

The interaction of thermal radiation on vertical oscillating plate with variable temperature and mass diffusion

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Abstract

Thermal radiation effects on unsteady free convective flow of a viscous incompressible flow past an infinite vertical oscillating plate with variable temperature and mass diffusion has been studied. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature is raised linearly with respect to time and the concentration level near the plate is also raised linearly with respect to time. An exact solution to the dimensionless governing equations has been obtained by the Laplace transform method, when the plate is oscillating harmonically in its own plane. The effects of velocity, temperature and concentration are studied for different parameters like phase angle, radiation parameter, Schmidt number, thermal Grashof number, mass Grashof number and time are studied. It is observed that the velocity increases with decreasing phase angle ωt .

Key words: radiation, heat and mass transfer, oscillating, vertical plate.

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List of symbols

a^*	absorption coefficient
A	constant
C'_w	concentration of the plate
C'_∞	concentration in the fluid far away from the plate
C'	species concentration in the fluid
C	dimensionless concentration
C_p	specific heat at constant pressure
D	mass diffusion coefficient
g	acceleration due to gravity
Gr	thermal Grashof number
Gc	mass Grashof number
Pr	Prandtl number
q_r	radiative heat flux in the y -direction
R	radiation parameter
Sc	Schmidt number
T_∞	temperature of the fluid far away from the plate
T_w	temperature of the plate
T	temperature of the fluid near the plate
t'	time
t	dimensionless time
u_0	amplitude of the oscillation
u	velocity component in x -direction
U	dimensionless velocity component in x -direction
x	spatial coordinate along the plate
y	spatial coordinate normal to the plate
Y	dimensionless spatial coordinate normal to the plate
α	thermal diffusivity
β	coefficient of volume expansion
β^*	volumetric coefficient of expansion with concentration
$erfc$	complementary error function
η	similarity parameter
μ	coefficient of viscosity
ν	kinematic viscosity
ωt	phase angle
θ	dimensionless temperature

1 Introduction

The interaction of convection and radiation in absorbing-emitting media occurs in many practical cases. Atmospheric phenomena, shock problems, rocket nozzles, industrial furnaces. Heat and mass transfer in the presence of thermal radiation play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications.

England and Emery[1] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along an isothermal vertical plate were studied by Hossain and Takhar[2]. Raptis and Perdikis[3] studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Das *et al*[4] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was solved by Soundalgekar [5]. The effect on the flow past a vertical oscillating plate due to a combination of concentration and temperature differences was studied extensively by Soundalgekar and Akolkar [6]. Free convection with thermal radiation of the oscillating flow past a vertical plate were analyzed by Mansour[7]. The governing equations are solved using perturbation technique. The effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux has been studied by Soundalgekar *et al* [8].

However the simultaneous heat and mass transfer effects on infinite oscillating vertical plate with variable temperature and mass diffusion, in the presence of thermal radiation is not studied in the literature. It is proposed to study thermal radiation effects on unsteady flow past an infinite oscillating vertical plate with variable temperature and mass diffusion. The dimensionless governing equations are solved using the Laplace transform technique.

2 Mathematical Analysis

Thermal radiation effects on unsteady flow of a viscous incompressible fluid past an infinite vertical oscillating plate with variable temperature and mass diffusion is studied. Consider the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞ . Here, the x -axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate starts oscillating in its own plane with frequency ω' and the temperature of the plate is raised to linearly with respect to time and the concentration level near the plate are also raised linearly with respect to time. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y'^2} \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \quad (3)$$

with the following initial and boundary conditions:

$$\begin{aligned} t' \leq 0 : \quad & u = 0, & T = T_\infty, \\ t' > 0 : \quad & u = u_0 \cos \omega t', & T = T_\infty + (T_w - T_\infty) A t', \\ & u = 0, & T \rightarrow T_\infty, \end{aligned} \quad (4)$$

$$\begin{aligned} C' &= C'_\infty \quad \text{for all } y \\ C' &= C'_\infty + (C'_{t_w} - C'_\infty) A t' \quad \text{for all } y = 0 \\ C' &\rightarrow C'_\infty \quad \text{for all } y \rightarrow \infty \end{aligned}$$

where $A = \frac{u_0^2}{\nu}$.

