# Flow around a sphere in an oscillating stream of a dusty fluid 

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#### Abstract

The oscillation of a stream of dusty fluid in the presence of a sphere has been discussed. The effect of interaction of curvature and viscosity has been included in the boundary layer equations. The dust particles slip on the surface of the sphere and the slip velocity is a function of relaxation time of the dust particles but is independent of mass concentration of the gas. Dust particles shift the steady secondary flow towards the main flow and increase the resistance on the sphere.


Keywords: Oscillating Stream, dusty fluid.

## 1 Introduction

It was shown by Schlichting [1] that small oscillations of a circular cylinder in a fluid induce characteristic secondary flows, imparting a steady motion to the fluid. Wang [2] derived the boundary layer equations in a manner in which the interaction of the curvature and viscosity was included in the

[^0]boundary layer equations and showed that this interaction was significant in calculating the resistance on the cylinder. Rauthan [3] discussed the flow around a sphere in an oscillating stream of an elastico-viscous liquid deriving the boundary layer equations by Wang's method.

The flow of a binary mixture of a fluid and solid particles has been subject of interest to engineers and scientists as such flows occur in powder technology, transport of liquid slurries in chemical and nuclear processing. In nature dust and smoke ladden air is also an example of such a mixture. The theory of two-phase system is applicable to physiological flows as blood is considered to be a suspension of red cells in liquid plasma (see, Srivastava and Srivastava [4]).

Saffman [5] proposed the following equations of dusty fluid and analysed the stability of a laminar parallel flow of such a mixture:

$$
\begin{gather*}
\rho\left[\frac{\partial \bar{u}}{\partial t}+(\bar{u} \cdot \nabla) \bar{u}\right]=-\operatorname{gradp}+\mu \nabla^{2} \bar{u}+k N\left(\bar{u}_{p}-\bar{u}\right),  \tag{1}\\
\operatorname{div} \bar{u}=0  \tag{2}\\
m\left[\frac{\partial \bar{u}_{p}}{\partial t}+\left(\bar{u}_{p} \cdot \nabla\right) \bar{u}_{p}\right]=k\left(\bar{u}-\bar{u}_{p}\right)  \tag{3}\\
\frac{\partial N}{\partial t}+\operatorname{div}\left(N \bar{u}_{p}\right)=0 \tag{4}
\end{gather*}
$$

where $\bar{u}$ and $\bar{u}_{p}$ are gas and dust particle velocities respectively, t is time, N is the number density of the dust particles, m is the mass of the dust particle, k is the stokes coefficient of the resistance and $\mathrm{p}, \rho, \mu$ the pressure, density and viscosity of the gas. The gas is taken as incompressible. If X is any entity for the gas then the corresponding entity for the dust particles is denoted by $X_{p}$. Using Saffman's model Michael [6] studied the motion of a sphere in a dusty fluid. Michael and Miller [7] studied the plane parallel flow of a dusty gas. Nag and Datta [8] studied the flow of a dusty gas in a rotating channel.

In this paper we have discussed the oscillations of a stream of dusty fluid in presence of a sphere by taking the equations (1) - (4) and deriving the boundary layer equations for the flow of gas and dust particles using Wang's [2] method of approximation.

## 2 Formulation of the problem

We choose spherical polar coordinates $(r, \theta, \varphi)$ with the origin fixed at the centre of the sphere and the surface of the sphere is taken as $r=a$. The flow is axi- symmetric, hence we take all quantities to be independent of $\varphi$ and also the velocity in the direction of $\varphi$ to be zero. Consider a stream of dusty gas oscillating with frequency $\omega$ at a large distance from the surface of the sphere, so its velocity is $V_{\infty} \cos \omega t$, where $V_{\infty}$ is constant (see, figure-1). As analysed by Michael [6] the potential flow of the gas in presence of the sphere is not changed for small values of the time of relaxation of the gas and so the velocity of the gas outside the boundary layer formed near the surface of the sphere is taken as:


Figure 1: Schematic of the problem

$$
\begin{equation*}
V^{\prime}(\theta, t)=\frac{3}{2} V_{\infty} \sin \theta \cos \omega t \tag{5}
\end{equation*}
$$

where $\theta$ is the angle measured from the forward stagnation point. Justification of this assumption is also given below.

