for J_q in the range $d_1 \leq y \leq d_2$ is obtained first and the expression for J_q in the range $0 \leq y \leq d_1$ is determined subsequently by matching the J_q expressions of the two regions at the interface. The expressions for momentum and energy fluxes P_{12} and J_q are as follows.

$$
\begin{aligned}
-P_{12}/L_s \;=\; & (U_\infty/d_1) + (mU_\infty^2/\nu x)[53d_1/160 - y + 3y^3/2d_1^2 \\
& -3y^4/2d_1^3 + 3y^5/4d_1^4 - y^6/4d_1^5 + y^7/28d_1^6] + (U_\infty^2 d_1'/\nu) \\
& [9/160 - 3y^3/2d_1^3 + 3y^4/d_1^4 - 9y^5/4d_1^5 + 3y^6/4d_1^6 \\
& -3y^7/28d_1^7] + [(\kappa B_0^2 U_\infty)/(\nu\rho)] \\
& [3y^2/2d_1 - y^3/d_1^2 + y^4/4d_1^3 - y + 2d_1/10], \quad (0 \le y \le d_1). \quad (31)
\end{aligned}
$$

$$
\begin{aligned}
-J_q/L_\lambda \;=\; & [Pr(U_\infty)(T_0 - T_\infty)/\nu][3y^3/2d_1d_2^2 - 9y^5/10d_1d_2^4 \\
& -9y^4/8d_1^2d_2^2 + 3y^6/4d_1^2d_2^4 + 3y^5/10d_1^3d_2^2 - 3y^7/14d_1^3d_2^4 \\
& +3d_1^2/40d_2^2 - 3d_1^4/280d_2^4 - 3/8]d_2' + [mPr(U_\infty)(T_0 - T_\infty)/\nu x] \\
& [3y^2/d_1 - 9y^3/4d_1d_2 + 3y^5/20d_1d_2^3 - 2y^3/d_1^2 + 15y^4/8d_1^2d_2 \\
& - y^6/4d_1^2d_2^3 + y^4/2d_1^3 - 21y^5/40d_1^3d_2 + 5y^7/56d_1^3d_2^3 \\
& + d_1/2 + 3d_1^2/20d_2 - 4d_1^4/35d_2^3 - 9d_2/8] + [Pr(U_\infty)(T_0 - T_\infty)/\nu] \\
& \times [-3y^3/4d_1^2d_2 + 9y^5/20d_1^2d_2^3 + 3y^4/4d_1^3d_2 - y^6/2d_1^3d_2^3 \\
& - 9y^5/40d_1^4d_2 + 9y^7/56d_1^4d_2^3 + 9d_1/40d_2 - 31d_1^3/280d_2^3]d_1' \\
& + [U_\infty^2 Pr/C_p][-9y/d_1^2 + 18y^2/d_1^3 - 18y^3/d_1^4 + 9y^4/d_1^5 - 9y^5/5d_1^6
\end{aligned}
$$

$$+ 9/5d_1] + [U_\infty^3 Pr/\nu x C_p][\xi(3y^2/2d_1 - 4y^3/d_1^2 + 19y^4/4d_1^3$$
$$- 3y^5/d_1^4 + y^6/d_1^5 - y^7/7d_1^6 - 3d_1/28) + m(3y^2/2d_1$$
$$- y^3/d_1^2 + y^4/4d_1^3 + d_1/4 - d_2)], \ (0 \le y \le d_1), \quad (Pr \le 1). \quad (32)$$

$$\begin{aligned} -J_q/L_\lambda &= [Pr(U_\infty)(T_0 - T_\infty)/\nu][3y^2/4d_2^2 - 3y^4/8d_2^4 - 3/8]d_2' \\ &+[mPr(U_\infty)(T_0 - T_\infty)/\nu x][2y - 3y^2/4d_2 - y^4/8d_2^3 \\ &-9d_2/8] + [mU_\infty^3 Pr/\nu x C_p][y - d_2].(d_1 \le y \le d_2), (Pr \le 1).(33) \end{aligned}$$

$$\begin{aligned} -J_q/L_\lambda &= [Pr(U_\infty)(T_0 - T_\infty)/\nu][3y^3/2d_1d_2^2 - 9y^5/10d_1d_2^4 - 9y^4/8d_1^2d_2^2 \\ &+3y^6/4d_1^2d_2^4 + 3y^5/10d_1^3d_2^2 - 3y^7/14d_1^3d_2^4 - 3d_2/5d_1 + 3d_2^2/8d_1^2 \\ &-3d_2^3/35d_1^3]d_2' + [mPr(U_\infty)(T_0 - T_\infty)/\nu x][3y^2/d_1 - 9y^3/4d_1d_2 \\ &+3y^5/20d_1d_2^3 - 2y^3/d_1^2 + 15y^4/8d_1^2d_2 - y^6/4d_1^2d_2^3 + y^4/2d_1^3 \quad (34) \end{aligned}$$