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma (T_\infty^4 - T^4) \quad (5)$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (6)$$

By using equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3 (T_\infty - T) \quad (7)$$

On introducing the following dimensionless quantities:

$$U = \frac{u}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad Y = \frac{y u_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$Gr = \frac{g \beta \nu (T_w - T_\infty)}{u_0^3}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad Gc = \frac{\nu g \beta^* (C'_w - C'_\infty)}{u_0^3}, \quad (8)$$

$$Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\nu}{D}, \quad R = \frac{16a^* \nu^2 \sigma T_\infty^3}{k u_0^2}, \quad \omega = \frac{\omega' \nu}{u_0^2}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial Y^2} \quad (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr} \theta \quad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} \quad (11)$$

The initial and boundary conditions in non-dimensional form are

$$t > 0 : \begin{array}{llll} U = 0, & \theta = 0, & C = 0, & \text{for all } Y, t \leq 0 \\ U = \cos \omega t, & \theta = t, & C = t, & \text{at } Y = 0 \\ U = 0, & \theta \rightarrow 0, & C \rightarrow 0 & \text{as } Y \rightarrow \infty \end{array} \quad (12)$$

All the physical variables are defined in the nomenclature. The solutions are obtained for hydrodynamic flow field in the presence of thermal radiation.

The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\begin{aligned} \theta = & \frac{t}{2} \left[\exp(2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{at}) + \right. \\ & \left. \exp(-2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{at}) \right] - \\ & \frac{\eta Pr \sqrt{t}}{2\sqrt{R}} \left[\exp(-2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{at}) - \right. \\ & \left. \exp(2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{at}) \right] \end{aligned} \quad (13)$$

$$C = t \left[(1 + 2\eta^2 Sc) \operatorname{erfc}(\eta\sqrt{Sc}) - 2\eta\sqrt{\frac{Sc}{\pi}} \exp(-\eta^2 Sc) \right] \quad (14)$$

$$\begin{aligned} U = & \frac{\exp(i\omega t)}{4} \left[\exp(2\eta\sqrt{i\omega t}) \operatorname{erfc}(\eta + \sqrt{i\omega t}) + \right. \\ & \left. \exp(-2\eta\sqrt{i\omega t}) \operatorname{erfc}(\eta - \sqrt{i\omega t}) \right] + \\ & \frac{\exp(-i\omega t)}{4} \left[\exp(2\eta\sqrt{-i\omega t}) \operatorname{erfc}(\eta + \sqrt{-i\omega t}) + \right. \\ & \left. \exp(-2\eta\sqrt{-i\omega t}) \operatorname{erfc}(\eta - \sqrt{-i\omega t}) \right] + \\ & \frac{Grt}{b(1-Pr)} \left[(1 + 2\eta^2) \operatorname{erfc}(\eta) - \frac{2\eta}{\sqrt{\pi}} \exp(-\eta^2) \right] + \\ & d \operatorname{erfc}(\eta) - \frac{d}{2} \exp(bt) \left[\exp(2\eta\sqrt{bt}) \operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{bt}) + \right. \\ & \left. \exp(-2\eta\sqrt{bt}) \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{bt}) \right] - \\ & \frac{Gc t^2}{6(1-Sc)} \left[(3 + 12\eta^2 + 4\eta^4) \operatorname{erfc}(\eta) - \right. \end{aligned}$$

$$\begin{aligned}
& \frac{\eta}{\sqrt{\pi}}(10 + 4 \eta^2) \exp(-\eta^2) - \\
& (3 + 12 \eta^2 Sc + 4 \eta^4 Sc^2) \operatorname{erfc}(\eta\sqrt{Sc}) + \\
& \frac{\eta\sqrt{Sc}}{\sqrt{\pi}}(10 + 4 \eta^2 Sc) \exp(-\eta^2 Sc) - \\
& \frac{d(1 + bt)}{2} \left[\exp(2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{at}) + \right. \\
& \left. \exp(-2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{at}) \right] + \\
& \frac{bd\eta Pr\sqrt{t}}{2\sqrt{R}} \left[\exp(-2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{at}) - \right. \\
& \left. \exp(2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{at}) \right] + \\
& \frac{bd}{2} \exp(bt) \left[\exp(-2\eta\sqrt{Pr(a+b)t}) \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{(a+b)t}) + \right. \\
& \left. \exp(2\eta\sqrt{Pr(a+b)t}) \operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{(a+b)t}) \right]
\end{aligned}$$

