Theoret. Appl. Mech., Vol.32, No.2, pp. 165–190, Belgrade 2005

Boundary layer of the dissociated gas flow over a porous wall under the conditions of equilibrium dissociation

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Abstract

This paper studies the ideally dissociated air flow in the boundary layer when the contour of the body within the fluid is porous. By means of adequate transformations, the governing boundary layer equations of the problem are brought to a general form. The obtained equations are numerically solved in a three-parametric localized approximation. Based on the obtained solutions, very important conclusions about behaviour of certain boundary layer physical values and characteristics have been drawn.

Keywords: boundary layer, dissociated gas, equilibrium dissociation, porous contour, general similarity method, porosity parameter

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Nomenclature

A, B	boundary layer characteristics
a, b	constants
C_i	mass concentration of any i component
c_p	specific heat of gas dissociated at constant pressure
$D_{21} = D_{12} = D$	coefficient of atomic component diffusion
D_i	diffusion coefficient of any i component
F_{dp}	characteristic boundary layer function
$f_1 = f$	first form parameter
f_k	set of form parameters
H	boundary layer characteristic
h	enthalpy
$\bar{h}$	nondimensional enthalpy
h_e	enthalpy at the outer edge of the boundary layer
h_i	enthalpy of the mass unit of any i component
h_w	enthalpy at the wall of the body within the fluid
h_1	enthalpy at the front stagnation point of the body within the fluid
Le	Lewis number
l	function
M	discrete point
Pr	Prandtl number
p	pressure
Q	nondimensional function
R_i	gas constant of any i component
Sm_i	Schmidt number (diffusion Prandtl number) of any
i	component
s	new longitudinal variable
u	longitudinal projection of velocity in the boundary layer
u_e	velocity at the boundary layer outer edge
V_w	conditional transversal velocity
v	transversal projection of velocity in the boundary layer
v_w	velocity of injection (or ejection) of the fluid
$\dot{W}_i$	mass formation rate of any i component
$x,\ y$	longitudinal and transversal coordinate
Z^{**}	function

z	new transversal variable
Δ^*	conditional displacement thicknesses
Δ^{**}	conditional momentum loss thickness
ζ	nondimensional friction function
η	nondimensional transversal coordinate
$\kappa = f_0$	local compressibility parameter
$\Lambda_1 = \Lambda$	first porosity parameter
Λ_k	set of porosity parameters
λ	thermal conductivity coefficient
μ	dynamic viscosity
μ_0	known values of dynamic viscosity of the dissociated gas
μ_w	given distributions of dynamic viscosity at the wall of the body within the fluid
ν_0	kinematic viscosity at a concrete point of the boundary layer
ρ	density of ideally dissociated gas
ρ_e	dissociated gas density at the outer edge of the boundary layer
ρ_0	known values of density of the dissociated gas
ρ_w	given distributions of density at the wall of the body within the fluid
τ_w	shear stress at the wall of the body within the fluid
Φ	nondimensional stream function
ψ	stream function
ψ^*	new stream function

1 Introduction

Governing equations of the considered problem

This paper investigates dissociated gas (air) flow in the boundary layer under conditions of equilibrium dissociation. To be more precise, it investigates laminar boundary layer on a body of arbitrary shape, whereas the dissociated gas flow is planar and the contour of the body within the fluid is porous.

The main goal of this investigation, as with our earlier studies, is to apply the *general similarity method* to obtain the so-called generalized boundary layer equations of the considered problem, and to solve them.

When the flow velocity of the gas (air) is high, as with supersonic flight of aircrafts through the Earth atmosphere, the temperature in the viscous boundary layer increases significantly. These high temperatures cause thermochemical reactions of dissociation and recombination. Due to the thermochemical processes in the boundary layer, the air becomes a multicomponent mixture of atomic and molecular components. Therefore, for the steady gas mixture flow followed with chemical reactions, the complete equation system of laminar planar boundary layer has the following form:

$$\rho u\,\frac{\partial u}{\partial x}+\rho v\,\frac{\partial u}{\partial y}=-\frac{dp}{dx}+\frac{\partial}{\partial y}\left(\mu\,\frac{\partial u}{\partial y}\right)\,,$$

$$\frac{\partial}{\partial x}\,(\rho u)+\frac{\partial}{\partial y}\,(\rho v)=0\,,$$

$$\rho u\,\frac{\partial C_i}{\partial x}+\rho v\,\frac{\partial C_i}{\partial y}=\frac{\partial}{\partial y}\left(\rho\,D_i\,\frac{\partial C_i}{\partial y}\right)+\dot{W}_i\,,\quad (i=1,\ 2,\ ...,\ q-1) \tag{1}$$

$$\rho\,u\frac{\partial h}{\partial x}+\rho v\frac{\partial h}{\partial y}\;=\;u\,\frac{dp}{dx}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\frac{\partial}{\partial y}\left(\frac{\mu}{\mathrm{Pr}}\,\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial y}\left[\sum_i\rho h_i D_i\left(1-\frac{Sm_i}{\mathrm{Pr}}\right)\frac{\partial C_i}{\partial y}\right]\,,$$

$$p=\rho\,\bar{R}T\,,\qquad \bar{R}=\sum_i C_i R_i\,,\quad \Big(\sum_i C_i=1\Big)\,.$$

The equations of the system (1) stand for: dynamic equation, mixture continuity equation, diffusion equation of (any) i component, energy equation and mixture state equation, respectively. Each of the mixture components ($i=1\div\ q-1$) has a corresponding diffusion equation.

