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The influence of variation of electroconductivity on ionized gas flow in the boundary layer along a porous wall

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Abstract

This paper investigates ionized gas flow in the boundary layer when its electroconductivity is varied. The flow is planar and the contour is porous. At first, it is assumed that the ionized gas electroconductivity σ depends only on the longitudinal variable. Then we adopt that it is a function of the ratio of the longitudinal velocity and the velocity at the outer edge of the boundary layer. For both electroconductivity variation laws, by application of the general similarity method, the governing boundary layer equations are brought to a generalized form and numerically solved in a four-parametric three times localized approximation. Based on many tabular solutions, we have shown diagrams of the most important nondimensional values and characteristic boundary layer functions for both of the assumed laws. Finally, some conclusions about influence of certain physical values on ionized gas flow in the boundary layer have been drawn.

Keywords: boundary layer, ionized gas, ionized gas electroconductivity, porous contour, general similarity method, porosity parameter.

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Nomenclature

$A,\ B$	boundary layer characteristics
$a,\ b$	constants
B_m	induction of outer magnetic field
c_p	specific heat of ionized gas at constant pressure
F_{mp}	characteristic boundary layer function
$f_1 = f$	first form parameter
f_k	set of form parameters
$g_1 = g$	first magnetic parameter
g_k	set of magnetic parameters
$H,\ H_1$	boundary layer characteristics
h	enthalpy
$\bar{h}$	nondimensional enthalpy
h_e	enthalpy at the outer edge of the boundary layer
h_w	enthalpy at the wall of the body within the fluid
h_1	enthalpy at the front stagnation point of the body within the fluid
$i,\ j$	iteration number
Le	Lewis number
M	discrete point
Pr	Prandtl number
p	pressure
Q	nondimensional function
s	new longitudinal variable
u	longitudinal projection of velocity in the boundary layer
u_e	velocity at the boundary layer outer edge
V_w	conditional transversal velocity
v	transversal projection of velocity in the boundary layer
v_w	velocity of injection (or ejection) of the fluid
$x,\ y$	longitudinal and transversal coordinate
Z^{**}	function
z	new transversal variable
Δ^*	conditional displacement thicknesses
Δ^{**}	conditional momentum loss thickness
$\Delta_1^*,\ \Delta_1^{**}$	conditional thicknesses
ζ	nondimensional friction function

η	nondimensional transversal coordinate
$\kappa = f_0$	local compressibility parameter
$\Lambda_1 = \Lambda$	first porosity parameter
Λ_k	set of porosity parameters
λ	thermal conductivity coefficient
μ	dynamic viscosity
μ_0	known values of dynamic viscosity of the ionized gas
μ_w	given distributions of dynamic viscosity at the wall of the body within the fluid
ν_0	kinematic viscosity at a concrete point of the boundary layer
ρ	ionized gas density
ρ_e	ionized gas density at the outer edge of the boundary layer
ρ_0	known values of density of the ionized gas
ρ_w	given distributions of density at the wall of the body within the fluid
σ	electroconductivity
τ_w	shear stress at the wall of the body within the fluid
Φ	nondimensional stream function
ψ	stream function
ψ^*	new stream function

1 Introduction

This paper investigates a very complex ionized gas (air) flow in the boundary layer around a porous contour. The solution of this problem is in a way similar to solution of other complex problems of fluid flow in the boundary layer for both incompressible [1] and compressible fluid. At supersonic flow velocities, gas dissociation is followed by ionization. As a result, gas becomes electroconductive. When ionized gas is under the influence of the magnetic field, an electric flow is formed in the gas. Due to this flow, the so-called Lorentz force and Joule heat generate. These two effects result in additional terms in the governing equations in which the ionized gas electroconductivity σ appears. Electroconductivity is one of the most important properties of ionized gas, so investigation of the influence of electroconductivity variation on this boundary layer flow is very important both from the aspect of theory and methodology and from the aspect of practical application.

Very important results in investigation of the dissociated gas flow are given in the book by Dorrance [2]. In his works, Loitsianskii [1,3] and the members of his school (e.g. Krivtsova [4,5]) performed a detailed investigation of the dissociated gas flow in the boundary layer. Investigators of the so-called Belgrade School of the Boundary Layer, led by Saljnikov [6], were also dedicated to investigation of these problems. Important are the works of Boricic et al. [7,8], Ivanovic [9] which investigate MHD boundary layer on a nonporous and porous contour of the body within the fluid, and the works of Miric-Milosavljevic and Pavlovic [e.g. 10] which investigate the temperature boundary layer. These and other papers have successfully applied the general similarity method. This method, developed by Loitsianskii and improved by Saljnikov, is analytical and numerical by nature and it represents a modern method for calculation of the laminar boundary layers. Applying this method, the governing equation system is in the first, analytic part, by means of suitable transformations, brought to a general form that defines universal general similarity solutions, and then, the obtained equation system is numerically solved in a corresponding parametric approximation. This way, a generalized analysis of the influence of certain parameters on the boundary layer development can be made without previously obtaining solutions of concrete examples.
The book [1] studies the ionized gas flow in the boundary layer along a flat nonporous plate when the magnetic field is present. Our paper, however, studies the laminar boundary layer on a body of an arbitrary shape, when the ionized gas flow is planar and steady and the contour of the body within the fluid is porous. Perpendicularly to the porous wall, the ionized gas of the same physical characteristics as the gas in the basic stream is injected i.e. ejected with the velocity $v_w(x)$. The outer magnetic field with the induction $\vec{B}_m$ is perpendicular to the wall of the body within the fluid, therefore $B_{mx} = 0$, while $B_{my} = B_m$. The thickness of the boundary layer is small, so based on [1], it is considered that the power of the field is $B_m = B_m(x)$ and that the magnetic Reynold's number is very small. Therefore, for the case of the ionized gas flow

in the magnetic field, the complete equation system of the steady planar laminar boundary layer in the conditions of equilibrium ionization, based on [1], is:

$$\frac{\partial}{\partial x}\,(\rho u) + \frac{\partial}{\partial y}\,(\rho v) = 0\,,$$

$$\rho u\,\frac{\partial u}{\partial x} + \rho v\,\frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\,\left(\mu\,\frac{\partial u}{\partial y}\right)\,\underline{-\sigma\,B_m^2\,u}\,,$$

$$\rho u\,\frac{\partial h}{\partial x} + \rho v\,\frac{\partial h}{\partial y} = u\,\frac{dp}{dx} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\,\left(\frac{\mu}{Pr}\,\frac{\partial h}{\partial y}\right)\,\underline{+\,\sigma\,B_m^2\,u^2}. \tag{1}$$

The term $\sigma\,B_m^2\,u$ represents Lorentz force and the term $\sigma\,B_m^2\,u^2$ Joule heat. The ionized gas electroconductivity σ appears only in these (underlined) terms, therefore only they will differ when the electroconductivity is varied.

The corresponding boundary layer conditions are:

$$\begin{aligned} &u = 0\,, \quad v = v_w(x)\,, \quad && h = h_w \quad && for \quad y = 0\,, \\ &u \to u_e(x)\,, && h \to h_e(x) \quad && for \quad y \to \infty, \end{aligned} \tag{2}$$

where $v = v_w(x)$ points out to a porous wall of the body within the fluid.

In the equation system (1) and in the boundary conditions (2) the notation common in the boundary layer theory is used: $u(x,\ y)$ - longitudinal projection of velocity in the boundary layer, $v\,(x,\ y)$ - transversal projection, ρ - ionized gas density, p - pressure, h - enthalpy, μ - coefficient of dynamic viscosity, σ - ionized gas electroconductivity and Pr - Prandtl number. The indices denote w - values on the wall of the body within the fluid and e - physical values at the outer edge of the boundary layer.