where, $a = \frac{R}{Pr}$, $b = \frac{R}{1 - Pr}$, $d = \frac{Gr}{b^2(1 - Pr)}$ and $\eta = \frac{Y}{2\sqrt{t}}$.

In order to get the physical insight into the problem, the numerical values of U have been computed from equation (15). While evaluating this expression, it is observed that the argument of the error function is complex and, hence, we have separated it into real and imaginary parts by using the following formula:

$$\begin{aligned}
\operatorname{erf}(a + ib) &= \operatorname{erf}(a) + \frac{\exp(-a^2)}{2a\pi} [1 - \cos(2ab) + i \sin(2ab)] \\
&+ \frac{2 \exp(-a^2)}{\pi} \sum_{n=1}^{\infty} \frac{\exp(-n^2/4)}{n^2 + 4a^2} [f_n(a, b) + i g_n(a, b)] + \epsilon(a, b)
\end{aligned}$$

where,

$$\begin{aligned}
f_n &= 2a - 2a \cosh(nb) \cos(2ab) + n \sinh(nb) \sin(2ab) \\
g_n &= 2a \cosh(nb) \sin(2ab) + n \sinh(nb) \cos(2ab) \\
|\epsilon(a, b)| &\approx \approx 10^{-16} |\operatorname{erf}(a + ib)|
\end{aligned}$$

3 Results and Discussion

In order to understand the physical situation of the problem the numerical values of the velocity, temperature and concentration for different values of the phase angle, radiation parameter, Schmidt number, thermal Grashof number, mass Grashof number and time. The purpose of the calculations given here is to assess the effect of different ωt , R , Gr , Gc , Sc and t upon the nature of the flow and transport. The computed values are illustrated in Figures 1 -8. The Laplace transform solutions are in terms of exponential and complementary error function.

Figure 1 is a graphical representation which depicts the concentration profiles for different values of the time ($t = 0.2, 0.4, 0.6, 0.8, 1$), $Sc = 2.01$ and $Pr = 0.71$. The trend shows that the plate concentration increases with increasing values of the time t . Figure 2 represents the effect of concentration profiles at time $t = 1$ for different Schmidt number ($Sc = 0.16, 0.3, 0.6, 2.01$). The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

The temperature profiles are calculated for different values of thermal radiation parameter ($R = 0.2, 2, 5, 10$) from Equation (13) and these are shown in Figure 3. for air ($Pr=0.71$) at time $t = 1$. The effect of thermal radiation parameter is important in temperature profiles. It is noticed that the temperature increases with decreasing radiation parameter. Figure 4 is a graphical representation which depicts the temperature profiles for different values of the time ($t = 0.2, 0.4, 0.6, 1$) and $Pr = 0.71$ in the presence of thermal radiation $R = 0.2$. It is clear that the temperature increases with increasing values of the time t .

The velocity profiles for different phase angles ($\omega t = 0, \pi/4, \pi/3, \pi/2$), $R = 5, Gr = 2, Gc = 2, Sc = 0.6, Pr = 0.71$ and $t = 0.2$ are shown in figure 5. It is clear that the velocity increases with decreasing phase angle ωt . The effect of velocity for different values of the radiation parameter ($R = 0.2, 5, 20$), $\omega t = \pi/4, Gr = 5, Gc = 2, Pr = 0.71, Sc = 0.6$ and $t = 0.4$ are shown in figure 6. The trend shows that the velocity increases with decreasing radiation parameter. It is observed that the velocity decreases in the presence of high thermal radiation.