We now assume that

$$
\begin{equation*}
\left|\frac{V^{\prime}}{a} \frac{\partial V^{\prime}}{\partial \theta}\right| \leq\left|\frac{\partial V^{\prime}}{\partial t}\right| \tag{6}
\end{equation*}
$$

which amounts to assuming that Strouhall number $S t=a \omega / V_{\infty}$ to be large. The Reynolds number $R e=\left(V_{\infty} a \rho / \mu\right)$ is taken of the same order as St and the ratio $(S t / R e)=n$ is taken of order unity. Here n can be interpreted as the measure of the ratio of the boundary layer thickness and the amplitude of the oscillations. Physically these assumptions mean that the unsteady forces are balanced by the pressure and viscous forces which are of order $(1 / \epsilon)=S t$ as compared with the inertia forces, where $\epsilon$ is a small quantity. Let $u^{\prime}, v^{\prime}$ and $u_{p}^{\prime}, v_{p}^{\prime}$ be the components of the velocity of the gas and dust particles in the direction of r and $\theta$ respectively in the boundary layer region, then the boundary conditions of the problem are:

$$
\begin{gather*}
u^{\prime}=0, \quad v^{\prime}=0, \quad u_{p}^{\prime}=0 \quad \text { at } \quad r=a .  \tag{7}\\
\left.\begin{array}{c}
u^{\prime} \rightarrow 0, \quad v^{\prime} \rightarrow V^{\prime} \quad \text { as } \quad r \rightarrow \infty \\
u_{p}^{\prime} \rightarrow 0, \quad v_{p}^{\prime} \rightarrow V_{p}^{\prime} \quad \text { as } \quad r \rightarrow \infty,
\end{array}\right\} \tag{8}
\end{gather*}
$$

where $V_{p}^{\prime}$ is the velocity of the dust particles outside the boundary layer region. No condition is prescribed on $v_{p}^{\prime}$ on the surface of the sphere as the order of the differential equations for the particles is one less than those for the gas.

## 3 Equations of motion

We define the following dimensionless quantities:

$$
\left.\begin{array}{l}
u^{\prime}=\in u V_{\infty}, \quad v^{\prime}=v V_{\infty}, \quad u_{p}^{\prime}=\in u_{p} V_{\infty},  \tag{9}\\
v_{p}^{\prime}=v_{p} V_{\infty}, \quad t=\frac{T}{\omega}, \quad r=a(1+\in y), \quad p=\rho a \omega V_{\infty} P .
\end{array}\right\}
$$

Writing (1) in spherical polar coordinates, taking all quantities independent of $\varphi$ and then applying the analysis of Wang [2] we get the following
equations for $u, v, P$ in the direction of $r, \theta$ respectively within the boundary layer region:

$$
\begin{gather*}
\frac{\partial v}{\partial T}+\in\left(u \frac{\partial v}{\partial y}+v \frac{\partial v}{\partial \theta}\right)=-(1-\in y) \frac{\partial P}{\partial \theta}+n\left(\frac{\partial^{2} v}{\partial y^{2}}+2 \in \frac{\partial v}{\partial y}\right)+\alpha \beta\left(v_{p}-v\right) \\
\frac{\partial P}{\partial y}=0\left(\epsilon^{2}\right) \tag{10}
\end{gather*}
$$

where $\alpha=\frac{N m}{\rho}$ is the relative mass concentration of the dust particles, $\beta=\frac{k}{m \omega}=\frac{1}{\tau \omega}, \tau=\frac{m}{k}$ is the time of relaxation of the dust particles and n is the ratio of the boundary layer thickness to the amplitude of oscillations.

Similarly the equation of continuity (2) becomes:

$$
\begin{equation*}
\frac{\partial u}{\partial y}+\frac{\partial v}{\partial \theta}+v \cot \theta+\epsilon\left(2 u-y \frac{\partial v}{\partial \theta}\right)=0 . \tag{12}
\end{equation*}
$$

The equation (3) yields the following equation for the dust particles in the direction of $\theta$ :

$$
\begin{equation*}
\frac{\partial v_{p}}{\partial T}+\in\left(u_{p} \frac{\partial v_{p}}{\partial y}+v_{p} \frac{\partial v_{p}}{\partial \theta}\right)=\beta\left(v-v_{p}\right) . \tag{13}
\end{equation*}
$$