In order to enrich the literature on the boundary layer, the ionized gas electroconductivity is varied in the paper. Since the exact law on variation of the ionized gas electroconductivity is not known, by analogy with MHD boundary layer, it is first assumed to be a function of the longitudinal coordinate [11], and then to be a function of the ratio of the velocity [12] in the boundary layer and the velocity at the outer edge of the boundary layer, i.e. that it is:

$$\begin{aligned} &a) \quad \sigma = \sigma(x)\,, \\ &b) \quad \sigma = \sigma_0 \left(1 - \frac{u}{u_e}\right)\,, \quad (\sigma_0 = const.). \end{aligned} \tag{3}$$

If the pressure is eliminated from the equation system (1), based on the conditions for the outer edge of the boundary layer ($u(x,\,y) \to u_e(x)\,,\ \left(\frac{\partial u}{\partial y}\right)_e \to 0\,,\ \rho \to \rho_e$), the system will take the following form:

$$\begin{aligned} &\frac{\partial}{\partial x}\,(\rho u) + \frac{\partial}{\partial y}\,(\rho v) = 0\,, \\ &\rho u\,\frac{\partial u}{\partial x} + \rho v\,\frac{\partial u}{\partial y} = \rho_e u_e \frac{du_e}{dx} + \frac{\partial}{\partial y}\left(\mu\,\frac{\partial u}{\partial y}\right) \underline{+\,\sigma B_m^2\,(u_e - u)}\,, \\ &\rho u\,\frac{\partial h}{\partial x} + \rho v\,\frac{\partial h}{\partial y} = -u\;\rho_e u_e \frac{du_e}{dx} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \\ &\qquad\qquad \frac{\partial}{\partial y}\left(\frac{\mu}{Pr}\,\frac{\partial h}{\partial y}\right) \underline{+\sigma B_m^2\,(u^2 - u u_e)}\,; \end{aligned} \tag{4}$$

where the boundary conditions (2) remain unchanged.

The equation system (4) applies to the electroconductivity variation law (3a). In case when the electroconductivity is a function of the velocity ratio, the underlined terms have the following form:

$$b) \quad \underline{-\,\sigma B_m^2\,u}\,, \qquad \underline{+\sigma B_m^2\,u^2}. \tag{4'}$$

2 Transformation of the boundary layer equations

Modern methods of solution of the boundary layer equations involve the usage of the momentum equation. In case of the ionized gas flow in the boundary layer along a porous wall, this equation will have the simplest form if, instead of the physical coordinates x and y, we introduce new variables [3] in the form of the following transformations:

$$s(x) = \frac{1}{\rho_0\,\mu_0} \int\limits_0^x \rho_w \mu_w \, dx\,, \qquad z(x,\,y) = \frac{1}{\rho_0} \int\limits_0^y \rho \; dy \tag{5}$$

and if we introduce the stream function $\psi(s,\,z)$ using the relations:

$$u = \frac{\partial \psi}{\partial z}, \qquad \tilde{v} = \frac{\rho_0 \mu_0}{\rho_w \mu_w} \left(u \frac{\partial z}{\partial x} + v \frac{\rho}{\rho_0} \right) = - \frac{\partial \psi}{\partial s}. \tag{6}$$

Here the values ρ_0 and μ_0 denote the known values of the density and the dynamic viscosity of the ionized gas (air) at a concrete point.

Applying transformations (5) and (6), the governing equation system, together with the boundary conditions, is transformed and bought to the form:

$$\begin{aligned}
&\frac{\partial \psi}{\partial z}\,\frac{\partial^2 \psi}{\partial s \partial z} - \frac{\partial \psi}{\partial s}\,\frac{\partial^2 \psi}{\partial z^2} = \frac{\rho_e}{\rho} u_e \frac{d u_e}{ds} + \nu_0 \frac{\partial}{\partial z} \left(Q\, \frac{\partial^2 \psi}{\partial z^2} \right) \\
&\qquad \underline{+ \frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\,\frac{\sigma B_m^2}{\rho} \left(u_e - \frac{\partial \psi}{\partial z} \right)}\,, \\
&\frac{\partial \psi}{\partial z}\,\frac{\partial h}{\partial s} - \frac{\partial \psi}{\partial s}\,\frac{\partial h}{\partial z} = - \frac{\rho_e}{\rho} u_e \frac{d u_e}{ds}\,\frac{\partial \psi}{\partial z} + \nu_0\, Q \left(\frac{\partial^2 \psi}{\partial z^2} \right)^2 + \\
&\nu_0 \frac{\partial}{\partial z} \left(\frac{Q}{Pr}\,\frac{\partial h}{\partial z} \right) \underline{+ \frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\,\frac{\sigma B_m^2}{\rho}\,\frac{\partial \psi}{\partial z} \left(\frac{\partial \psi}{\partial z} - u_e \right)}\,;
\end{aligned} \tag{7}$$

$$\frac{\partial \psi}{\partial z} = 0\,, \quad \frac{\partial \psi}{\partial s} = - \frac{\mu_0}{\mu_w}\, v_w = - \,\tilde{v}_w\,, \qquad h = h_w \qquad for \quad z = 0\,,$$

$$\frac{\partial \psi}{\partial z} \to u_e(s)\,, \qquad\qquad h \to h_e(s) \quad for \quad z \to \infty\,.$$

The equation system (7) applies to the case when the electroconductivity is varied in accordance with the law ($3a$). For the electroconductivity variation law ($3b$), the obtained equations differ from the ones above only in the underlined terms. In that case these terms are:

$$b) \qquad \underline{-\frac{\sigma_0 B_m^2}{\rho u_e}\frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\left(u_e-\frac{\partial\psi}{\partial z}\right)\frac{\partial\psi}{\partial z}}\,,$$

$$\underline{+\frac{\sigma_0 B_m^2}{\rho u_e}\frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\left(u_e-\frac{\partial\psi}{\partial z}\right)\left(\frac{\partial\psi}{\partial z}\right)^2}. \qquad (7')$$

In the transformed equations (7), the nondimensional function Q and Prandtl number Pr are determined with the expressions:

$$Q=\frac{\rho\,\mu}{\rho_w\mu_w}\,,\quad \mathrm{Pr}=\frac{\mu\,c_p}{\lambda}\,, \qquad (8)$$

where λ - thermal conductivity coefficient and c_p - specific heat of ionized gas at constant pressure.

In order to solve the equation system (4), it is necessary to derive the momentum equation of the ionized gas on a body with a porous contour. It has been derived in this paper, it has the same form for both of the assumed forms of the electroconductivity variation law, and it is:

$$\frac{dZ^{**}}{ds}\;=\;\frac{F_{mp}}{u_e}\,. \qquad (9)$$

While obtaining the momentum equation, the usual values are introduced: parameter of the form $f(s)$, magnetic parameter $g(s)$, conditional displacement thickness Δ^*, conditional momentum loss thickness Δ^{**}, conditional thicknesses Δ_1^* and Δ_1^{**}, shear stress at the wall of the body within the fluid τ_w, nondimensional friction function $\zeta(s)$, nondimensional values H and H_1 and characteristic function of the boundary layer on a porous wall F_{mp}. For the ionized gas flow, these values are for both electroconductivity variation laws, defined by the relations:

$$Z^{**}=\frac{\Delta^{**2}}{\nu_0}\,,\qquad f(s)=f_1(s)=\frac{u_e'\,\Delta^{**^2}}{\nu_0}=u_e'\,Z^{**},$$

$$g\,(s)=g_1\,(s)=N_\sigma Z^{**},\quad N_\sigma=\frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\;\bar{N},$$

$$\Delta^*(s) = \int_0^\infty \left(\frac{\rho_e}{\rho} - \frac{u}{u_e}\right) dz, \quad \Delta^{**}(s) = \int_0^\infty \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right) dz\,,$$

$$\tau_w(s) = \left(\mu\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\rho_w\,\mu_w}{\rho_0}\,\frac{u_e}{\Delta^{**}}\zeta\,;$$