The notations common in the boundary layer theory are used for certain physical values in these equations [1, 2].

According to Lighthill, gas mixture flow can be replaced with a binary mixture model consisting only of an atomic and a molecular component. Ideally dissociated gas is defined this way. Air at temperatures around 2 000 K and even up to around 8 000 K can be considered [1] an ideally dissociated gas. An ideally dissociated gas model can be applied to the boundary layer flow. Then the mass concentration of the atomic component is defined as $C_1 = \rho_1/\rho = \rho_A/\rho = C_A = \alpha$; while the mass concentration of the molecular component is $C_2 = \rho_2/\rho = \rho_M/\rho = C_M = 1 - \alpha$, where the subscripts A and M stand for atomic, i.e., molecular component of the ideally dissociated gas.

When dissociation and recombination velocities are high enough, thermochemical *equilibrium* is established in the boundary layer. In that case the concentration $C_1 = \alpha$ is directly related to the absolute temperature i.e. enthalpy.

If it is assumed that the thermochemical equilibrium is established in the whole boundary layer area, then [3, 4], the boundary layer equations (15) can be written in the following form:

$$
\begin{aligned}
&\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0\,, \\
&\rho\, u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = \rho_e u_e\,\frac{du_e}{dx} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \\
&\rho\, u\frac{\partial h}{\partial x} + \rho v\frac{\partial h}{\partial y} = -u\,\rho_e u_e\,\frac{du_e}{dx} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \\
&\qquad + \frac{\partial}{\partial y}\left[\frac{\mu}{\mathrm{Pr}}\,(1+l)\,\frac{\partial h}{\partial y}\right].
\end{aligned}
\tag{2}
$$

This equation system (2) was solved and analyzed by Krivcova in her papers [3, 4], analytically by development of power series with respect to so-called parameters, or numerically by application of the so-called parametric method. The equations were solved under boundary conditions for the *nonporous contour* of the body within the fluid.

Compressible fluid flow problems have been investigated by scientists all around the world as well as in Serbia, especially by Saljnikov [10] together with the members of the so-called Belgrade School of Boundary Layer. To our knowledge, the most important results obtained by investigation of the dissociated gas flow were presented by Dorrance in [1]. Members of the School led by Loitsianskii [8] also obtained some important results in the field of dissociated gas flow in the boundary layer.

This paper presents the results of investigation of the ideally dissociated gas (air) flow under conditions of equilibrium dissociation where the wall of the body within the fluid is *porous*. These results were obtained by application of the general similarity method firstly suggested by Loitsianskii, and later improved by Saljnikov, but mostly for its application to incompressible fluid flow in the boundary layer.

The corresponding boundary conditions of the considered flow problem are:

$$u=0\,,\qquad \underline{v=v_w(x)}\,,\quad h=h_w \qquad \text{for}\ \ y=0\,,$$
$$u\to u_e(x)\,,\qquad\qquad h\to h_e(x)\quad \text{for}\ \ y\to\infty\,. \tag{3}$$

In the governing equations of the system (2), as well as in the boundary conditions (3), the usual notations are used: $u(x,\ y)$ - longitudinal projection of velocity in the boundary layer, $v(x,\ y)$ - transversal projection, ρ - density of ideally dissociated gas (mixture), μ - dynamic viscosity, h - enthalpy, Pr - Prandtl number, and x, y - longitudinal and transversal coordinate. The Function $l=l(p,\ h)$ for the equilibrium bicomponential mixture is determined by the expression [3],

$$l=(Le-1)\ (h_A-h_M)\ \left(\frac{\partial C_A}{\partial h}\right)_p\,, \tag{4}$$

where p - denotes pressure, and Le is Lewis number. The subscript "e" stands for physical values at the outer edge of the boundary layer and the subscript "w" for values at the wall of the body within the fluid. It is pointed out that $v_w(x)$ represents the given velocity with which the dissociated gas flows transversally through a solid porous wall of the body within the fluid (Fig.1). Here, $v_w > 0$ at injection, and $v_w < 0$ at ejection of the gas.