Assuming that N is constant which is consistent with other equations (see, Saffman [5] ), the equation (4) can be written as :

$$
\begin{equation*}
\frac{\partial u_{p}}{\partial y}+\frac{\partial v_{p}}{\partial \theta}+v_{p} \cot \theta+\in\left(2 u_{p}-y \frac{\partial v_{p}}{\partial \theta}\right)=0 . \tag{14}
\end{equation*}
$$

The equation (11) shows that the pressure gradient perpendicular to the surface of the sphere is of $0\left(\epsilon^{2}\right)$ and is negligible, hence as usual the pressure gradient in the direction of $\theta$ in the boundary layer region is taken to be the same as in the potential flow region. The differential equations for the potential flow region are:

$$
\begin{equation*}
\frac{\partial V}{\partial T}+\in V \frac{\partial V}{\partial \theta}=-(1-\in y) \frac{\partial P}{\partial \theta}+\alpha \beta\left(V_{p}-V\right) \tag{15}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial V_{p}}{\partial T}+\in V_{p} \frac{\partial V_{p}}{\partial \theta}=\beta\left(V-V_{p}\right) \tag{16}
\end{equation*}
$$

where $V^{\prime}=V_{\infty} V, V_{p}^{\prime}=V_{\infty} V_{p}$.

## 4 Solution of eqoations

Multiplying (16) by $\alpha$, adding (15) to it and then taking $V=V_{p}$ in the first approximation as $\tau \rightarrow 0$, we get

$$
\begin{equation*}
(1+\alpha)\left(\frac{\partial V}{\partial T}+\in V \frac{\partial V}{\partial \theta}\right)=-(1-\in y) \frac{\partial P}{\partial \theta} \tag{17}
\end{equation*}
$$

Hence the potential flow velocity of the dusty fluid is same as that for the clear fluid in the first approximation ( see, Michael [6] ) i.e.

$$
\begin{equation*}
V=\frac{3}{2} \cos T \sin \theta \tag{18}
\end{equation*}
$$

Equation (16) gives $V_{p}$ in the first approximation as (neglecting $\in$ );

$$
\begin{equation*}
V_{p}=\frac{3}{2} \cos \xi \cos (T-\xi) \sin \theta \tag{19}
\end{equation*}
$$

where $\tan \xi=1 / \beta$.
We write all quantities:

$$
\left.\begin{array}{l}
u=u_{0}+\epsilon u_{1}+0\left(\epsilon^{2}\right), \quad v=v_{0}+\epsilon v_{1}+0\left(\epsilon^{2}\right),  \tag{20}\\
u_{p}=u_{p_{o}}+\epsilon u_{p_{1}}+0\left(\epsilon^{2}\right), \\
v_{p}=v_{p_{o}}+\epsilon v_{p_{1}}+0\left(\epsilon^{2}\right), \quad P=P_{0}+\in P_{1}+0\left(\epsilon^{2}\right)
\end{array}\right\}
$$

The boundary conditions (7) and (8) can be written as

$$
\begin{gather*}
u_{0}=0, \quad v_{0}=0, \quad u_{p_{o}}=0, \quad u_{1}=0, \quad v_{1}=0, \quad u_{p_{1}}=0 \quad \text { at } y=0  \tag{21}\\
v_{0} \rightarrow \frac{3}{2} \sin \theta \cos T, \quad u_{0} \rightarrow 0, \quad v_{1} \rightarrow 0, \quad u_{1} \rightarrow 0,
\end{gather*}
$$

$$
\begin{equation*}
v_{p_{o}} \rightarrow \frac{3}{2} \sin \theta \cos (T-\xi) \cos \xi, v_{p_{1}} \rightarrow 0, u_{p_{o}} \rightarrow 0, u_{p_{1}} \rightarrow 0 \quad \text { as } \quad y \rightarrow \infty \tag{22}
\end{equation*}
$$