$$\zeta(s) = \left[\frac{\partial\,(u/u_e)}{\partial\,(z/\Delta^{**})}\right]_{z=0}, \qquad H = \frac{\Delta^*}{\Delta^{**}}\;; \tag{10}$$

$$a) \quad \bar{N} = \frac{\sigma B_m^2}{\rho_e}, \quad H_1 = \frac{\Delta_1^*}{\Delta^{**}}\,, \quad \Delta_1^*(s) = \int_0^\infty \frac{\rho_e}{\rho}\left(1 - \frac{u}{u_e}\right) dz\,,$$

$$F_{mp} = 2\,[\,\zeta - (2 + H)\;f] - 2gH_1 - 2\Lambda\,,$$

$$b) \quad \bar{N} = \frac{\sigma_0 B_m^2}{\rho_e}, \quad H_1 = \frac{\Delta_1^{**}}{\Delta^{**}}\,, \quad \Delta_1^{**}(s) = \int_0^\infty \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right)\frac{\rho_e}{\rho} dz\,,$$

$$F_{mp} = 2\,[\,\zeta - (2 + H)\;f] + 2gH_1 - 2\Lambda\,.$$

As a result of the porous wall of the body within the fluid, an addend appears in the momentum equation. It is now necessary to introduce a new parameter – the so-called porosity parameter $\Lambda(s)$:

$$\Lambda = -\frac{\mu_0}{\mu_w}\frac{v_w\Delta^{**}}{\nu_0} = -\frac{V_w\Delta^{**}}{\nu_0} = \Lambda(s) \tag{11}$$

defined with the value

$$V_w(s) = \frac{\mu_0}{\mu_w}\,v_w = \tilde{v}_w$$

which is in our investigations [11] called a conditional transversal velocity at the inner edge of the boundary layer when the wall of the body within the fluid is porous.

For the application of the general similarity method, it is very important that the boundary conditions and the stream function on the wall of the body within the fluid, for both electroconductivity variation laws, remain the same as with the nonporous wall. That is why, as with incompressible fluid, a new stream function $\psi^*(s,\, z)$ is introduced in the form of the relation

$$\psi(s,\, z) = \psi_w(s) + \psi^*(s,\, z)\,, \quad \psi^*(s,\, 0) = 0 \tag{12}$$

where $\psi(s,\, 0) = \psi_w(s)$ stands for the stream function along the wall of the body within the fluid.

Applying the relations (12), the system (7) is transformed into the equation system:

$$\frac{\partial \psi^*}{\partial z}\,\frac{\partial^2 \psi^*}{\partial s \partial z} - \frac{\partial \psi^*}{\partial s}\,\frac{\partial^2 \psi^*}{\partial z^2} - \frac{d\psi_w}{ds}\,\frac{\partial^2 \psi^*}{\partial z^2} =$$

$$\frac{\rho_e}{\rho} u_e u_e' + \nu_0\,\frac{\partial}{\partial z}\left(Q\,\frac{\partial^2 \psi^*}{\partial z^2}\right) + \underline{\frac{\sigma B_m^2}{\rho}\,\frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\left(u_e - \frac{\partial \psi^*}{\partial z}\right)},$$

$$\frac{\partial \psi^*}{\partial z}\,\frac{\partial h}{\partial s} - \frac{\partial \psi^*}{\partial s}\,\frac{\partial h}{\partial z} - \frac{d\psi_w}{ds}\,\frac{\partial h}{\partial z} = -\frac{\rho_e}{\rho} u_e u_e'\,\frac{\partial \psi^*}{\partial z} + \nu_0\, Q\left(\frac{\partial^2 \psi^*}{\partial z^2}\right)^2 +$$

$$\nu_0\,\frac{\partial}{\partial z}\left(\frac{Q}{Pr}\,\frac{\partial h}{\partial z}\right) + \underline{\frac{\sigma B_m^2}{\rho}\,\frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\left[\frac{\partial \psi^*}{\partial z}\left(\frac{\partial \psi^*}{\partial z} - u_e\right)\right]};$$

$$\psi^* = 0\,, \quad \frac{\partial \psi^*}{\partial z} = 0\,, \quad h = h_w \quad for \quad z = 0\,,$$

$$\frac{\partial \psi^*}{\partial z} \to u_e(s)\,, \quad h \to h_e(s) \quad z \to \infty\,. \tag{13}$$

It applies to the electroconductivity variation law $(3a)$. For the form $(3b)$ the underlined terms in the dynamic and energy equation are:

$$b) \quad \underline{-\frac{\sigma_0 B_m^2}{\rho u_e}\,\frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\left(u_e - \frac{\partial \psi^*}{\partial z}\right)}\frac{\partial \psi^*}{\partial z},$$

$$\underline{+\frac{\sigma_0 B_m^2}{\rho u_e}\,\frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\left(u_e - \frac{\partial \psi^*}{\partial z}\right)\left(\frac{\partial \psi^*}{\partial z}\right)^2}. \tag{13'}$$

3 General mathematical model

In order to obtain the generalized boundary layer equations, it is necessary from the very beginning to introduce new transformations in the form of the expressions:

$$s = s, \quad \eta(s,\, z) = \frac{u_e^{b/2}}{K(s)} z,$$

$$\psi^*(s,\, z) = u_e^{1-b/2} K(s)\, \Phi\, \left[\eta\,,\ \kappa\,,\ (f_k)\,,\ (g_k)\,,\ (\Lambda_k)\right]\,,$$

$$h(s,\, z) = h_1\, \cdot \bar{h}\, \left[\eta\,,\ \kappa\,,\ (f_k)\,,\ (g_k)\,,\ (\Lambda_k)\right]\,; \tag{14}$$

$$h_e + \tfrac{u_e^2}{2} = h_1 = const.,$$

$$K(s) = \left(a\nu_0 \int\limits_0^s u_e^{b-1} ds \right)^{1/2}; \qquad a,\ b = const.$$

In thus defined similarity transformations, the common notation is used: $\eta(s,\, z)$ - newly introduced transversal variable, Φ - newly introduced stream function and $\bar{h}$ - nondimensional enthalpy.

Based on the expressions for the variable $\eta(s,\, z)$, certain important values and characteristics of the boundary layer (10) can be written in the form of suitable relations:

$$u = u_e \,\frac{\partial \Phi}{\partial \eta}\,, \qquad \Delta^{**}(s) = \frac{K(s)}{u_e^{b/2}}\, B(s)\,, \quad B(s) = \int\limits_0^\infty \frac{\partial \Phi}{\partial \eta} \left(1 \, - \, \frac{\partial \Phi}{\partial \eta}\right) d\eta\,,$$

$$\frac{\Delta^*(s)}{\Delta^{**}(s)} = H = \frac{A(s)}{B(s)}\,, \quad A(s) = \int\limits_0^\infty \left(\frac{\rho_e}{\rho} \, - \, \frac{\partial \Phi}{\partial \eta}\right) d\eta\,,$$

$$\zeta = B \left(\frac{\partial^2 \Phi}{\partial \eta^2}\right)_{\eta=0}, \quad \frac{f}{B^2} = \frac{a\, u_e'}{u_e^b} \int\limits_0^s u_e^{b-1}\, ds\,; \tag{15}$$

$$a) \quad \frac{\Delta_1^*(s)}{\Delta^{**}(s)} = H_1 = \frac{A_1(s)}{B(s)}\,; \quad A_1(s) = \int\limits_0^\infty \frac{\rho_e}{\rho} \left(1 - \frac{\partial \Phi}{\partial \eta}\right) d\eta\,,$$

$$b)\qquad \frac{\Delta_1^{**}(s)}{\Delta^{**}(s)} = H_1 = \frac{A_1(s)}{B(s)}\,; \quad A_1(s) = \int\limits_0^\infty \frac{\partial\Phi}{\partial\eta}\,\left(1- \frac{\partial\Phi}{\partial\eta}\right)\frac{\rho_e}{\rho}\; d\eta\,.$$

In the nondimensional functions Φ and $\bar{h}$ and in the generalized similarity transformations (14), we introduce: local parameter of the ionized gas compressibility $\kappa = f_0$, set of parameters of the form f_k of Loitsianskii type [3], set of magnetic parameters g_k and a set of porosity parameters Λ_k [13]:

$$\kappa = f_0(s) = \frac{u_e^2}{2h_1}\,, \qquad f_k(s) = u_e^{k-1}\, u_e^{(k)}\, Z^{**k},$$

$$g_k(s) = u_e^{k-1}\, N_\sigma^{(k-1)}\, Z^{**k}, \tag{16}$$

$$\Lambda_k(s) = -u_e^{k-1}\,\left(\tfrac{V_w}{\sqrt{\nu_0}}\right)^{(k-1)}\, Z^{**k-1/2} \quad (\,k = 1,\ 2,\ 3,\ \dots)\,.$$

They represent new independent variables, instead of the longitudinal variable s.