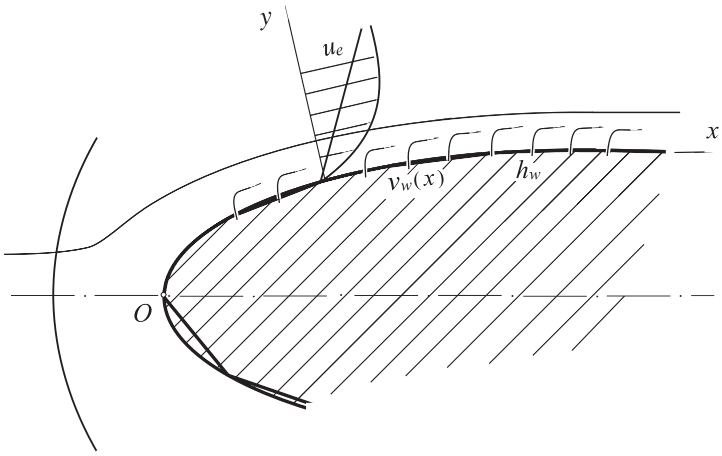

Figure 1: Flow in the boundary layer

The nondimensional transfer coefficient, Prandtl and Lewis number are defined with the known expressions:

$$Pr = \frac{\mu\, c_p}{\lambda}\,, \qquad Le = \frac{\rho\, c_p D}{\lambda} \tag{5}$$

where λ is thermal conductivity coefficient, c_p - specific heat of gas dissociated at constant pressure and $D_{21} = D_{12} = D$ - coefficient of atomic component diffusion. These numbers can be regarded as constant values [1, 3]. In our further studies Prandtl number is considered to be $\mathrm{Pr} = 0.712$.

2 Transformation of boundary layer equations

In order to apply the *general similarity method*, instead of physical coordinates x, y, by analogy with other already solved problems of compressible fluid flow [6, 7], we introduce new variables in the form of the following transformations:

$$s = \frac{1}{\rho_0\,\mu_0} \int\limits_0^x \rho_w \mu_w \partial x = s(x); \qquad z = \frac{1}{\rho_0} \int\limits_0^y \rho \partial y = z(x, y), \tag{6}$$

The stream function $\psi(s,\, z)$ is also introduced in accordance with the relations:

$$u = \frac{\partial \psi}{\partial z}\,, \qquad \tilde{v} = \frac{\rho_0\,\mu_0}{\rho_w \mu_w} \left(u\,\frac{\partial z}{\partial x} + v\,\frac{\rho}{\rho_0} \right) = -\,\frac{\partial \psi}{\partial s}, \tag{7}$$

which result from the continuity equation.

In the expressions (6) and (7) ρ_0 and $\mu_0 = \rho_0\,\nu_0$ stand for the known values of density and dynamic viscosity of the dissociated gas (air). Here, ρ_w and μ_w denote the given distributions of these values at the wall of the body within the fluid, and ν_0 stands for kinematical viscosity at a concrete point of the boundary layer.

From the first two equations of the system (2), by a usual procedure – by integration transversally to the boundary layer and by transformation of the variables, the momentum equation of the considered problem is obtained. In its all three forms, the corresponding momentum equation is:

$$\frac{dZ^{**}}{ds} = \frac{F_{dp}}{u_e}\,, \qquad \frac{df}{ds} = \frac{u_e'}{u_e} F_{dp} + \frac{u_e''}{u_e'} f, \qquad \frac{1}{\Delta^{**}} \frac{d\Delta^{**}}{ds} = \frac{u_e'}{u_e} \frac{F_{dp}}{2f}\ . \tag{8}$$

While obtaining the momentum equation, the following values are introduced: parameter of the form f, value Z^{**}, conditional displacement thicknesses $\Delta^*(s)$ and $\Delta^{**}(s)$ conditional momentum loss thickness, nondimensional friction function $\zeta(s)$, *porosity parameter* $\Lambda(s)$, characteristic function F_{dp} and nondimensional value H. With this flow problem we have:

$$f(s) = \frac{u_e' \Delta^{**2}}{\nu_0} = u_e'\,Z^{**} = f_1\,, \quad Z^{**} = \frac{\Delta^{**2}}{\nu_0},$$

$$\Delta^*(s) = \int\limits_0^\infty \left(\frac{\rho_e}{\rho} - \frac{u}{u_e} \right) dz\,,$$

$$\Delta^{**}(s) = \int_0^\infty \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right) dz\,,$$

$$\varsigma(s) = \left[\frac{\partial(u/u_e)}{\partial(z/\Delta^{**})}\right]_{z=0}, \quad H = \frac{\Delta^*}{\Delta^{**}}, \tag{9}$$

$$F_{dp} = 2[\varsigma - (2+H)f] - 2\Lambda,$$

$$\underline{\Lambda(s) = -\frac{v_w \mu_0}{\mu_w}\,\frac{\Delta^{**}}{\nu_0} = -\frac{V_w \Delta^{**}}{\nu_0} = \Lambda_1,}$$

$$V_w = \frac{\mu_0}{\mu_w} v_w;$$

where the value $V_w(s)$ can be named *conditional transversal velocity* at the inner edge of the boundary layer. (In these expressions and further on ' stands for a derivative with respect to the variable s).