The velocity profiles for different thermal Grashof number ($Gr = 2, 5$), mass Grashof number ($Gc = 2, 10$), $\omega t = \pi/4$, $R = 0.2$, $Sc = 0.6$, $Pr = 0.71$ and time $t = 0.3$ are shown in Figure 7. It is clear that the velocity increases with increasing thermal Grashof number or mass Grashof number. The effect of velocity profiles for different time ($t = 0.2, 0.3, 0.4$), $R = 0.2$, $\omega t = \pi/4$, $Gr = 5$, $Gc = 2$, $Pr = 0.71$, $Sc = 0.6$ are shown in Figure 8. In this case, the velocity increases gradually with respect to time t .

4 Conclusions

Theoretical solution of thermal radiation effects on unsteady flow past an infinite vertical oscillating plate, in the presence of variable temperature and mass diffusion is studied. The dimensionless governing equations are solved using Laplace transform technique. The effect of velocity, temperature and concentration for different physical parameters like ωt , R , Gr , Gc , Sc and t are studied. It is observed that the velocity increases with decreasing phase angle ωt and radiation parameter R . The trend is just reversed with respect to time t . The temperature decreases due to high thermal radiation. It is also observed that the concentration increases with decreasing Schmidt number.

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**Interakcija termičke radijacije na vertikalnu
oscilujuću ploču sa promenljivom temperature i
difuzijom mase**

UDK 537.84

Proučavaju se uticaji termičke radijacije na viskozno nestišljivo tečenje preko beskonačne vertikalne oscilujuće ploče sa promenljivom temperaturom i difuzijom mase. Smatra se da je fluid siv, apsorbuje i emituje radijaciju ali da nema raspršivanja. Temperatura ploče raste linearno sa vremenom a nivo koncentracije blizu ploče takodje raste linearno sa vremenom. Egzaktno rešenje bezdimenzionih jednačina problema se dobija