Substituting (20) in (10), (12), (13) and (10) and equating terms independent of $\in$ and coefficient of $\in$ on both sides of the equations we get differential equations for $\left(u_{0}, v_{0}, u_{p_{o}}, v_{p_{o}}, P_{0}\right)$ and $\left(u_{1}, v_{1}, u_{p_{1}}, v_{p_{1}}, P_{1}\right)$. The solution of these equations under the boundary conditions (21) and (22) are given by

$$
\begin{gather*}
v_{0}=\frac{3}{2}\left[\cos T-e^{-A \eta} \cos (T-B \eta)\right] \sin \theta,  \tag{23}\\
u_{0}=-\frac{3}{K}\left[\eta \cos T+e^{-A \eta} \cos (T-\psi-B \eta)-\cos (T-\psi)\right] \cos \theta,  \tag{24}\\
v_{p_{o}}=\frac{3}{2}\left[\cos (T-\xi)+e^{-A \eta} \cos (T-\xi-B \eta)\right] \cos \xi \sin \theta,  \tag{25}\\
u_{p_{o}}=-\frac{3}{K}\left[\eta \cos (T-\xi)+e^{-A \eta} \cos (T-\xi-B \eta-\psi)-\right. \\
\cos (T-\psi-\xi)] \cos \xi \cos \theta, \tag{26}
\end{gather*}
$$

where

$$
\left.\begin{array}{l}
A=\cos \psi=[(b+\alpha \beta) /(2 b)]^{1 / 2} \\
B=\cos \psi=[(b-\alpha \beta) /(2 b)]^{1 / 2}  \tag{28}\\
b=\left[\alpha^{2} \beta^{2}+\left(1+\alpha \beta^{2}+\beta^{2}\right)^{2}\right]^{1 / 2} \\
y=a \eta, K=\left[\frac{1+(\alpha \beta+\beta)^{2}}{n^{2}\left(1+\beta^{2}\right)}\right]^{1 / 4}
\end{array}\right\}
$$

For calculating the second approximation it is seen that the convective terms in the equation contribute terms with $\cos ^{2} T$. These terms can be reduced to terms with $\cos 2 T$ and steady state (time independent) term. Moreover the interaction viscous and curvature terms contributes terms with $\cos T$. Hence the differential equations for second approximation are solved by assuming $v_{1}$ and $v_{p_{1}}$ of the following forms:

$$
\begin{align*}
& v_{1}=\frac{9}{8}\left[f(\eta)+g(\eta) e^{2 i T}\right] \sin 2 \theta+\frac{3}{2} h(\eta) \sin \theta e^{i T}  \tag{29}\\
& v_{p_{1}}=\frac{9}{8}\left[F(\eta)+G(\eta) e^{2 i T}\right] \sin 2 \theta+\frac{3}{2} H(\eta) \sin \theta e^{i T} \tag{30}
\end{align*}
$$

Complex notation is adopted herewith the convention that only real part of a complex quantities will has physical meaning. The term $h(\eta) e^{i T} \sin \theta$ representing the effect of curvature is important as it contributes in calculating the resistance on the surface of the sphere. This terms will be absent if we adopt Schlichting's method of calculation. On substituting the expression for the first order solution and above form of expression for $v_{1}$ and $v_{p_{1}}$ in the differential equation for the second approximation we get the following solution:

$$
\begin{align*}
& 2 f(\eta)=-\sin 2 \psi\left[\frac{1-2 \cos \psi}{4 A^{2}}\left(1-e^{-2 A \eta}\right)+2\left(1-e^{-A \eta} \cos B \eta\right)-\right. \\
&\left.6\left\{\cos 2 \psi-e^{-A \eta} \cos (B \eta+2 \psi)\right\}+2 \eta e^{-A \eta} \cos (B \eta+\psi)\right],  \tag{31}\\
& 2 F(\eta)= 2 f(\eta)-\frac{\beta}{1+\beta^{2}}[1-2\{\eta \cos (\psi-B \eta)- \\
&\left.\cos (2 \psi-B \eta)+\cos B \eta\} e^{-A \eta}+(1-\cos 2 \psi)^{-2 A \eta}\right],  \tag{32}\\
& 2 g(\eta)=-\frac{1}{n K^{2}}\left[1+\frac{\alpha \beta e^{-2 i \xi} \cos ^{2} \xi}{\beta+2 i}\right] \\
& {\left[\frac{e^{-2(A+i B) \eta}-e^{-(C+i D) \eta}}{4 e^{2 i \psi}-e^{2 i \chi}}+\frac{4 e^{2 i \psi}\left(e^{-(A+i B) \eta}-e^{-(C+i D) \eta}\right)}{\left(e^{2 i \psi}-e^{2 i \chi}\right)^{2}}\right.} \\
&\left.+2 \eta e^{i \psi} \frac{e^{-(A+i B) \eta}}{e^{2 i \psi}-e^{2 i \chi}}\right],  \tag{33}\\
& 2 G(\eta)= \frac{1}{\beta+2 i}\left[2 \beta g(\eta)+\left\{e^{-2(A+i B) \eta}+\right.\right. \\
&\left.\left.2 e^{-(A+i B) \eta+i \psi}\right\} e^{-2 i \xi} \cos ^{2} \xi\right], \tag{34}
\end{align*}
$$