Applying the transformations of the variables (6) and the stream function (7), the governing system (2), (3) of the considered dissociated gas flow problem comes down to the following equation system:

$$\frac{\partial\psi}{\partial z}\,\frac{\partial^2\psi}{\partial s\partial z} - \frac{\partial\psi}{\partial s}\,\frac{\partial^2\psi}{\partial z^2} = \frac{\rho_e}{\rho} u_e u_e' + \nu_0 \frac{\partial}{\partial z}\left(Q\,\frac{\partial^2\psi}{\partial z^2}\right),$$

$$\frac{\partial\psi}{\partial z}\,\frac{\partial h}{\partial s} - \frac{\partial\psi}{\partial s}\,\frac{\partial h}{\partial z} = -\frac{\rho_e}{\rho} u_e u_e' \frac{\partial\psi}{\partial z} + \nu_0 Q\left(\frac{\partial^2\psi}{\partial z^2}\right)^2 + \nu_0 \frac{\partial}{\partial z}\left[\frac{Q}{Pr}\,(1+l)\,\frac{\partial h}{\partial z}\right]; \tag{10}$$

$$\frac{\partial\psi}{\partial z} = 0\,, \quad \underline{\frac{\partial\psi}{\partial s} = -\frac{\mu_0}{\mu_w} v_w} = -V_w, \quad h = h_w \quad for \quad z = 0\,,$$

$$\frac{\partial\psi}{\partial z} \to u_e(s)\,, \qquad h \to h_e(s) \quad for \quad z \to \infty\,.$$

The nondimensional function Q is determined as:

$$Q = \frac{\rho\,\mu}{\rho_w\,\mu_w}\,; \qquad Q = 1 \quad for \quad z = 0\,,$$
$$Q \to \frac{\rho_e\,\mu_e}{\rho_w\,\mu_w} = Q(s) \quad for \quad z \to \infty\,. \tag{11}$$

As seen from the expression (8), the momentum equation is by its form the same as the momentum equation of incompressible fluid [8]. And the dynamic equation of the system (10) has a similar form to the corresponding equation of incompressible fluid.

However, it is noticed that the (underlined) boundary layer condition for a partial derivative is $\partial\psi/\partial s \neq 0$. With the application of the general similarity method, it is important that this boundary layer condition should equal zero. Therefore, as with incompressible fluid, [8] the stream function $\psi\,(s,\,z)$ is divided into two parts. If the notation $\psi\,(s,\,0) = \psi_w\,(s)$ is introduced for the stream function for the flow along the wall of the body ($z = 0$), then the stream function for this flow problem can be written in the form of the relation

$$\psi\,(s,z) = \psi_w\,(s) + \psi^*\,(s,z)\,, \qquad \psi^*\,(s,\,0) = 0. \tag{12}$$

where $\psi^*\,(s,\,z)$ is a new stream function.

When we apply the relation (12), the equation system (10) transforms into the system:

$$\frac{\partial\psi^*}{\partial z}\,\frac{\partial^2\psi^*}{\partial s\partial z} - \frac{\partial\psi^*}{\partial s}\,\frac{\partial^2\psi^*}{\partial z^2} - \underline{\frac{d\psi_w}{ds}\frac{\partial^2\psi^*}{\partial z^2}} = \frac{\rho_e}{\rho}u_e u_e' + \nu_0\,\frac{\partial}{\partial z}\,\left(Q\,\frac{\partial^2\psi^*}{\partial z^2}\right)\,,$$

$$\frac{\partial\psi^*}{\partial z}\,\frac{\partial h}{\partial s} - \frac{\partial\psi^*}{\partial s}\,\frac{\partial h}{\partial z} - \underline{\frac{d\psi_w}{ds}\frac{\partial h}{\partial z}} = -\frac{\rho_e}{\rho}u_e\,u_e'\frac{\partial\psi^*}{\partial z} + \nu_0\,Q\,\left(\frac{\partial^2\psi^*}{\partial z^2}\right)^2 +$$
$$\nu_0\frac{\partial}{\partial z}\,\left[\frac{Q}{Pr}(1+l)\frac{\partial h}{\partial z}\right]\,; \tag{13}$$

$$\psi^*(s,\ z) = 0\,, \qquad \frac{\partial \psi^*}{\partial z} = 0\,, \qquad h = h_w \qquad for \quad z = 0\,,$$

$$\frac{\partial \psi^*}{\partial z} \to u_e(s)\,, \quad h \to h_e(s) \quad for \quad z \to \infty\,.$$

Each of the equations of the system (13) contains one (underlined) term on the left, where $d\psi_w/ds$ appears. It is noticed that

$$\frac{d\psi_w(s)}{ds} \;=\; \left(\frac{\partial \psi}{\partial s}\right)_{z=0} = -\frac{\mu_0}{\mu_w} v_w = -\ V_w\,. \tag{14}$$

In the case of a nonporous wall of the body within the fluid (for which $v_w = 0$), in the equations of the system (13) the underlined terms equal zero, therefore the obtained equation system is exactly the same as the corresponding system [3]. The characteristic function F_{dp} comes down to the corresponding function F.