The local compressibility parameter $\kappa = f_0$ and the stated sets of parameters satisfy the corresponding simple recurrent differential equations in the form of:

$$\frac{u_e}{u_e'} f_1 \frac{d\kappa}{ds} = 2\,\kappa\, f_1 = \theta_0\,,$$

$$\frac{u_e}{u_e'} f_1 \frac{df_k}{ds} = [\,(k-1)f_1 + kF_{mp}\,]\, f_k + f_{k+1} = \theta_k,$$

$$\frac{u_e}{u_e'}\; f_1\, \frac{dg_k}{ds} g_k' = [\,(k-1)\, f_1 + k\, F_{mp}\,]\;\; g_k + g_{k+1} = \gamma_k,$$

$$\frac{u_e}{u_e'} f_1 \frac{d\Lambda_k}{ds} = \{\;(k-1)f_1 + [\,(2k-1)/2\,]\; F_{mp}\}\;\; \Lambda_k + \Lambda_{k+1} = \chi_k. \qquad (k = 1, 2, 3, \dots) \tag{17}$$

Applying the similarity transformations (14) and (16) to the system (13), i.e. (13'), the boundary layer equation system is obtained as follows:

$$
\begin{aligned}
&\frac{\partial}{\partial\eta}\left(Q\,\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)\,f_1}{2\,B^2}\;\Phi\,\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f_1}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]\\
&\qquad\underline{+\frac{g_1}{B^2}\,\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)}+\frac{\Lambda_1}{B}\,\frac{\partial^2\Phi}{\partial\eta^2}=\\
&\frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial^2\Phi}{\partial\eta\,\partial f_k}-\frac{\partial\Phi}{\partial f_k}\,\frac{\partial^2\Phi}{\partial\eta^2}\right)+\right.\\
&\quad+\sum_{k=1}^{\infty}\gamma_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial^2\Phi}{\partial\eta\,\partial g_k}-\frac{\partial\Phi}{\partial g_k}\,\frac{\partial^2\Phi}{\partial\eta^2}\right)\\
&\quad\left.+\sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\,\partial\Lambda_k}-\frac{\partial\Phi}{\partial\Lambda_k}\frac{\partial^2\Phi}{\partial\eta^2}\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial}{\partial\eta}\left(\frac{Q}{Pr}\,\frac{\partial\bar{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)\,f_1}{2\,B^2}\;\Phi\,\frac{\partial\bar{h}}{\partial\eta}-\\
&\frac{2\kappa f_1}{B^2}\,\frac{\rho_e}{\rho}\,\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2-\\
&\underline{-\frac{2\kappa g_1}{B^2}\,\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}}+\frac{\Lambda_1}{B}\,\frac{\partial\bar{h}}{\partial\eta}= \qquad (18)\\
&\frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial\bar{h}}{\partial f_k}-\frac{\partial\Phi}{\partial f_k}\,\frac{\partial\bar{h}}{\partial\eta}\right)+\right.\\
&\quad+\sum_{k=1}^{\infty}\gamma_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial\bar{h}}{\partial g_k}-\frac{\partial\Phi}{\partial g_k}\,\frac{\partial\bar{h}}{\partial\eta}\right)+\\
&\quad\left.\sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial\bar{h}}{\partial\Lambda_k}-\frac{\partial\Phi}{\partial\Lambda_k}\,\frac{\partial\bar{h}}{\partial\eta}\right)\right];
\end{aligned}
$$

$$b) \qquad \underline{-\frac{g_1}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}}\,,$$

$$\underline{+\frac{2\kappa g_1}{B^2}\,\frac{\rho_e}{\rho}\,\left(1-\frac{\partial\Phi}{\partial\eta}\right)\left(\frac{\partial\Phi}{\partial\eta}\right)^2}. \tag{18'}$$

The transformed boundary conditions are:

$$\Phi=\frac{\partial\Phi}{\partial\eta}=0\ , \qquad \bar{h}=\bar{h}_w=const. \qquad for \qquad \eta=0\,,$$

$$\frac{\partial\Phi}{\partial\eta}\to 1\,, \qquad \bar{h}\to\bar{h}_e=1-\kappa \qquad for \qquad \eta\to\infty\ . \tag{19}$$

It can be seen that the outer velocity of the boundary layer $u_e(s)$ appears explicitly neither in the obtained equation system (18), i.e. (18') nor in the boundary conditions (19). Therefore, this equation system is generalized and *it represents a general mathematical model of the ionized gas flow along a porous wall of the body within the fluid for both of the assumed electroconductivity variation laws.*

In the equation system (18) i.e. (18'), we can notice that there are terms that depend on the porosity parameter and that there is a sum of terms that are multiplied with the function χ_k. In the case of a nonporous wall of the body within the fluid, all the porosity parameters equal zero. Therefore, all the mentioned terms equal zero. In that case, the generalized system with the boundary conditions comes down to the corresponding equation system [14] that refers to the case of a flow along a nonporous wall of the body within the fluid.

4 Numerical solution

In order for the system (18), i.e. (18'), with the boundary conditions (19), to be solved numerically, a finite number of parameters is adopted. Then, the solution is obtained in an n - parametric approximation. Due to numerous difficulties encountered in the course of solution of this system, only a relatively small number of parameters can be included, even with modern computers. If we assume that all the similarity parameters, from the second one onwards, equal zero, i.e. if:

$$\begin{aligned} &\kappa = f_0 \neq 0\,, \qquad f_1 = f \neq 0\,, \qquad g_1 = g \neq 0\,, \qquad \Lambda_1 = \Lambda \neq 0\,; \\ &f_2 = f_3 = ... = 0\,, \qquad g_2 = g_3 = ... = 0\,, \qquad \Lambda_2 = \Lambda_3 = ... = 0\,, \end{aligned} \tag{20}$$

the obtained equation system is significantly simplified. The so-called localization is also performed because of these difficulties. If the derivatives per the compressibility, magnetic and porosity parameters are neglected ($\partial/\partial\kappa = 0$, $\partial/\partial g_1 = 0$, $\partial/\partial\Lambda_1 = 0$), the system (18) is considerably simplified, and in the four-parametric three times localized approximation, it is:

$$\begin{aligned} &\frac{\partial}{\partial\eta}\left(Q\,\frac{\partial^2\Phi}{\partial\eta^2}\right) + \frac{aB^2+(2-b)\,f}{2\,B^2}\;\Phi\;\frac{\partial^2\Phi}{\partial\eta^2} + \\ &\frac{f}{B^2}\left[\frac{\rho_e}{\rho} - \left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]\;\underline{+\frac{g}{B^2}\,\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)} + \\ &+\frac{\Lambda}{B}\,\frac{\partial^2\Phi}{\partial\eta^2} = \frac{F_{mp}f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial^2\Phi}{\partial\eta\,\partial f} - \frac{\partial\Phi}{\partial f}\,\frac{\partial^2\Phi}{\partial\eta^2}\right), \\ &\frac{\partial}{\partial\eta}\left(\frac{Q}{Pr}\,\frac{\partial\bar{h}}{\partial\eta}\right) + \frac{aB^2+(2-b)\,f}{2\,B^2}\;\Phi\;\frac{\partial\bar{h}}{\partial\eta} - \\ &\frac{2\kappa f}{B^2}\,\frac{\rho_e}{\rho}\,\frac{\partial\Phi}{\partial\eta} + 2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2 - \\ &\underline{-\frac{2\kappa g}{B^2}\,\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}} + \\ &\frac{\Lambda}{B}\,\frac{\partial\bar{h}}{\partial\eta} = \frac{F_{mp}f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial\bar{h}}{\partial f} - \frac{\partial\Phi}{\partial f}\,\frac{\partial\bar{h}}{\partial\eta}\right); \end{aligned} \tag{21}$$

$$b)\quad \underline{-\frac{g}{B^2}\,\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}},$$

$$
\underline{+\frac{2\kappa g}{B^2}\,\frac{\rho_e}{\rho}\,\left(1-\frac{\partial\Phi}{\partial\eta}\right)\left(\frac{\partial\Phi}{\partial\eta}\right)^2}. \tag{21'}
$$