The prime indicates the differentiation with respect to x. Using the expressions of P_{12} and J_q along with the velocity and temperature functions(29) the variational principle (26) is formulated independently for $Pr \le 1$ and $Pr \ge 1$ respectively. After performing the integration with respect to y one can obtain the variational principle in the following forms respectively

$$\delta \int_0^l L_1[d_1, d_2, d_1', d_2']dx = 0, \quad (Pr \le 1) \text{ and} \quad (35)$$

$$\delta \int_0^l L_2[d_1, d_2, d_1', d_2']dx = 0. \quad (Pr \ge 1) \quad (36)$$

The variational principles (36) and (37) are found identical when $d_1 = d_2$. Accordingly, the Euler-Lagrange equations are

$$(\partial L_{1,2}/\partial d_1) - (d/dx)(\partial L_{1,2}/\partial d_1') = 0, \tag{37}$$

$$\text{and} \quad (\partial L_{1,2}/\partial d_2) - (d/dx)(\partial L_{1,2}/\partial d_2') = 0. \quad (Pr \leq 1, Pr \geq 1) \tag{38}$$

The equations (38) and (39) are second order ordinary differential equations in terms of d_1 and d_2 respectively. The procedure for solving equations (38) and (39) can be considerably simplified by introducing the non dimensional boundary layer thicknesses d_1^* and d_2^* given by

$$d_1 = d_1^* \sqrt{\nu x/U_\infty} \quad \text{and} \quad d_2 = d_2^* \sqrt{\nu x/U_\infty}. \tag{39}$$

The variational principles (36) and (37) subject to transformations (40) and the resulting Euler-Lagrange equations are obtained as simple polynomial equations

$$(\partial L_{1,2}/\partial d_1^*) = 0, \tag{40}$$

$$(\partial L_{1,2}/\partial d_2^*) = 0. \qquad (Pr \leq 1, Pr \geq 1) \tag{41}$$

The coefficients of these equations(41) and (42) depend on the independent parameters Pr, m and ξ and Ec, where

$$Re = (U_\infty x/\nu), \text{ (Renolds number)}$$

$$Pr = (\nu/\alpha), \text{ (Prandtl number)}$$

$$Ec = U_\infty^2/\{C_p[T_o(x) - T_\infty]\}, \text{ (Eckert number)}$$

$$\xi = (\kappa B_0^2 x)/(\rho U_\infty). \text{ (Magnetic parameter)}$$

Equation(41) is a simple polynomial equation in terms of boundary layer thickness whose coefficients depend on the wedge angle parameter m, and the magnetic parameter ξ. This equation is solved easily for any given combinations of m and ξ and corresponding hydrodynamical boundary layer thickness d_1^* is obtained as the only positive root. The polynomial equation(42) is solved for the values of Pr, m, ξ and Ec and it is found that for any value of Pr there corresponds only one real root d_2^*.

4 Results and discussions

After getting d_1^* and d_2^* for given values of Pr, m, ξ and Ec the local shear stress values and local heat transfer values are calculated with the help of the following relations respectively.

$$\tau_w^* = \sqrt{\nu x/U_\infty^3}\ (-P_{12}/L_s)_{y=0}, \tag{42}$$

$$N_{ul} = \sqrt{\nu x/U_\infty(T_0 - T_\infty)^2}\ (-J_q/L_\lambda)_{y=0}. \tag{43}$$