3 Generalized boundary layer equations of the considered flow problem

In accordance with the ideas followed with the application of the general similarity method to different flow problems for both compressible and incompressible fluid [6, 7], in these studies we introduced new variables and a new stream function $\Phi(s,\,\eta)$. However, after comprehensive and rather complicated numerical transformations, it has been determined that, here also, new transformations should be introduced in the form of the following expressions:

$$s \;=\; s\,; \quad \eta(s,z) = \frac{u_e^{b/2}(s)}{K(s)}\ z\ ,$$

$$K(s) \;=\; \left(a\nu_0 \int\limits_0^s u_e^{b-1} ds\right)^{1/2}, \quad a,\ b = const.$$

$$\psi^*(s,z) = u_e^{1-b/2} K(s) \cdot \Phi(\eta,\ \kappa,\ f_1, f_2, f_3, \ldots,\ \Lambda_1, \Lambda_2, \Lambda_3, \ldots), \quad (15)$$

$$h(s,z) = h_1 \cdot \bar{h}(\eta,\ \kappa,\ f_1, f_2, f_3, \ldots,\ \Lambda_1, \Lambda_2, \Lambda_3, \ldots),$$

$$\left(h_e + \frac{u_e^2}{2} = h_1 = const. \right)$$

In defined so-called similarity transformations, the following notations are used: $\eta(s,\ z)$ - newly introduced transversal variable, Φ - new stream function, $\bar{h}$- nondimensional enthalpy, and h_1 - enthalpy at the front stagnation point of the body within the fluid.

Here also, based on the newly introduced transversal variable $\eta(s,\ z)$, important values and characteristics of the boundary layer (9) can be written in the form of suitable relations:

$$\Delta^{**} = \frac{K(s)}{u_e^{b/2}} B(s), \qquad \frac{\Delta^*}{\Delta^{**}} = H = \frac{A(s)}{B(s)}, \qquad \frac{f}{B^2} = \frac{a\, u_e'}{u_e^b} \int_0^s u_e^{b-1}\, ds,$$

$$\zeta = B \left(\frac{\partial^2 \Phi}{\partial \eta^2} \right)_{\eta=0}, \quad \tau_w = \left(\mu \frac{\partial u}{\partial y} \right)_{y=0} = \frac{\rho_w \mu_w}{\rho_0} \frac{u_e}{\Delta^{**}} \zeta, \qquad (16)$$

$$A(s) = \int_0^\infty \left(\frac{\rho_e}{\rho} - \frac{\partial \Phi}{\partial \eta} \right) d\eta, \quad B(s) = \int_0^\infty \frac{\partial \Phi}{\partial \eta} \left(1 - \frac{\partial \Phi}{\partial \eta} \right) d\eta;$$

where the values A and B are assumed to be continual functions of the longitudinal variable s. The local parameter of dissociated gas compressibility [4] $\kappa = f_0$, the set of parameters of the form $f_k(s)$ of Loitsianskii's type [8], as well as the set of *porous wall parameters* [9] $\Lambda_k(s)$ in the functions Φ and $\bar{h}$, newly introduced in the general similarity transformations (15), are determined with the expressions:

$$\kappa = f_0(s) = \tfrac{u_e^2}{2h_1}\ , \qquad f_k(s) = \ u_e^{k-1} u_e^{(k)}\, Z^{**k},$$
$$\Lambda_k(s) = -u_e^{k-1}\ \left(\frac{V_w}{\sqrt{\nu_0}}\right)^{(k-1)} Z^{**k-1/2} \qquad k = 1,\ 2,\ 3\ ...). \tag{17}$$

For $k = 1$ we obtain $f_1(s) = \ u_e'\ Z^{**}$- a form parameter already known in the boundary layer theory, while the porosity parameter $\Lambda_1 = -(V_w \Delta^{**}/\nu_0)$ is the same as the earlier defined parameter (9). Parameters of the sets (17) satisfy recurrent simple differential equations of the form:

$$\frac{u_e}{u_e'} f_1 \frac{d\kappa}{ds} = 2\kappa\, f_1\ \equiv \theta_0\,,$$
$$\frac{u_e}{u_e'} f_1 \frac{df_k}{ds} = [\,(k-1)\ f_1 + k\, F_{dp}\,]\ \ f_k + f_{k+1} \equiv \theta_k, \tag{18}$$
$$\frac{u_e}{u_e'} f_1 \frac{d\Lambda_k}{ds} = \{\,(k-1)\ f_1 + [\,(2k-1)\,/2\,]\ F_{dp}\}\ \ \Lambda_k + \Lambda_{k+1} \equiv \chi_k.$$

Applying the similarity transformations (15), (17) to the equation system (13) the *generalized boundary layer equation system* has been obtained. The outer velocity $u_e(s)$ appears explicitly in neither of the equations of the obtained system.