Here, the boundary conditions (19) remain unchanged.

In the equations of the system (21), i.e. (21'), the index 1 is left out in the first parameters. Each of the equations contains a different term that characterizes a porous wall of the body within the fluid.

For numerical integration of the obtained system of differential partial equations of the third order, it is necessary to decrease the order of the differential equation. Applying the usual transformation [15]

$$
\frac{u}{u_e}=\frac{\partial\Phi}{\partial\eta}=\varphi=\varphi\,(\eta\,,\ \kappa\,,\ f,\ g,\ \Lambda)\,, \tag{22}
$$

the order of the differential equation of this system has been decreased, and now the system, together with the boundary conditions, has the following form:

$$
\begin{aligned}
&\frac{\partial}{\partial\eta}\left(Q\frac{\partial\varphi}{\partial\eta}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial\varphi}{\partial\eta}+\\
&\frac{f}{B^2}\left[\frac{\rho_e}{\rho}-\varphi^2\right]\underline{+\frac{g}{B^2}\frac{\rho_e}{\rho}(1-\varphi)}+\\
&\frac{\Lambda}{B}\,\frac{\partial\varphi}{\partial\eta}=\frac{F_{mp}f}{B^2}\left(\varphi\frac{\partial\varphi}{\partial f}-\frac{\partial\Phi}{\partial f}\frac{\partial\varphi}{\partial\eta}\right),
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial}{\partial\eta}\left(\frac{Q}{\mathrm{Pr}}\frac{\partial\overline{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial\overline{h}}{\partial\eta}-\\
&\frac{2\,\kappa\,f}{B^2}\frac{\rho_e}{\rho}\varphi+2\kappa Q\,\left(\frac{\partial\varphi}{\partial\eta}\right)^2\underline{-\frac{2\kappa g}{B^2}\frac{\rho_e}{\rho}(1-\varphi)\,\varphi}+\\
&\frac{\Lambda}{B}\,\frac{\partial\overline{h}}{\partial\eta}=\frac{F_{mp}f}{B^2}\left(\varphi\frac{\partial\overline{h}}{\partial f}-\frac{\partial\Phi}{\partial f}\frac{\partial\overline{h}}{\partial\eta}\right);
\end{aligned} \tag{23}
$$

$$\Phi \quad = \quad \varphi = 0, \qquad \bar{h} = \bar{h}_w = const. \qquad for \qquad \eta = 0,$$

$$\varphi \quad \to \quad 1\,, \qquad \bar{h} \to \bar{h}_e = 1 - \kappa \qquad for \qquad \eta \to \infty\,.$$

In case when the electroconductivity is varied according to the law (3 b), the underlined terms are:

$$b) \qquad \underline{-\frac{g}{B^2}\frac{\rho_e}{\rho}(1-\varphi)\,\varphi\,;} \qquad \underline{+\frac{2\kappa g}{B^2}\frac{\rho_e}{\rho}\,(1-\varphi)\;\varphi^2.} \qquad (23')$$

For the nondimensional function Q [14] and the density ratio ρ_e/ρ [4] that appear in the equations (23), approximation formulae are used:

$$Q = Q\,(\bar{h}) = \left(\frac{\bar{h}_w}{\bar{h}}\right)^{1/3}, \qquad \frac{\rho_e}{\rho} \approx \frac{\bar{h}}{1-\kappa}\,. \qquad (24)$$

A concrete numerical solution of the obtained system of nonlinear and conjugated differential partial equations (23) is done by finite differences method, i.e., "passage method" or TDA method. Based on the planar integration grid, the derivatives of the functions φ, Φ and $\bar{h}$ are substituted with finite differences ratios, and the equation system (23) is brought down to the following system of algebraic equations:

$$(I) \quad a^i_{M,K+1}\varphi^i_{M-1,K+1} - 2b^i_{M,K+1}\varphi^i_{M,K+1} + c^i_{M,K+1}\varphi^i_{M+1,K+1} = g^i_{M,K+1},$$

$$(II) \quad a^j_{M,K+1}\overline{h}^j_{M-1,K+1} - 2b^j_{M,K+1}\overline{h}^j_{M,K+1} + c^j_{M,K+1}\overline{h}^j_{M+1,K+1} = g^j_{M,K+1};$$

$$M = 2,3,...,N-1; \quad K = 0,1,2,...; i,j = 0,1,2,...$$

$$\Phi^i_{1,K+1} = \varphi^i_{1,K+1} = 0, \quad \overline{h}^j_{1,K+1} = \overline{h}_w = const. \quad for \quad M = 1\,,$$

$$\varphi^i_{N,K+1} = 1,\; \overline{h}^j_{N,K+1} = 1-\kappa \qquad for \quad M = N, \qquad (25)$$

that apply for both forms of the electroconductivity variation law. The algebraic equation system (25) consists of two subsystems – dynamic

(I) and thermodynamic (II). When the electroconductivity is varied according to the law (3 a), the coefficients $a^i_{M,K+1}$, $b^i_{M,K+1}$,$c^i_{M,K+1}$ and $g^i_{M,K+1}$ of the dynamic subsystem are determined by the expressions:

$$a^i_{M,K+1} = Q^{j-1}_{M,K+1} - \frac{1}{4}(Q^{j-1}_{M+1,K+1} - Q^{j-1}_{M-1,K+1}) - \frac{\Delta\eta}{2(B^{i-1}_{K+1})^2}$$

$$\left\{ \left[a(B^{i-1}_{K+1})^2 + (2-b)f_{K+1}\right] \frac{\Phi^{i-1}_{M,K+1}}{2} + F^{i-1}_{mp,K+1} f_{K+1} \frac{\Phi^{i-1}_{M,K+1} - \Phi_{M,K}}{\Delta f} \right\} - \frac{\Delta\eta}{2} \frac{\Lambda}{B^{i-1}_{K+1}},$$

$$b^i_{M,K+1} = Q^{j-1}_{M,K+1} + \frac{(\Delta\eta)^2}{2(B^{i-1}_{K+1})^2} \left[f_{K+1}\varphi^{i-1}_{M,K+1} \left(1 + \frac{F^{i-1}_{mp,K+1}}{\Delta f}\right) + g\frac{\bar{h}^{j-1}_{M,K+1}}{1-\kappa} \right], \tag{26}$$

$$c^i_{M,K+1} = Q^{j-1}_{M,K+1} + \frac{1}{4}(Q^{j-1}_{M+1,K+1} - Q^{j-1}_{M-1,K+1}) + \frac{\Delta\eta}{2(B^{i-1}_{K+1})^2}$$

$$\left\{ \left[a(B^{i-1}_{K+1})^2 + (2-b)f_{K+1}\right] \frac{\Phi^{i-1}_{M,K+1}}{2} + F^{i-1}_{mp,K+1} f_{K+1} \frac{\Phi^{i-1}_{M,K+1} - \Phi_{M,K}}{\Delta f} \right\} + \frac{\Delta\eta}{2} \frac{\Lambda}{B^{i-1}_{K+1}},$$

$$g^i_{M,K+1} = -\frac{(\Delta\eta)^2}{(B^{i-1}_{K+1})^2} \left[(f_{K+1} + g)\ \frac{\bar{h}^{j-1}_{M,K+1}}{1-\kappa} + F^{i-1}_{mp,K+1} f_{K+1} \varphi^{i-1}_{M,K+1} \frac{\varphi_{M,K}}{\Delta f} \right].$$