The main results of engineering interest are skin friction(shear stress) and wall heat transfer(Nusselt Number) and hence these two important characteristics of the problem are analyzed here. Tables 1 and 2 represent the values of skin friction for various values of m when ξ=0 and for various values of ξ when m=1 respectively. In order to verify the accuracy of the obtained results by using the present technique, the obtained numerical results are compared with Cebeci and Bradshaw[4], Arial[3], Lin and Lin[7] and Yih[20]. As given in Table 1, it is evidently observed that when the values of m increase, the values of skin friction also increase. When the wedge becomes a flat plate (m=0) the surface skin friction value is very small and low. While the wedge angle becomes large the values of skin friction also increase rapidly. This circumstance remains the same for any given value of m. When m=1 and for any given values of the magnetic parameter ξ, the shear stress values are tabulated in Table 2. Since m=1, this type is a particular case of stagnation flow. From this table, it is revealed that the skin friction values increase with the increasing values of ξ with uniform interval. The increasing of the magnetic parameter ξ, also increases the skin friction and heat transfer values. Table 3 exhibits the heat transfer values for various values of Prandtl number when m= 0, 1, Ec=0 and ξ=0. From this table, the heat transfer values are increasing with the Prandtl numbers and the increase is rapid for higher Prandtl numbers. Numerical results for heat transfer are presented in Table 4 for various values of Prandtl number, Eckert number and magnetic parameter ξ, for given m=0. From these tables the obtained results are well comparable with known results and the comparison shows in good agreement.

Skin friction values are presented graphically for various values of ξ and m in Fig. 1. In this figure, the pressure gradient parameter m ranges from 0 to 9, and the magnetic parameter ξ ranges from 0 to 10. Figs. 2-4 represent the heat transfer values for various values of m when Ec=0, 0.5 and 1 for given values of Pr=0.1. Similarly for the cases of Ec=0, 0.5 and 1 when Pr=1 are given

Table 1: Comparison of Skin friction values for various values of m when $\xi = 0$.

m	Present results	Cebeci and Bradshaw[4]
0.0	0.33206	0.337164446
1/3	0.75745	0.759123053
1.0	1.23259	1.23334938

Table 2: Comparison of Skin friction values for various values of ξ when $m = 1$.

ξ	Present results	Arial[3]	Yih [20]
0	1.23334938	1.232588	1.232588
1	1.58635749	1.585331	1.585331
4	2.34901650	2.346663	2.346663
9	3.24428992	3.240950	3.240950
25	5.15282465	5.147965	5.147964
100	10.0833987	10.074741	10.074741

Table 3: Comparison of Heat transfer values for various values of Pr when $m = 0, 1$, $Ec = 0$ and $\xi = 0$.

Pr	Present results		Lin and Lin [7]	
	m=0	m=1	m=0	m=1
0.0001	0.0056490	0.007990	0.00558768	0.00793796
0.001	0.017800	0.025240	0.0173157	0.0248294
0.01	0.054680	0.079086	0.0515902	0.0759726
0.1	0.146750	0.232550	0.140032	0.219505
1	0.344540	0.568590	0.332058	0.570466
10	0.778350	1.323490	0.728148	1.33880
100	1.71100	2.942610	1.57186	2.98634
1000	3.71945	6.427350	3.38710	6.52914
10000	8.04602	13.93341	7.29742	14.1583

Table 4: Comparison of Heat transfer values for various values of Pr, Ec and ξ when $m = 0$.

		Present results		Yih [20]	
Pr	ξ	Ec=0	Ec=1	Ec=0	Ec=1
0.733	0.0	0.302071269	0.194863613	0.297526	0.170272
	0.5	0.346883447	0.198323111	0.357022	0.210072
	1.0	0.370572675	0.210918412	0.382588	0.228813
	1.5	0.386374354	0.215030718	0.398264	0.240798
	2.0	0.398053480	0.215510246	0.409168	0.249316
1	0.0	0.344541458	0.145509029	0.332057	0.166029
	0.5	0.388043942	0.186647276	0.402864	0.201452
	1.0	0.411101757	0.192692338	0.433607	0.216814
	1.5	0.426067133	0.198764352	0.452634	0.226323
	2.0	0.436983831	0.205348438	0.465987	0.232998

graphically in Figs. 5-7 respectively. Figs. 8-11, display the heat transfer values for various Prandtl numbers for given Ec=0.1 when m=0.111111, 0.333333, 0.5 and 1. From these figures the local skin friction and heat transfer values increase as the pressure gradient parameter m increases.