The obtained generalized equation system, together with the transformed boundary conditions, is:

$$\frac{\partial}{\partial\eta}\left(Q\,\frac{\partial^2\Phi}{\partial\eta^2}\right) + \frac{aB^2 + (2-b)\,f_1}{2\,B^2}\ \Phi\ \frac{\partial^2\Phi}{\partial\eta^2} + \frac{f_1}{B^2}\left[\frac{\rho_e}{\rho} - \left(\frac{\partial\Phi}{\partial\eta}\right)^2\right] +$$
$$\underline{\frac{\Lambda_1}{B}\ \frac{\partial^2\Phi}{\partial\eta^2}} = \frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\ \frac{\partial^2\Phi}{\partial\eta\,\partial f_k} - \frac{\partial\Phi}{\partial f_k}\ \frac{\partial^2\Phi}{\partial\eta^2}\right) + \right.$$
$$\left.\underline{\sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\ \frac{\partial^2\Phi}{\partial\eta\,\partial\Lambda_k} - \frac{\partial\Phi}{\partial\Lambda_k}\ \frac{\partial^2\Phi}{\partial\eta^2}\right)}\right],$$

$$\frac{\partial}{\partial \eta}\left[\frac{Q}{Pr}(1+l)\frac{\partial \bar{h}}{\partial \eta}\right]+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial \bar{h}}{\partial \eta}-$$

$$\frac{2\kappa f_1}{B^2}\frac{\rho_e}{\rho}\frac{\partial \Phi}{\partial \eta}+2\kappa Q\left(\frac{\partial^2 \Phi}{\partial \eta^2}\right)^2+\underline{\frac{\Lambda_1}{B}\frac{\partial \bar{h}}{\partial \eta}}=$$

$$\frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial \Phi}{\partial \eta}\frac{\partial \bar{h}}{\partial f_k}-\frac{\partial \Phi}{\partial f_k}\frac{\partial \bar{h}}{\partial \eta}\right)+\right. \tag{19}$$

$$\left.\underline{\sum_{k=1}^{\infty}\chi_k\left(\frac{\partial \Phi}{\partial \eta}\frac{\partial \bar{h}}{\partial \Lambda_k}-\frac{\partial \Phi}{\partial \Lambda_k}\frac{\partial \bar{h}}{\partial \eta}\right)}\right];$$

$$\Phi=0\,,\quad \frac{\partial \Phi}{\partial \eta}=0\,,\quad \bar{h}=\bar{h}_w=const.\quad for\quad \eta=0\,,$$

$$\frac{\partial \Phi}{\partial \eta}\to 1\,,\quad \bar{h}\to \bar{h}_e(s)=1-\kappa\quad for\quad \eta\to\infty\,.$$

Both equations of the system (19), on the left hand-side, contain one term that depends on the porosity parameter Λ_1. On the right hand-side, each of the equations contains a sum of terms that are multiplied with the function χ_k. In the case of a non-porous wall ($v_w=0$, $V_w=0$) all the porosity parameters equal zero, therefore these terms also equal zero. In that case, the obtained equations take the form of the corresponding equations [3] for the case of a flow along a non-porous wall.

Because of numerous parameters, the obtained equations are solved, as with other flow problems, in a so-called $n-$ parametric approximation. In the case when all the parameters are $f_k=0$ and $\Lambda_k=0$ when $k\geq 2$, the obtained equation system (19) comes down to a system of partial equations with four independent variables: η, κ, f_1, Λ_1, and this represents a three-parametric approximation. Furthermore, when the general similarity method is applied, a so-called localization is also done. The first derivatives with respect to the parameters κ and f_1 are ignored. As seen in earlier studies [3, 4], the influence of the compressibility parameter κ to the nondimensional enthalpy $\bar{h}$ is significant. For a more correct calculation, localization with respect to the parameter

κ can be performed in relation to the nondimensional total enthalpy $g=(h+u^2/2)/h_1$, i.e., it is justified to assume that $\partial g/\partial\kappa\approx 0$. Therefore in the three-parametric ($f_0=\kappa\neq 0$, $f_1=f\neq 0$, $\Lambda_1=\Lambda\neq 0$, $f_2=f_3=...=0, \Lambda_2=\Lambda_3=...=0$) twice localized approximation ($\partial/\partial\kappa=0$, $\partial/\partial\Lambda_1=0$), the corresponding boundary layer equation system of the considered flow problem has the following form:

$$\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)\,f}{2\,B^2}\;\Phi\;\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]+$$

$$\underline{\frac{\Lambda}{B}\frac{\partial^2\Phi}{\partial\eta^2}}=\frac{F_{dp}f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\;\frac{\partial^2\Phi}{\partial\eta\,\partial f}-\frac{\partial\Phi}{\partial f}\;\frac{\partial^2\Phi}{\partial\eta^2}\right),$$