Applying the law (3b), the coefficients $a^i_{M,K+1}$ and $c^i_{M,K+1}$ are the same as in the expressions (26), while $b^i_{M,K+1}$ and $g^i_{M,K+1}$ differ and they are:

$$b^i_{M,K+1} = Q^{j-1}_{M,K+1} + \frac{(\Delta\eta)^2}{2(B^{i-1}_{K+1})^2}\left[f_{K+1}\varphi^{i-1}_{M,K+1}\left(1+\frac{F^{i-1}_{mp,K+1}}{\Delta f}\right)+ g\frac{\bar{h}^{j-1}_{M,K+1}}{1-\kappa}(1-\varphi^{i-1}_{M,K+1})\right],$$

$$g^i_{M,K+1} = -\frac{(\Delta\eta)^2}{(B^{i-1}_{K+1})^2}\; f_{K+1}\left[\frac{\bar{h}^{j-1}_{M,K+1}}{1-\kappa} + F^{i-1}_{mp,K+1}\varphi^{i-1}_{M,K+1}\frac{\varphi_{M,K}}{\Delta f}\right]. \quad (26')$$

In the thermodynamic subsystem, the corresponding coefficients for the electroconductivity variation law ($3a$) are:

$$a^j_{M,K+1} = \frac{Q^{j-1}_{M,K+1}}{\mathrm{Pr}} - \frac{1}{4\,\mathrm{Pr}}(Q^{j-1}_{M+1,K+1} - Q^{j-1}_{M-1,K+1}) - \frac{\Delta\eta}{2(B^{i-1}_{K+1})^2}$$

$$\left\{\left[a(B^{i-1}_{K+1})^2+(2-b)f_{K+1}\right]\frac{\Phi^{i-1}_{M,K+1}}{2}+ F^{i-1}_{mp,K+1}f_{K+1}\frac{\Phi^{i-1}_{M,K+1}-\Phi_{M,K}}{\Delta f}\right\} - \frac{\Delta\eta}{2}\,\frac{\Lambda}{B^{i-1}_{K+1}},$$

$$b^j_{M,K+1} = \frac{Q^{j-1}_{M,K+1}}{\mathrm{Pr}} + \frac{(\Delta\eta)^2}{2(B^{i-1}_{K+1})^2}\left[f_{K+1}\varphi^i_{M,K+1}\left(\frac{2\kappa}{1-\kappa}+\frac{F^{i-1}_{mp,K+1}}{\Delta f}\right)+ \frac{2\kappa g}{1-\kappa}\varphi^{i-1}_{M,K+1}(1-\varphi^i_{M,K+1})\right], \quad (27)$$

$$c^j_{M,K+1} = \frac{Q^{j-1}_{M,K+1}}{\mathrm{Pr}} + \frac{1}{4\,\mathrm{Pr}}(Q^{j-1}_{M+1,K+1} - Q^{j-1}_{M-1,K+1}) + \frac{\Delta\eta}{2(B^{i-1}_{K+1})^2}\left\{\left[a(B^{i-1}_{K+1})^2+(2-b)f_{K+1}\right]\frac{\Phi^{i-1}_{M,K+1}}{2}+\right.$$

$$F^{i-1}_{mp,K+1} f_{K+1} \frac{\Phi^{i-1}_{M,K+1} - \Phi_{M,K}}{\Delta f} \Bigg\} + \frac{\Delta\eta}{2} \frac{\Lambda}{B^{i-1}_{K+1}},$$

$$\begin{aligned} g^j_{M,K+1} &= -\frac{(\Delta\eta)^2}{(B^{i-1}_{K+1})^2} F^{i-1}_{mp,K+1} f_{K+1} \varphi^i_{M,K+1} \frac{\overline{h}_{M,K}}{\Delta f} - \\ &\quad \frac{\kappa}{2} Q^{j-1}_{M+1,K+1} (\varphi^i_{M+1,K+1} - \varphi^i_{M-1,K+1})^2. \end{aligned}$$

If the law ($3b$) is applied, only the coefficient $b^i_{M,K+1}$ will be different. Here it is:

$$\begin{aligned} b^j_{M,K+1} = \frac{Q^{j-1}_{M,K+1}}{\Pr} + \frac{(\Delta\eta)^2}{2(B^{i-1}_{K+1})^2} \Bigg[f_{K+1}\, \varphi^{i-1}_{M,K+1} \left(\frac{2\kappa}{1-\kappa} + \frac{F^{i-1}_{mp,K+1}}{\Delta f} \right) - \\ \frac{2\kappa g}{1-\kappa} (\varphi^{i-1}_{M,K+1})^2 (1 - \varphi^{i-1}_{M,K+1}) \Bigg]. \end{aligned} \qquad (27')$$

The system of algebraic equations (25) comes down to a more suitable form:

$$\begin{aligned} &\varphi^i_{N,K+1} = 1\,, \\ &\varphi^i_{M,K+1} = K^i_{M,K+1} + L^i_{M,K+1} \varphi^i_{M+1,K+1}, \\ &\varphi^i_{1,K+1} = 0\,; \\ &\bar{h}^j_{N,K+1} = 1 - \kappa, \\ &\bar{h}^j_{M,K+1} = K^j_{M,K+1} + L^j_{M,K+1} \bar{h}^j_{M+1,K+1}, \\ &\bar{h}^j_{1,K+1} = \bar{h}_w = const., \\ &(M = N-1,\ N-2,\ ...,\ 3,\ 2) \\ &i,\ j = 1,\ 2,\ 3, ... \end{aligned} \qquad (28)$$

In the recurrent relations (28) the passage coefficients for the dynamic subsystem are:

$$K^i_{M,K+1} = \frac{a^i_{M,K+1} K^i_{M-1,K+1} - g^i_{M,K+1}}{2b^i_{M,K+1} - a^i_{M,K+1} L^i_{M-1,K+1}}, \qquad K^i_{1,K+1} = \varphi^i_{1,K+1} = 0\,,$$

$$L^i_{M,K+1} = \frac{c^i_{M,K+1}}{2b^i_{M,K+1} - a^i_{M,K+1} L^i_{M-1,K+1}}, \qquad L^i_{1,K+1} = 0\,. \tag{29}$$

These coefficients for the thermodynamic subsystem have the same form but are essentially different:

$$\begin{aligned} K^j_{M,K+1} &= \frac{a^j_{M,K+1} K^j_{M-1,K+1} - g^j_{M,K+1}}{2b^j_{M,K+1} - a^j_{M,K+1} L^j_{M-1,K+1}}, \\ K^j_{1,K+1} &= \bar{h}^j_{1,K+1} = \bar{h}_w = const., \end{aligned}$$

$$L^j_{M,K+1} = \frac{c^j_{M,K+1}}{2b^j_{M,K+1} - a^j_{M,K+1} L^j_{M-1,K+1}}, \qquad L^j_{1,K+1} = 0\,. \tag{30}$$

$$(M = 2,\ 3,\ ...,\ N-2,\ N-1)$$

Based on the recurrent formulae, according to the mentioned procedure, the passage coefficients are calculated in direction of increase of the index M. Having passed through all discrete points of the calculating layer twice, solutions of the functions φ and $\bar{h}$ are obtained for that layer. The procedure is repeated for the next calculating layer of the planar integration grid until the integration is done for the whole range of possible changes of the parameter of the form f. In the course of integration, for each calculating layer, based on [16, 17], a number of knobs has been determined as $N = 401$.