$$
\begin{gather*}
2 h(\eta)=-\frac{\eta}{K}\left[\sin 2 \psi-e^{-A \eta} \cos B \eta\right]  \tag{35}\\
2 H(\eta)=-\frac{\eta}{K}\left[\sin (2 \psi+\xi)-e^{-A \eta} \cos (B \eta+\xi)\right] \cos \xi \tag{36}
\end{gather*}
$$

where,

$$
\left.\begin{array}{l}
C=\cos \chi=[(q+2 \alpha \beta) /(2 q)]^{1 / 2},  \tag{37}\\
D=\sin \chi=[(q-2 \alpha \beta) /(2 q)]^{1 / 2}, \\
q=\left[4 \alpha^{2} \beta^{2}+\left(4+\alpha \beta^{2}+\beta^{2}\right)^{2}\right]^{1 / 2}
\end{array}\right\}
$$

## 5 Discussions and conclusions

The total non-dimensional resistance Q on the surface of the sphere is given by:

$$
Q=\frac{1}{\pi \rho V_{\infty}^{2} a^{2}} \int_{0}^{\pi}\left(\tau_{r \theta} \sin \theta+P \cos \theta\right)_{\eta=0} 2 \pi a^{2} \sin \theta d \theta
$$

Since the boundary layer is assumed to be thin, $\int_{0}^{\pi} P \cos \theta \sin \theta d \theta$ can be taken as zero. The expression for $\left(\tau_{r \theta}\right)_{\eta=0}$ can be written as:

$$
\begin{equation*}
\left(\tau_{r \theta}\right)_{\eta=0}=\rho n V_{\infty}^{2}\left(\frac{\partial v_{0}}{\partial \eta}+\epsilon \frac{\partial v_{1}}{\partial \eta}\right)_{\eta=0}+0\left(\epsilon^{2}\right) \tag{38}
\end{equation*}
$$

Substituting (23) and (29) in (38), Q is given by :

$$
\begin{equation*}
(Q / n)=4[(K \cos \psi-\epsilon \sin 2 \psi+\in) \cos T-(K \sin \psi-\in \cos 2 \psi) \sin T] \tag{39}
\end{equation*}
$$

The amplitude of oscillations $|Q|$ is given by:

$$
\begin{equation*}
|Q|^{2}=16 n^{2}\left[K^{2}+2 \in^{2}-2 K \in B\left(3-4 B^{2}\right)+2 A \in(K-2 \in B)\right] \tag{40}
\end{equation*}
$$

Taking $\in=1 / 5$ and $n=1$, the values of $|Q|$ has been given in table- 1 for $\alpha=0.05,0.10,0.15,0.20,0.25,0.30$ and $\beta=1,2,3,4,5,10,15$.

Table 1: The values of $|Q|$ for various values of $\alpha$ and $\beta$.

| $\alpha \beta$ | 1 | 2 | 3 | 4 | 5 | 10 | 15 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.05 | 4.0369 | 4.0689 | 4.0812 | 4.0869 | 4.0900 | 4.0952 | 4.0966 |
| 0.10 | 4.0756 | 4.1383 | 4.1618 | 4.1726 | 4.1785 | 4.1884 | 4.1910 |
| 0.15 | 4.1189 | 4.2089 | 4.2417 | 4.2572 | 4.2656 | 4.2797 | 4.2835 |
| 0.20 | 4.0578 | 4.2773 | 4.3208 | 4.3405 | 4.3512 | 4.3693 | 4.3742 |
| 0.25 | 4.2048 | 4.3467 | 4.3991 | 4.4228 | 4.4356 | 4.4572 | 4.4629 |
| 0.30 | 4.2452 | 1.4160 | 4.4765 | 4.5038 | 4.5186 | 4.5434 | 4.5501 |