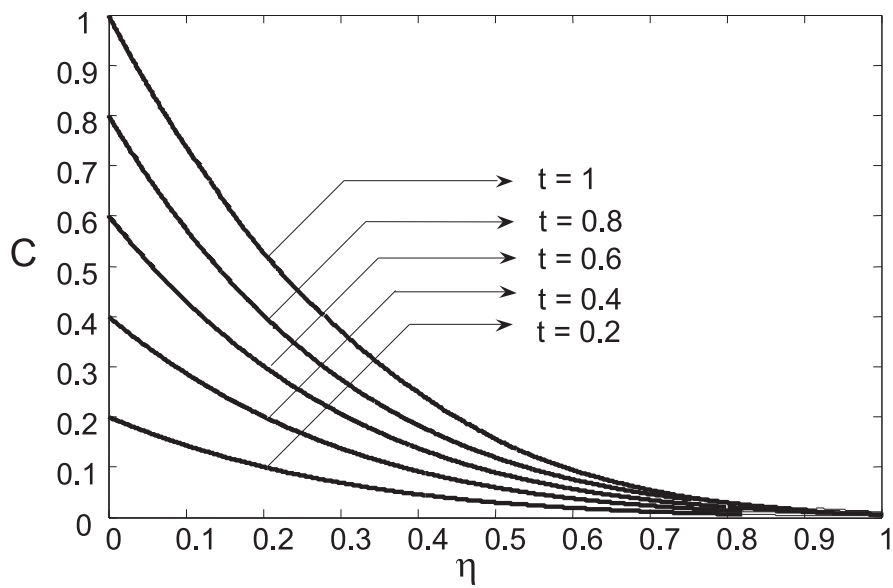


Figure 1: Concentration profiles for different values of t

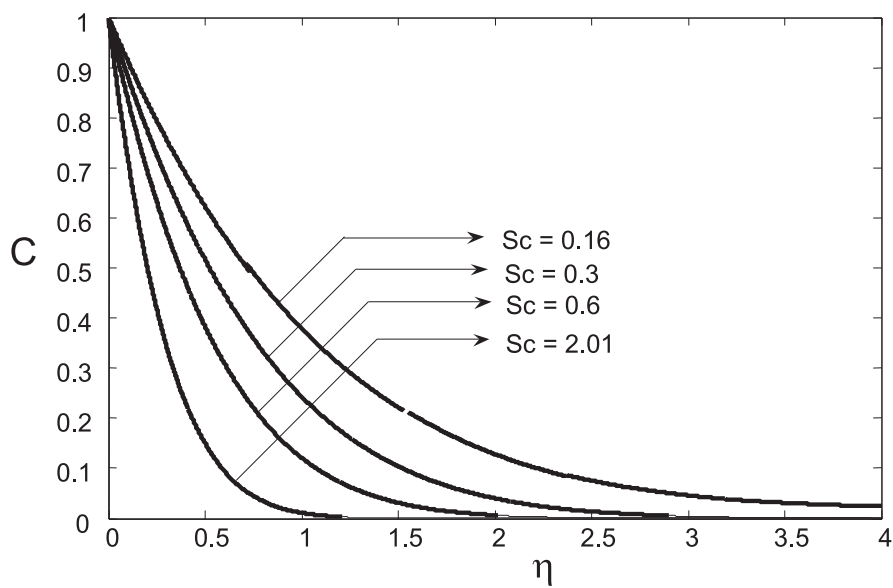
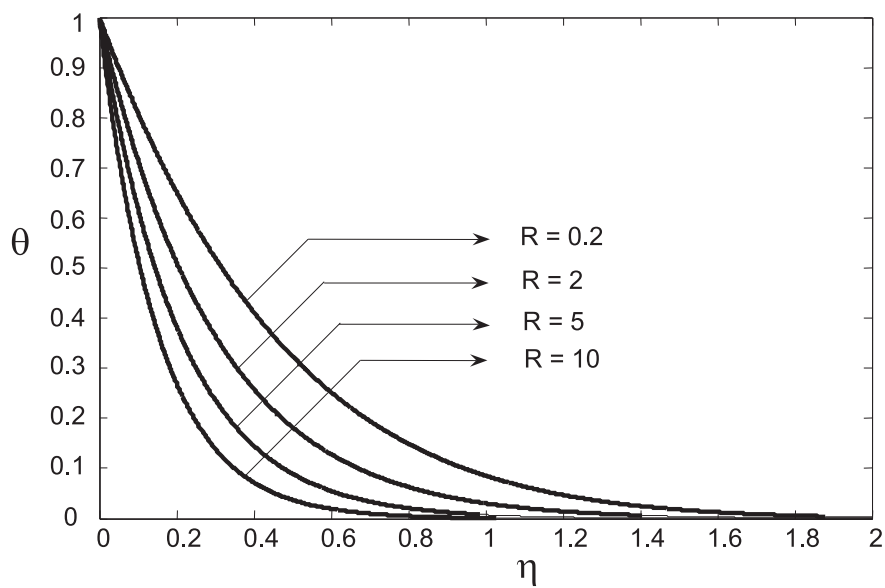
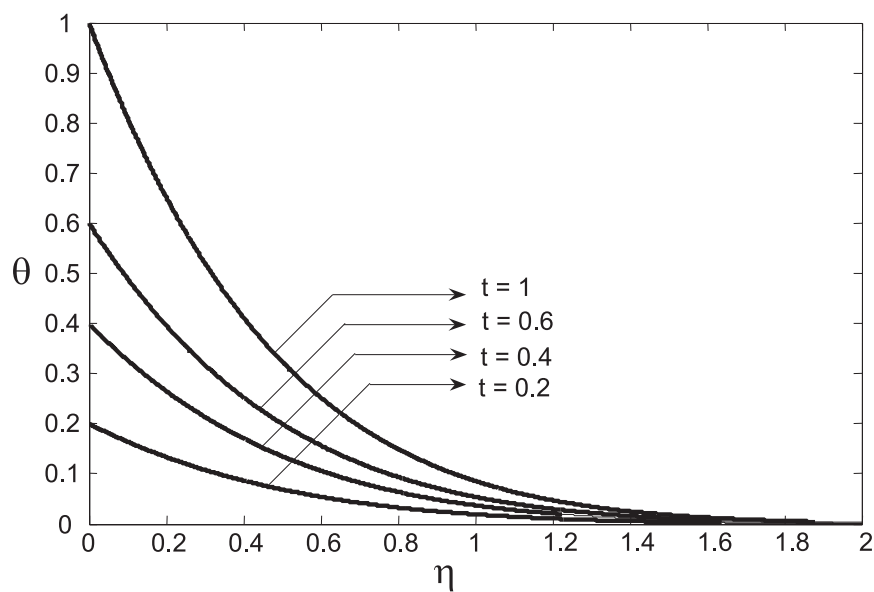