In the equation system (23), Prandtl number appears explicitly. Since by nature it depends little on the temperature, we have accepted that the value of Prandtl number is constant and that for air it is

Pr = 0.712. The constants a and b are analyzed in the paper [15], and their values are found to be: $a = 0.4408\,;\ \ b = 5.7140$.

The numerical solution of the equation system (23), i.e. (23'), is done in the programming language FORTRAN. Therefore, a rather complicated program has been written based on the methodology used in the paper [15].

Due to localization per the compressibility, porosity and magnetic parameters, the first derivatives per these parameters are neglected, and the program enables the equations to be solved for in advance given values of these, now, simple parameters. Numerical solutions, obtained in output, are in the form of tables.

5 Obtained results and conclusions

This paper gives only the most important results in the form of corresponding diagrams for both ionized gas electroconductivity variation laws. Figs. 1 and 2 show diagrams of the nondimensional velocity u/u_e, Figs. 3 and 4 show diagrams of the nondimensional enthalpy $\bar{h}$, while Figs. 5 and 6 give distributions of the nondimensional friction function ζ. Diagrams of the nondimensional friction function ζ (Figs. 7 and 8) and the characteristic boundary layer function F_{mp} (Figs. 9 and 10) for different values of the porosity parameter Λ are also given. Based on this and other [14] diagrams that are not presented here, the following conclusions have been drawn:

- Behaviour of the characteristics of the ionized gas boundary layer in case of a porous contour of the body within the fluid, for both electroconductivity variation laws is similar to the behaviour of the corresponding characteristics of similar compressible fluid flow problems [4].

- The nondimensional velocity u/u_e in different cross-sections of the boundary layer very quickly converges towards unity (Figs. 1 and 2).

- Based on these (Figs. 5 and 6), and other diagrams [14] that are not presented in this paper, a significant influence of the magnetic parameter g on the characteristics of the boundary layer

A, B, F_{mp} and ζ is noticed. The influence of this parameter on the nondimensional friction function ζ, and hence on the boundary layer separation point is pointed out. Increasing the values of this parameter, the boundary layer separation point moves up the stream (law $3a$, Fig. 5). When the electroconductivity is varied in accordance with the law $3b$, the increase of the magnetic parameter delays the boundary layer separation point (Fig. 6).

- The compressibility parameter of the ionized gas $\kappa = f_0$ does not have a significant influence on the nondimensional friction function ζ.

- At low values, the porosity parameter Λ has a minor influence on profiles of the nondimensional velocities u/u_e, while at higher values this influence is more significant.

- A change in compressibility parameter has a great influence on distribution of the nondimensional enthalpy $\bar{h}$ in the boundary layer of the ionized gas, which is also the case with other compressible fluid flow problems [4].

- The porosity parameter Λ however, has a greater influence on the nondimensional friction function ζ (Fig. 7), as well as on the characteristic function F_{mp}. Therefore, the porosity parameter also has a significant influence on the boundary layer separation point. It is noticed that injection of the ionized gas moves the boundary layer separation point down the stream. Here, variation of the electroconductivity has a significant influence. Applying the electroconductivity variation law ($3b$), separation of the boundary layer is delayed significantly.

Finally, in order to obtain more accurate and correct results, it is desirable to solve the corresponding equation system (18), i.e. (18') without localization per compressibility $\kappa = f_0$, magnetic g, and especially per the porosity parameter Λ. Clearly, this will cause additional problems in numerical solution. It would also be interesting to study the influence of the electroconductivity σ variation on certain values and characteristics of the boundary layer. The obtained results would be more correct, which would make a remarkable contribution to the boundary layer theory.

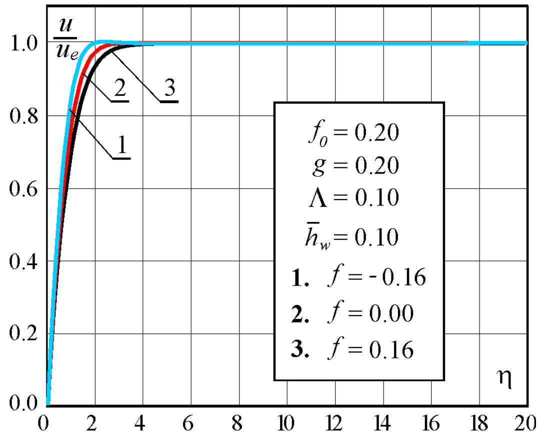

Figure 1: Diagram of the nondimensional velocity u/u_e (a)

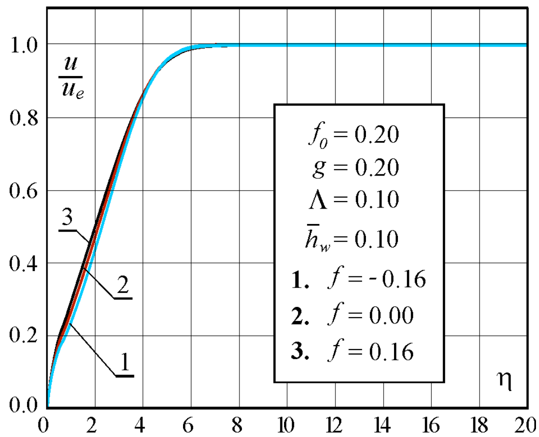

Figure 2: Diagram of the nondimensional velocity u/u_e (b)

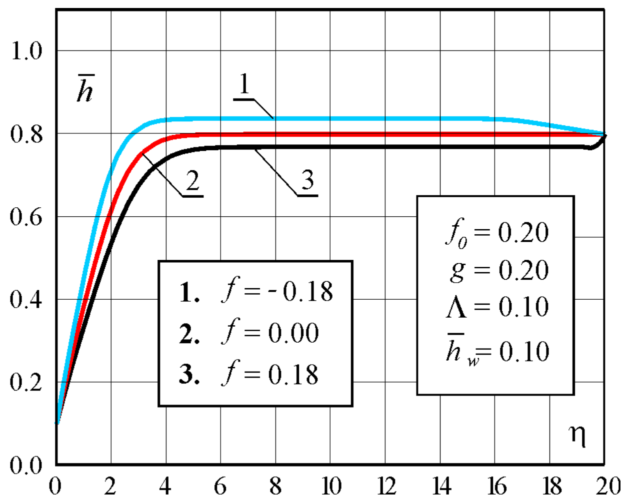

Figure 3: Diagram of the nondimensional enthalpy $\bar{h}$ (a)

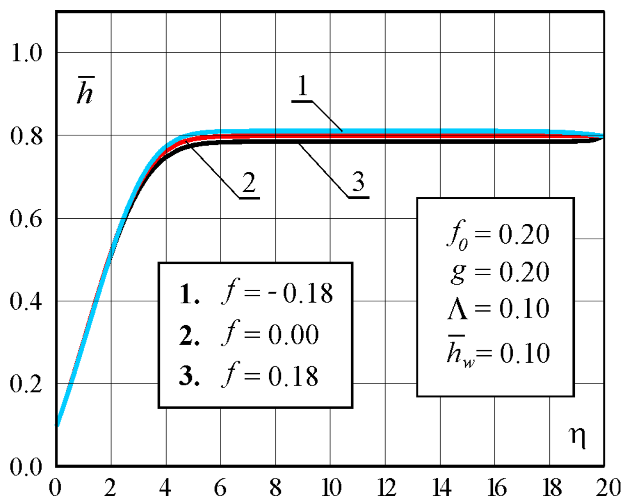

Figure 4: Diagram of the nondimensional enthalpy $\bar{h}$ (b)