$$\frac{\partial}{\partial\eta}\left[\frac{Q}{Pr}\;(1+l)\,\frac{\partial\bar{h}}{\partial\eta}\right]+\frac{aB^2+(2-b)\;f}{2\,B^2}\;\Phi\;\frac{\partial\bar{h}}{\partial\eta}-$$

$$\frac{2\kappa\,f}{B^2}\;\frac{\partial\Phi}{\partial\eta}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]+2\kappa\,Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2+$$

$$+\underline{\frac{\Lambda}{B}\frac{\partial\bar{h}}{\partial\eta}}=\frac{F_{dp}f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial f}-\frac{\partial\Phi}{\partial f}\;\frac{\partial\bar{h}}{\partial\eta}\right);\qquad(20)$$

$$\Phi=0\,,\qquad\frac{\partial\Phi}{\partial\eta}=0\,,\qquad\bar{h}=\bar{h}_w=const.\qquad for\qquad\eta=0\;,$$

$$\frac{\partial\Phi}{\partial\eta}\rightarrow 1,\qquad\bar{h}\rightarrow\bar{h}_e(s)=1-\kappa\qquad for\qquad\eta\rightarrow\infty\;.$$

Therefore, the problem of the planar flow of ideally dissociated gas (air) in the boundary layer under conditions of equilibrium dissociation is defined with the generalized equation system (20). Hence, investigation of this problem comes down to solution of the obtained approximate equation system.

4 Numerical solution of the transformed equation system. Obtained results

In papers [3, 4] it is stated that for dissociated air $Le \approx 1$. It follows that the function l (which is a part of the energy equation) is defined with the expression (4) and that it equals zero. Furthermore, in the papers written by the same author, based on the tables of the thermodynamic functions for air, it is proven that the following very correct approximate formula can be applied for the wide range of pressure changes:

$$Q(\bar{h}) = \left(\frac{\bar{h}_w}{\bar{h}}\right)^{1/3} , \tag{21}$$

and it is used in this paper.

For the density ratio in this paper, we used the approximation $\rho_e/\rho \approx \bar{h}/(1-\kappa)$ obtained from the corresponding rather complicated formula stated in [3].

Besides the previous relations for certain physical values, for numerical integration of the system (20), it is necessary to decrease the order of the dynamic equation. Introducing the transformation

$$\frac{u}{u_e} = \frac{\partial \Phi}{\partial \eta} = \varphi = \varphi\ (\eta\ ,\ \ \kappa\ ,\ \ f\ ,\ \ \Lambda)\ , \tag{22}$$

the order of the dynamic equation is decreased; therefore the corresponding equation system of the considered dissociated air flow problem takes the following form:

$$\frac{\partial}{\partial \eta}\left(Q\frac{\partial \varphi}{\partial \eta}\right) + \frac{aB^2 + (2-b)\,f}{2\,B^2}\ \Phi\ \frac{\partial \varphi}{\partial \eta} + \frac{f}{B^2}\ \left(\frac{\bar{h}}{1-\kappa}\ -\ \varphi^2\right) + \frac{\Lambda}{B}\ \frac{\partial \varphi}{\partial \eta} =$$

$$= \frac{F_{dp} f}{B^2}\ \left(\varphi\ \frac{\partial \varphi}{\partial f}\ -\ \frac{\partial \Phi}{\partial f}\ \frac{\partial \varphi}{\partial \eta}\right) ,$$

$$\frac{\partial}{\partial \eta}\ \left(\frac{Q}{Pr}\ \frac{\partial \bar{h}}{\partial \eta}\right) + \frac{aB^2 + (2-b)\ f}{2\,B^2}\ \Phi\ \frac{\partial \bar{h}}{\partial \eta} -$$

$$\frac{2\kappa f}{B^2}\ \varphi\ \left(\frac{\bar{h}}{1-\kappa} - \varphi^2\right) + 2\kappa\, Q\ \left(\frac{\partial \varphi}{\partial \eta}\right)^2 + \frac{\Lambda}{B}\frac{\partial \bar{h}}{\partial \eta} =$$

$$= \frac{F_{dp} f}{B^2} \left(\varphi \frac{\partial \bar{h}}{\partial f} - \frac{\partial \Phi}{\partial f} \, \frac{\partial \bar{h}}{\partial \eta} \right) ; \tag{23}$$

$$\Phi = 0 \, , \qquad \varphi = 0 \, , \qquad \bar{h} = \bar{h}_w = const. \qquad for \quad \eta = 0 \ ,$$

$$\varphi \to 1 \ , \qquad \bar{h} \to \bar{h}_e(s) = 1 - \kappa \qquad for \quad \eta \to \infty \ .$$

Numerical solution of the obtained system of nonlinear and conjugated differential partial equations (23) is performed by the finite differences method, i.e., so-called "passage method". According to the usual scheme of the finite differences, the equation system (23) is firstly transformed into an equivalent system of algebraic equations, which is solved by iterative procedure, taking into consideration the order of calculation of certain functions and linearization. The values of the functions φ, Φ, $\bar{h}$ are calculated at discrete points M of the half planar integration grid, i.e., at discrete points of each calculating layer $(K+1)$. For each calculating layer with this, as well as other complicated boundary layer fluid flow problem [6, 7], the number of discrete points $N = 401$ has been determined.