Figure 2: Concentration profiles for different values of Sc

Figure 3: Temperature Profiles for different values of R Figure 4: Temperature Profiles for different values of t

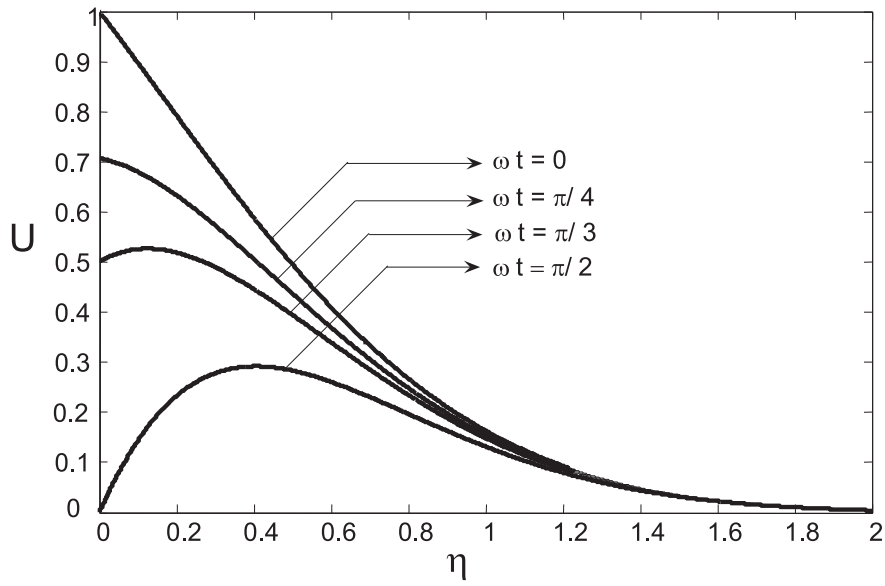


Figure 5: Velocity profiles for different values of ωt

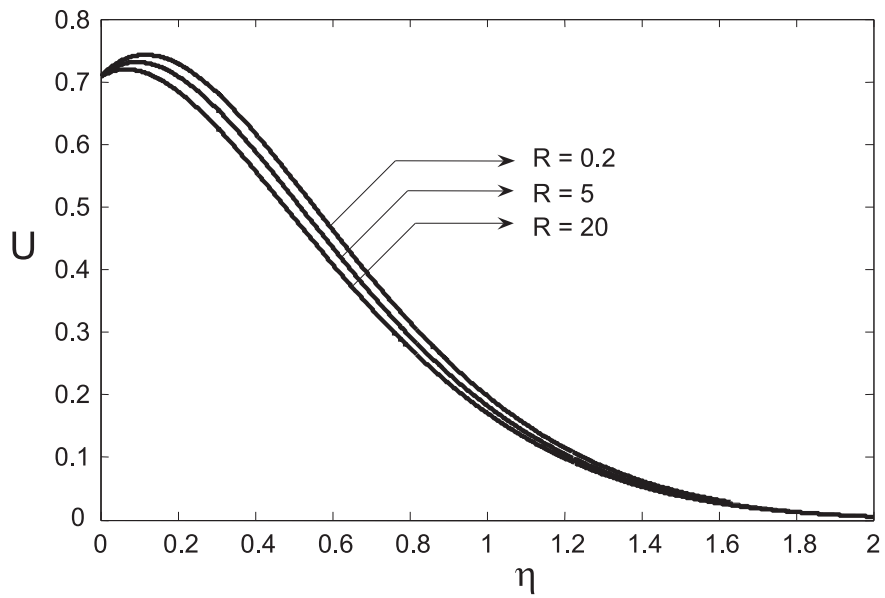


Figure 6: Velocity profiles for different values of R

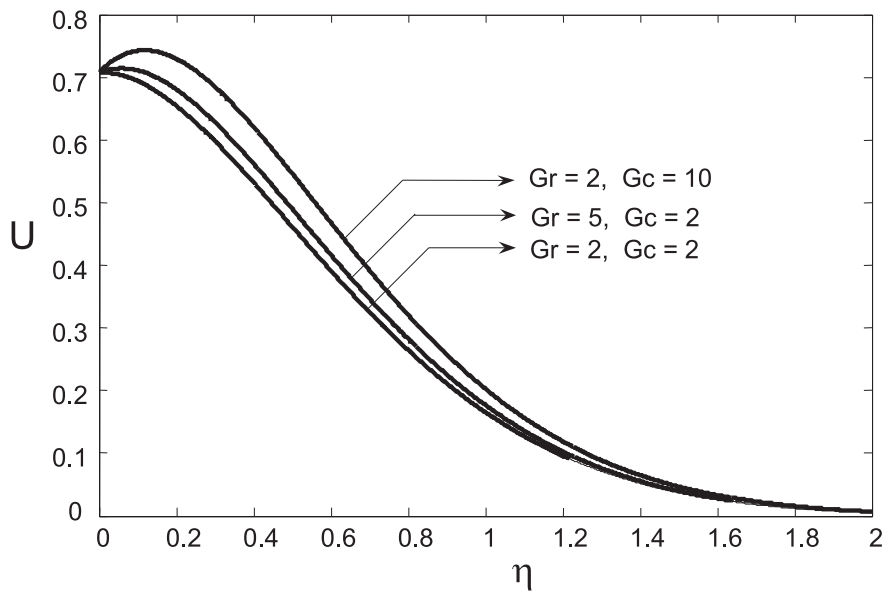


Figure 7: Velocity profiles for different values of Gr, Gc