Table-1 shows that the amplitude of oscillations of the resistance on the surface of the sphere increases with the increase of $\alpha$, the relative mass concentration of the dust particles and also with the increase of $\beta$. Hence the amplitude of resistance decreases with the increase of oscillation frequency $\omega$ or relaxation time of the dust particles. Rauthan [3] has found that in a similar case the non-linear in the constitutive equation increases the amplitude of resistance which implies that adding dust particles to a clear fluid makes it to behave like a elastico-viscous fluid.

Also we observe that $v_{p_{o}}(0)=0$ and $v_{p_{1}}(0)=\frac{9}{8} G(0) e^{2 i T} \sin 2 \theta$, where

$$
G(0)=\frac{e^{-2 i \xi} \cos ^{2} \xi}{2(\beta+2 i)}
$$

Hence

$$
\begin{equation*}
\operatorname{realv}_{p}^{\prime}(0)=\frac{9 V_{\infty} \in \beta^{2} \cos (2 T-\zeta-2 \xi)}{16\left(1+\beta^{2}\right) \sqrt{4+\beta^{2}}} \sin 2 \theta \tag{41}
\end{equation*}
$$

where $\tan \zeta=2 / \beta$.
So we conclude that dust particles slip along the surface of the sphere. The slip velocity is zero at $\theta=0, \pi / 2,3 \pi / 2, \pi$ and it is maximum at $\theta=\pi / 4,3 \pi / 4,5 \pi / 4,7 \pi / 4$. The maximum amplitude of oscillation $A_{m}$ of the slip velocity is given by:

$$
\begin{equation*}
A_{m}=\frac{9}{16} \frac{V_{\infty} \in \beta^{2}}{\left(1+\beta^{2}\right) \sqrt{4+\beta^{2}}} \approx \frac{9}{16} \frac{V_{\infty} \in}{\beta}=\frac{9}{16} \frac{V_{\infty}^{2} \tau}{a} \quad, \text { for large } \beta \tag{42}
\end{equation*}
$$

This shows that for small values of time of relaxation of the dust particles $\tau$, the amplitude of the slip velocity is propotional to $\tau$ and inversely propotional to the radius of sphere and it is independent of the frequency of oscillation $\omega$ of the main stream. The slip velocity has a phase lag of $\tan ^{-1}(2 \omega \tau)+2 \tan ^{-1}(\omega \tau)$ with the oscillations of the main stream at all points on the surface of sphere. For small values of $\omega \tau$ this phase lag is approximately $4 \omega \tau$.

It can be seen from (31) and (32) that $f(\eta)$ and $F(\eta)$ do not satisfy the outer boundary conditions and hence with the present perturbation scheme a steady secondary inflow for the gas as well as for the dust particles persists outside the shear wave region. The steady flow of the gas and dust particle at a large distance from the surface of the sphere are given by

$$
\begin{gather*}
2 f(\infty)=\frac{B}{2 A}\left(48 A^{4}-28 A^{2}-3\right)  \tag{43}\\
2 F(\infty)=2 f(\infty)-\frac{\beta}{1+\beta^{2}} \tag{44}
\end{gather*}
$$

The values of $-f(\infty)$ have been given in table- 2 for $\alpha=0.05,0.10,0.15$, $0.20,0.25,0.30$ and $\beta=1,2,3,4,5,10,15$. When $\alpha=0, \beta=0$, the value of $f(\infty)$ is -1.25 , hence dust particles reduce the magnitude of this flow. This result is similar to that of Rauthan[3] in case of elastico-viscous fluid.