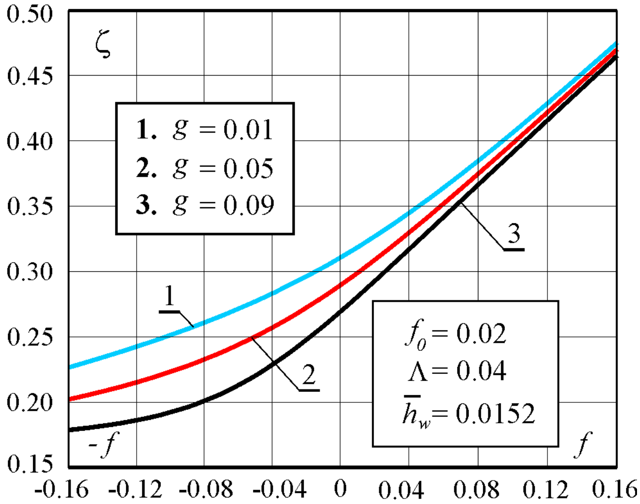

Figure 5: Distribution of the nondimensional friction function ζ (a)

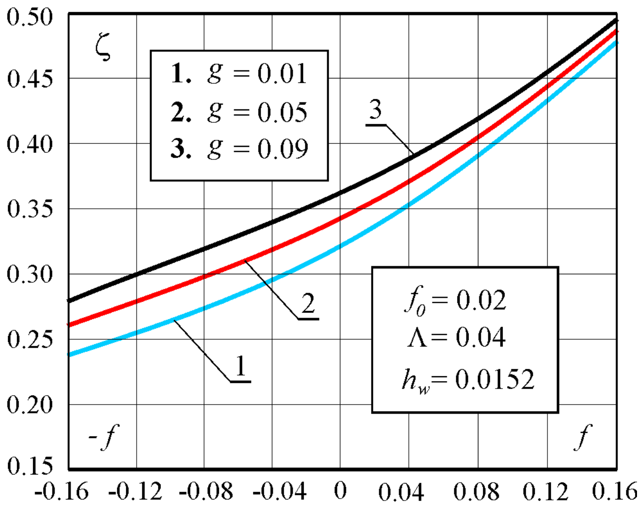

Figure 6: Distribution of the nondimensional friction function ζ (b)

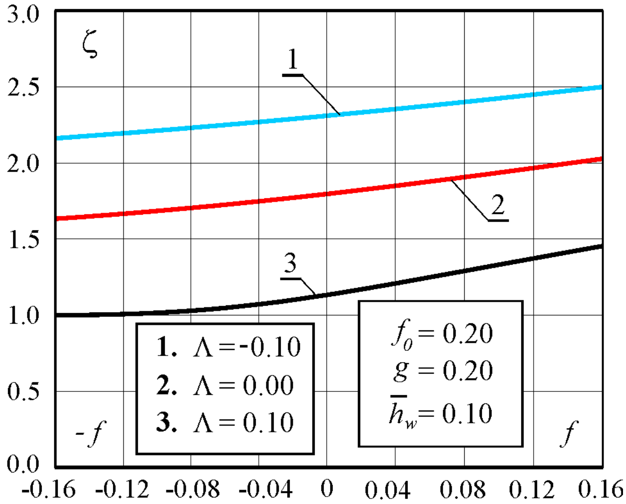

Figure 7: Distribution of the nondimensional friction function $\zeta(\Lambda)$ (a)

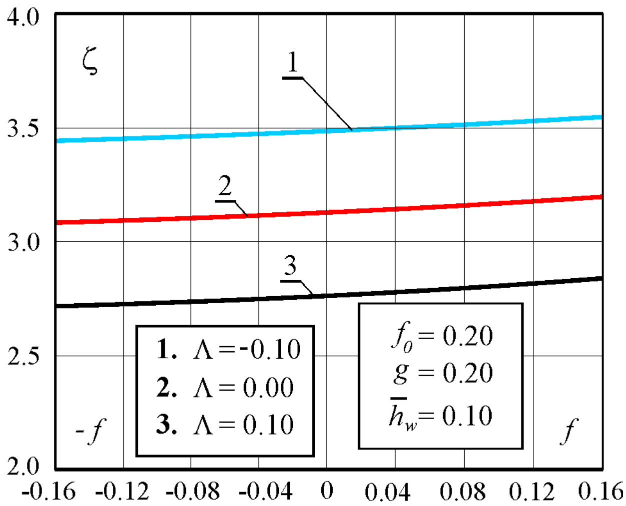

Figure 8: Distribution of the nondimensional friction function $\zeta(\Lambda)$ (b)

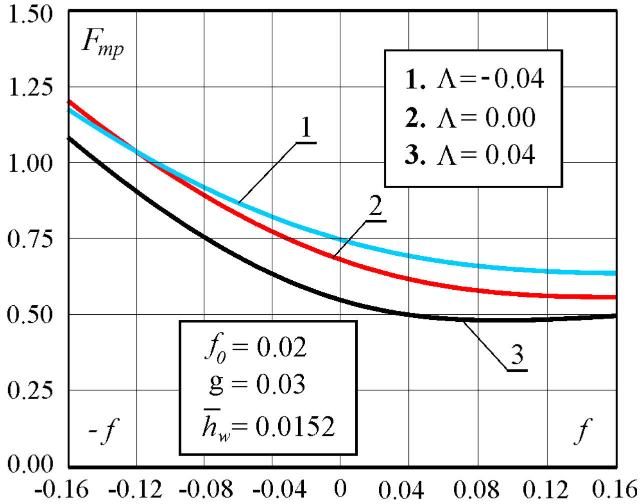

Figure 9: Distribution of the characteristic function $F_{mp}(\Lambda)$ (a)

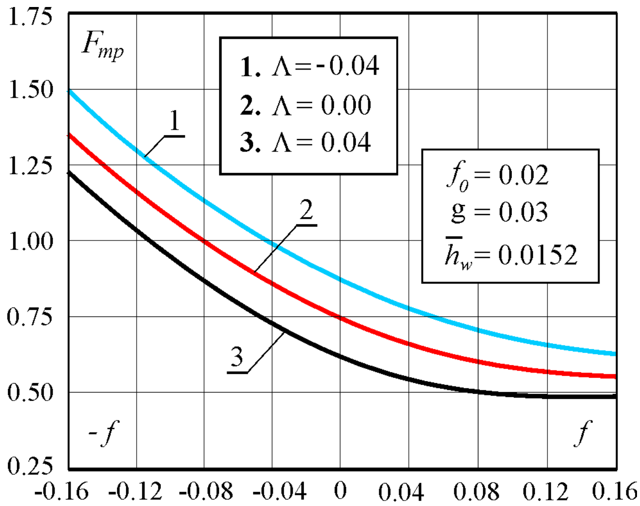

Figure 10: Distribution of the nondimensional function $F_{mp}(\Lambda)$ (b)

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Uticaj variranja elektroprovodnosti na strujanje jonizovanog gasa u graničnom sloju duž poroznog zida

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U radu se istražuje strujanje jonizovanog gasa u graničnom sloju za slučaj da se varira njegova elektroprovodnost. Strujanje je ravansko a kontura opstrujavanog tela je porozna. Prvo je pretpostavljeno da elektroprovodnost σ jonizovanog gasa zavisi samo od podužne promenljive. Zatim je usvojeno da je ista funkcija odnosa podužne brzine i brzine na spoljašnjoj granici graničnog sloja. Za oba navedena zakona promene elektroprovodnosti polazne jednačine graničnog sloja su, primenom metode uopštene sličnosti, dovedene na uopšteni oblik i numerički rešene u četvoroparametarskom tri puta lokalizovanom približenju. Od mnoštva dobijenih rezultata u obliku tabela prikazani su dijagrami najvažnijih bezdimenzijskih veličina i karakterističnih funkcija graničnog sloja i to za oba pretpostavljena zakona elektroprovodnosti. Na kraju su izvedeni i zaključci o uticaju pojedinih fizičkih veličina na strujanje jonizovanog gasa u graničnom sloju.