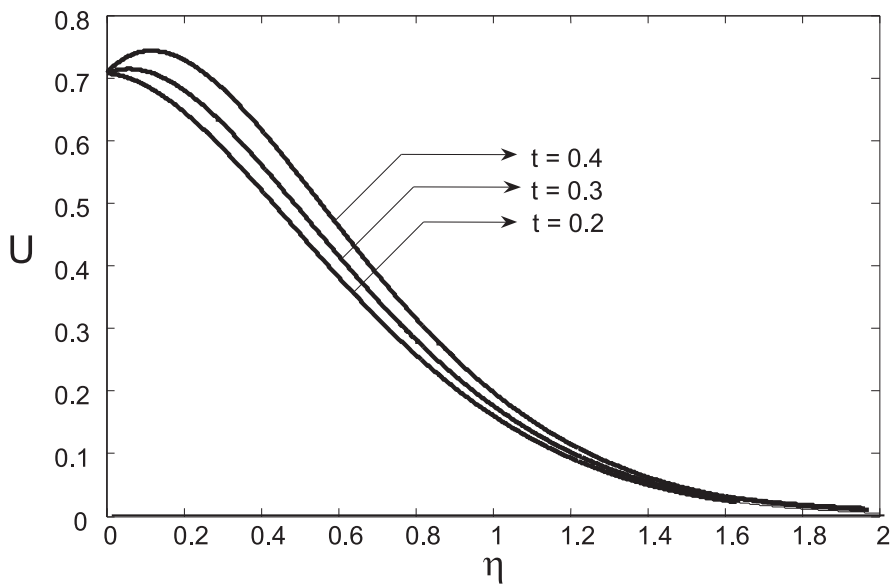


Figure 8: Velocity profiles for different values of t

metodom Laplasove transformacije kada ploča harmonijski osciluje u sopstvenoj ravni. Efekti brzine, temperature i koncentracije se proučavaju pri raznim vrednostima parametara faznog ugla, radijacionog parametra, Šmitovog termičkog Grashofovog broja, masenog Grashofovog broja i vremena. Uočava se porast brzine sa opadanjem faznog ugla ωt .