The above table shows that the more we add the dust the lesser is the inflow. The inflow also decreases with decrease of $\beta$ i.e. increase of time of relaxation or frequency of oscillation of the main stream. Equation (44) shows that the steady velocity of the dust particle relative to the gas dose not depend on the relative mass concentration and is a function of $\beta$. It increases with the increase of $\beta$ and becomes maximum when $\beta=$ 1 and then again decreases. To understand the steady flow of the gas in detail a stream function is introduced for this flow in the form

Table 2: The values of $-f(\infty)$ for various values of $\alpha$ and $\beta$.

| $\alpha \beta$ | 1 | 2 | 3 | 4 | 5 | 10 | 15 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.05 | 1.2074 | 1.2164 | 1.2249 | 1.2303 | 1.2339 | 1.2417 | 1.2445 |
| 0.10 | 1.1671 | 1.1854 | 1.2019 | 1.2124 | 1.2193 | 1.2343 | 1.2394 |
| 0.15 | 1.1288 | 1.1568 | 1.1809 | 1.1960 | 1.2060 | 1.2274 | 1.2348 |
| 0.20 | 1.0927 | 1.1302 | 1.1615 | 1.1809 | 1.1937 | 1.2211 | 1.2306 |
| 0.25 | 1.0585 | 1.1056 | 1.1435 | 1.1671 | 1.1824 | 1.2153 | 1.2267 |
| 0.30 | 1.0261 | 1.0827 | 1.1270 | 1.1542 | 1.1720 | 1.2100 | 1.2232 |

$$
\begin{align*}
\Psi(\theta, \eta) & =\frac{9}{8} \sin 2 \theta \int f(\eta d \eta \\
\Psi(\theta, \eta) & =\frac{9 B}{32 K A} M(\eta) \sin 2 \theta \tag{45}
\end{align*}
$$

where

$$
\begin{align*}
M(\eta)= & {\left[2 \eta\left(48 A^{4}-28 A^{2}-3\right)+8 A^{3}\left(16 A^{2}-13\right)\left(e^{-A \eta} \cos B \eta-1\right)+\right.} \\
& 8 A^{2} B\left(16 B^{2}-11\right) e^{-A \eta} \sin B \eta-8 A^{2}\left\{\left(2 A^{2}-1\right) \cos B \eta+\right. \\
& \left.2 A B \sin B \eta\} \eta e^{-A \eta}-\left(\frac{3-4 A^{2}}{2 A}\right)\left(e^{-2 A \eta}-1\right)\right] \tag{46}
\end{align*}
$$

When $\alpha=0, \beta=0$, we get

$$
\begin{align*}
\Psi(\theta, \eta)= & -\frac{9 \sqrt{2}}{64}\left[10 \eta+20\left(e^{-\eta} \cos \eta-1\right)+\right. \\
& \left.12 e^{-\eta} \sin \eta+8 \eta e^{-\eta} \sin \eta+e^{-2 \eta}-1\right] \sin 2 \theta \tag{47}
\end{align*}
$$

Stream lines are drawn in fig 2. for $\alpha=0, \beta=0$ and in fig 3. for $\alpha=0.05, \beta=5$ respectively it can be seen from (46) that in the region $0<\theta<\pi / 2$ the function $\Psi>0($ or $<0)$ according as $M(\eta)>0(o r<0)$ and $\Psi=0$ for $M(\eta)=0$. Fig 2. reveals that $M(\eta)$ vanishes when $\eta=$ 1.628 for $\alpha=0, \beta=0$. Thus in the region $0<\eta<1.628$ the stream lines corresponds to the positive values of $\Psi$, while in the region $\eta>1.628$ they corresponds to the negative values of $\Psi$. This situation is reversed in the
region $\pi / 2<\theta<\pi$ and we take $\alpha=0.05, \beta=5, M(\eta)$ vanishes when $\eta$ $=2.395$, which shows that the dust particles shift the entire flow patterns towards the main flow. Rauthan [3] founds similar effects of elasticity of liquid when an elastico-viscous liquid oscillates near the surface of the sphere.


Figure 2: Streamline pattern for the steady secondary flow for $\alpha=0, \beta=$ 0.

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Figure 3: Streamline pattern for the steady secondary flow for $\alpha=$ $0.05, \beta=5$.

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## Tečenje oko sfere u oscilujućoj struji prašnjavog fluida

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Razmatra se oscilacija struje prašnjavog fluida u prisustvu sfere. Efekt interakcije krivine i viskoznosti je uključen u jednačine graničnog sloja. Prašnjave čestice klizaju po sferi pa je brzina klizanja funkcija vremena relaksacije prašnjavih čestica ali je nezavisna od masene koncentracije gasa. Prašnjave čestice pomeraju stacionarno sekundarno tečenje i povećavaju otpor na sferi.


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