5 Conclusions

This study deals with the effects of transverse magnetic field, viscous dissipation, stress work, shear stress and surface heat transfer over a non-isothermal wedge. The governing partial differential equations are reduced to simple polynomial equations, the coefficients of which are functions of independent parameter Pr, m, ξ and Ec. These equations offer a practising engineer, a rapid way of obtaining skin friction and heat transfer values for any values of Pr, m, ξ, and Ec. The great advantage involved in the present technique is that the results are obtained with remarkable accuracy and the amount of calculation is considerably less when compared with more conventional exact methods. Hence the practising engineers and scientists can employ this unique approximate analytical method as a powerful tool for solving heat transfer and boundary layer flow problems.

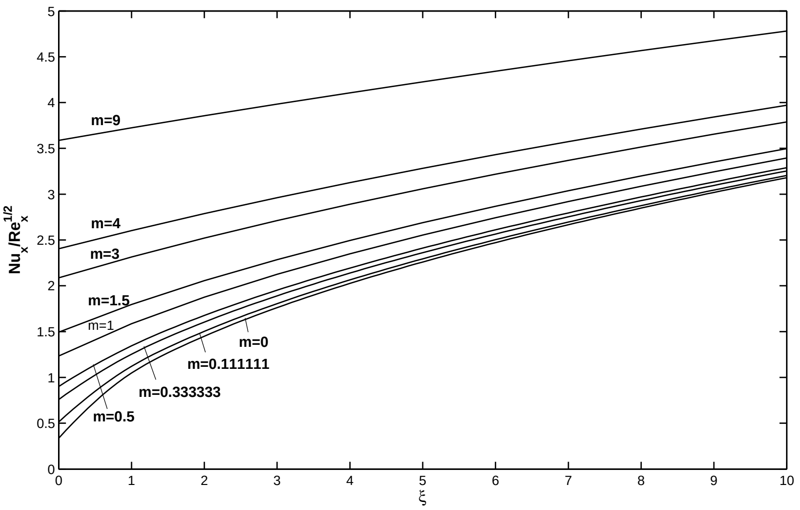

Figure 1: Skin friction as a function of ξ for various m

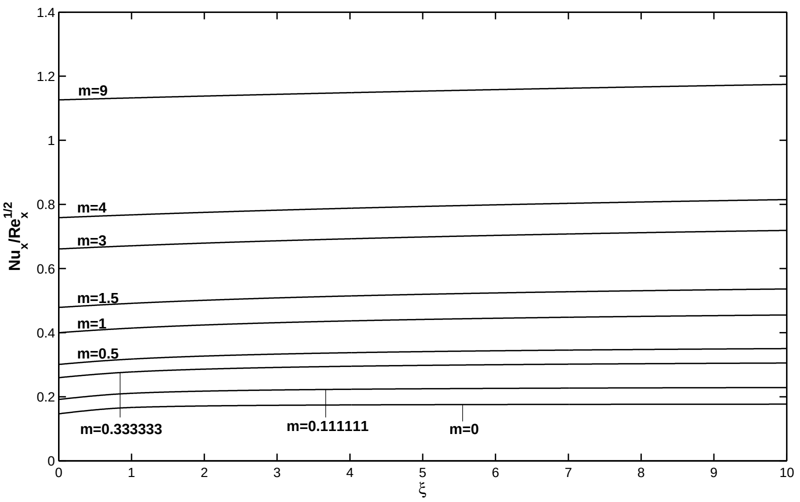

Figure 2: Local Nusselt number as a function of ξ for various m when $Ec = 0$ and $Pr = 0.1$

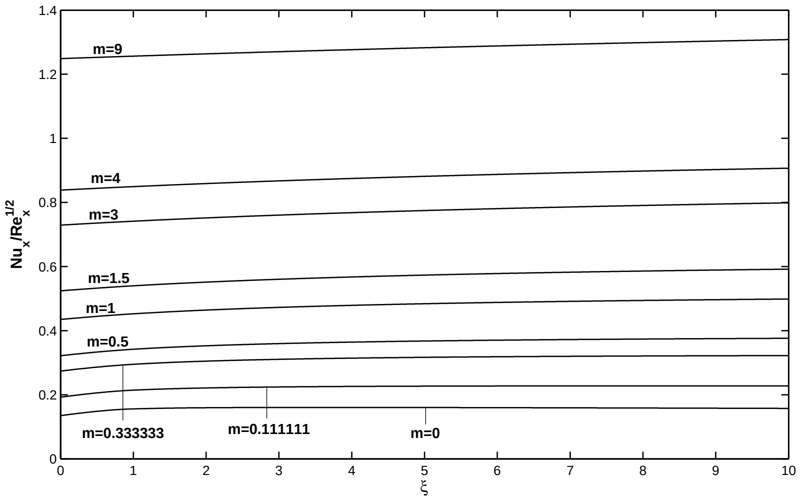

Figure 3: Local Nusselt number as a function of ξ for various m when $Ec = 0.5$ and $Pr = 0.1$

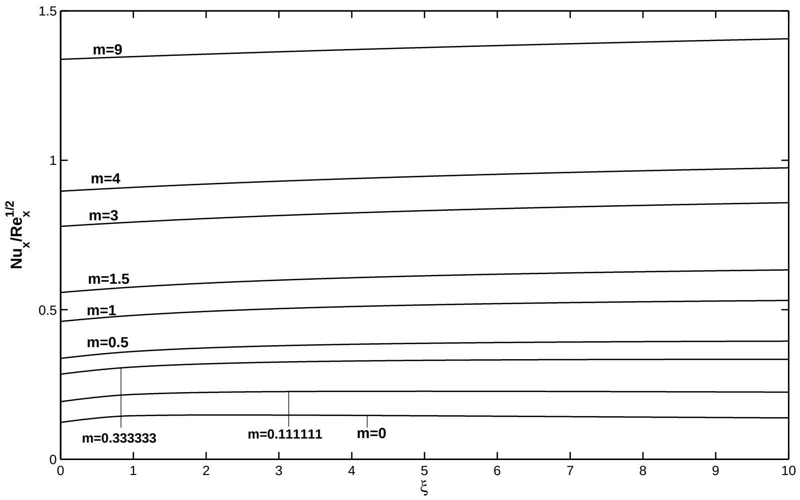

Figure 4: Local Nusselt number as a function of ξ for various m when $Ec = 1$ and $Pr = 0.1$

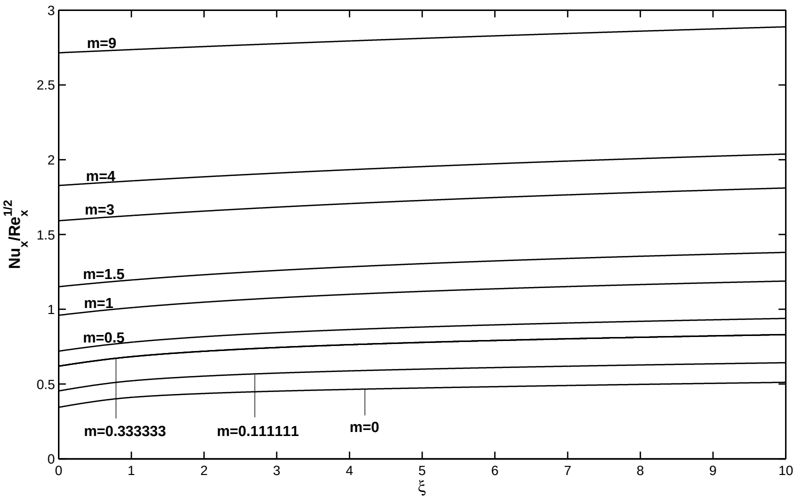

Figure 5: Local Nusselt number as a function of ξ for various m when $Ec = 0$ and $Pr = 1$

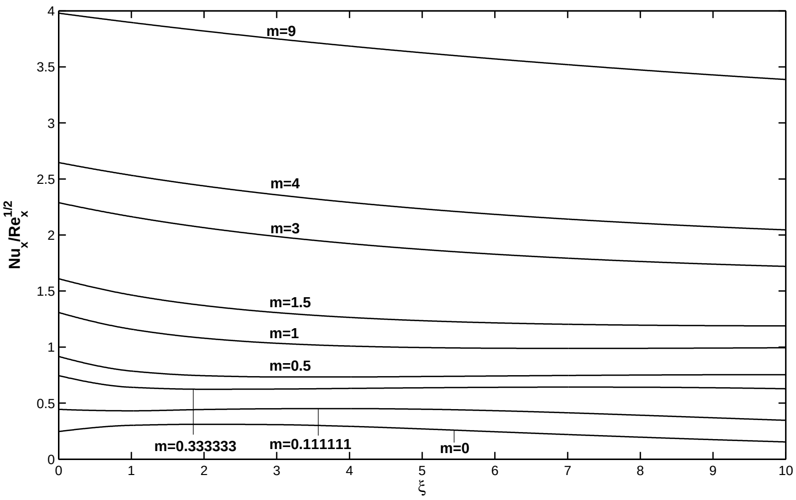

Figure 6: Local Nusselt number as a function of ξ for various m when $Ec = 0.5$ and $Pr = 1$

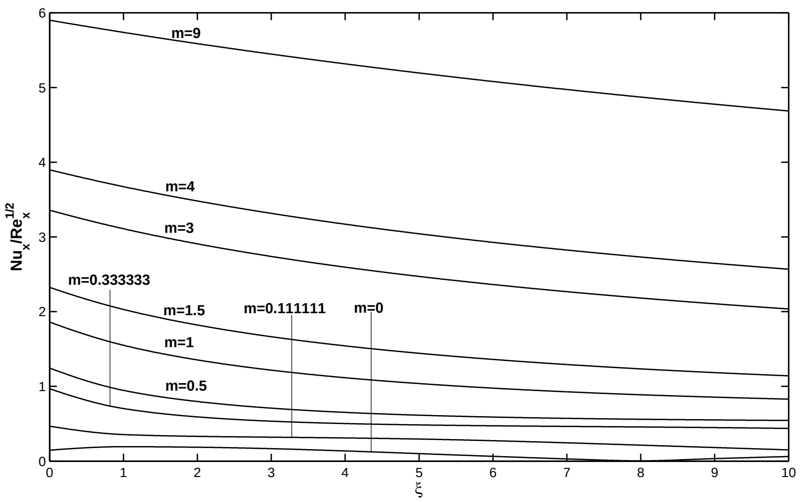

Figure 7: Local Nusselt number as a function of ξ for various m when $Ec = 1$ and $Pr = 0.1$

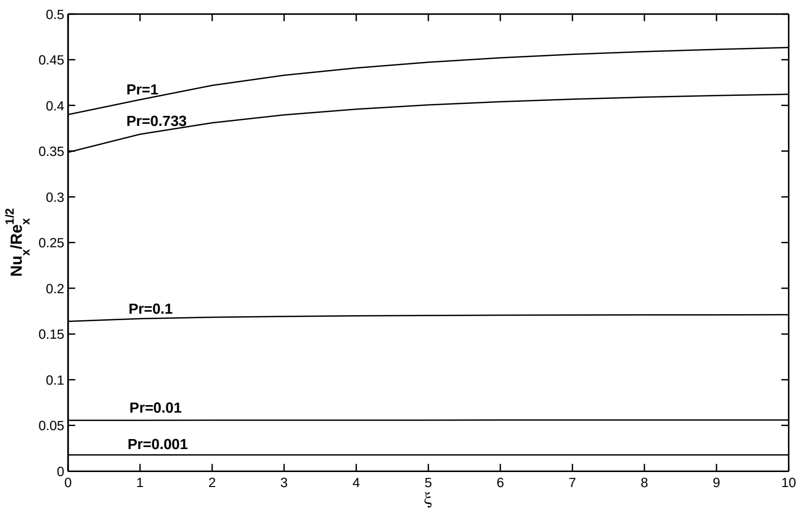

Figure 8: Local Nusselt number as a function of ξ for various Pr when $Ec = 0.1$ and $m = 0$

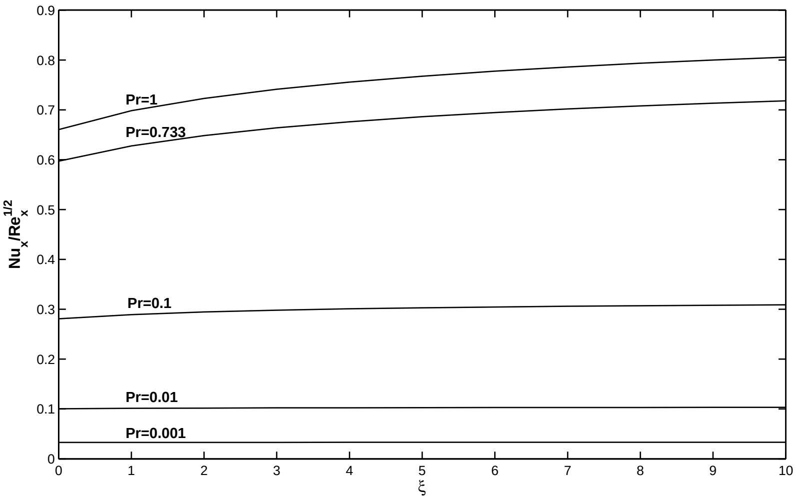

Figure 9: Local Nusselt number as a function of ξ for various Pr when $Ec = 0.1$ and $m = 0.33333$

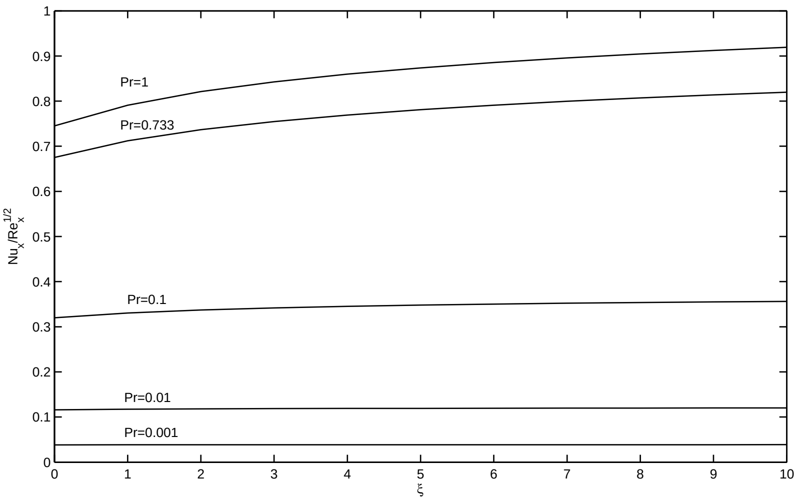

Figure 10: Local Nusselt number as a function of ξ for various Pr when $Ec = 0.1$ and $m = 0.5$

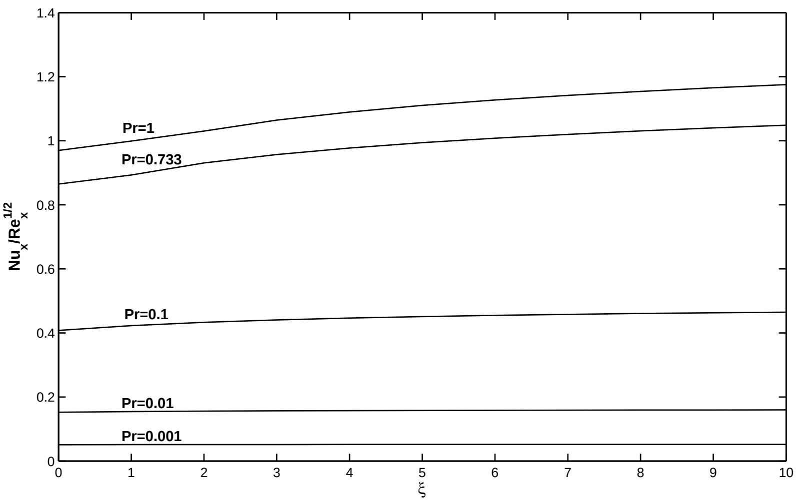

Figure 11: Local Nusselt number as a function of ξ for various Pr when $Ec = 0.1$ and $m = 1$

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Thermodinamičko modeliranje viskozne disipacije pri MHD tečenju

Cilj je studija efekata viskozne disipacije na MHD prinudnu konvekciju u okolini neizotermnog klina. Tu se primenjuje originalni Djarmatijev varijacioni princip koji unutar termodinamike ireverzibilnih procesa ujedinjuje zahteve tehničkih, bioloških i nauka prirodne okoline. Za formulaciju tog varijacionog principa se rasporedi brzine i temperature unutar graničnog sloja posmatraju kao proste polinomijalne funkcije Ojler-Lagranževe se redukuju na proste polinomijalne jednačine preko debljine graničnog sloja. Vrednosti koeficijenta trenja na zidu i Nuseltovog broja su prikazane za različite vrednosti parametra ugla klina m, eksponenta temperature zida $2m$, magnetskog parametra ξ, Prantlovog broja (Pr) i Ekertovog broja (Ec). Rezultati rada su poredjeni sa poznatim dostupnim rezultatima. Uporedjenje je zadovaljavajuće sa veoma dobrom tačnošću.

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