For the concrete solution of the equation system (23), i.e., for the solution of the corresponding algebraic system, the necessary program in FORTRAN has been written. It is based on the program used in [10]. All the necessary calculations have been made for the concrete values of a and b , and they are: $a = 0.4408$; $b = 5.7140$; which, according to [10], represent the optimal values. For the Prandtl number, as already stated, for the case of dissociated air flow $\Pr = 0.712$. While calculating the characteristic functions B and F_{dp} , at a zero iteration, the values $B^0_{K+1} = 0.469$ and $F^0_{dp,K+1} = 0.4411$, were accepted, as already done in the paper [10].

Numerical solutions of the equation system (23) obtained in these studies are given in the form of suitable tables. Table 1 represents, for example, the solution of the boundary layer equations for the case defined with $f = 0.0$ and $\Lambda = 0.02$.

Table 1: One of the solutions of the dissociated gas boundary layer equations

$f = 0.0000000D + 00$	$\Lambda = 0.02$	$\Delta f = 0.1000000D - 02$
$A = -0.1758993$	$B = 0.5415912$	$B^2 = 0.2933210$
$f/B^2 = 0.0000000$	$H = -0.3247825$	$\partial^2\Phi/\partial\eta^2 = 0.6111155$
$F_{dp} = 0.6219525$	$\kappa = 0.04$	$\varsigma = 0.3309762$

M	η	u/u_e	Φ	$\bar{h}$	Q
1	0.0000000	0.0000000	0.0000000	0.0152000	1.0000000
9	0.4000000	0.1040232	0.0220045	0.0901270	0.5524966
17	0.8000000	0.2288988	0.0876944	0.1956200	0.4267204
25	1.2000000	0.3737805	0.2077863	0.3188060	0.3626094
33	1.6000000	0.5250726	0.3875796	0.4496218	0.3233444
41	2.0000000	0.6683210	0.6267695	0.5777911	0.2974114
49	2.4000000	0.7898973	0.9193251	0.6932084	0.2798938
57	2.8000000	0.8809334	1.2545788	0.7878932	0.2682000
65	3.2000000	0.9402648	1.6198119	0.8580680	0.2606797
25	1.2000000	0.3737805	0.2077863	0.3188060	0.3626094
33	1.6000000	0.5250726	0.3875796	0.4496218	0.3233444
41	2.0000000	0.6683210	0.6267695	0.5777911	0.2974114
49	2.4000000	0.7898973	0.9193251	0.6932084	0.2798938
57	2.8000000	0.8809334	1.2545788	0.7878932	0.2682000
65	3.2000000	0.9402648	1.6198119	0.8580680	0.2606797
73	3.6000001	0.9736425	2.0033170	0.9048068	0.2561116
81	4.0000001	0.9898044	2.3964360	0.9327349	0.2535295
89	4.4000001	0.9965468	2.7939177	0.9477225	0.2521859
97	4.8000001	0.9989764	3.1931097	0.9549622	0.2515470
105	5.2000001	0.9997346	3.5928826	0.9581170	0.2512706
113	5.6000001	0.9999398	3.9928267	0.9593592	0.2511621
121	6.0000001	0.9999881	4.3928146	0.9598016	0.2511235
129	6.4000001	0.9999979	4.7928124	0.9599442	0.2511111
137	6.8000001	0.9999997	5.1928120	0.9599857	0.2511074
145	7.2000001	1.0000000	5.5928120	0.9599967	0.2511065
153	7.6000001	1.0000000	5.9928120	0.9599993	0.2511063
161	8.0000001	1.0000000	6.3928120	0.9599999	0.2511062
169	8.4000001	1.0000000	6.7928120	0.9600000	0.2511062

M	η	u/u_e	Φ	$\overline{h}$	Q
177	8.8000001	1.0000000	7.1928120	0.9600000	0.2511062
185	9.2000001	1.0000000	7.5928120	0.9600000	0.2511062
193	9.6000001	1.0000000	7.9928120	0.9600000	0.2511062
201	10.0000001	1.0000000	8.3928120	0.9600000	0.2511062
209	10.4000002	1.0000000	8.7928120	0.9600000	0.2511062
217	10.8000002	1.0000000	9.1928120	0.9600000	0.2511062
225	11.2000002	1.0000000	9.5928120	0.9600000	0.2511062
233	11.6000002	1.0000000	9.9928120	0.9600000	0.2511062
241	12.0000002	1.0000000	10.3928120	0.9600000	0.2511062
249	12.4000002	1.0000000	10.7928120	0.9600000	0.2511062
257	12.8000002	1.0000000	11.1928120	0.9600000	0.2511062
265	13.2000002	1.0000000	11.5928120	0.9600000	0.2511062
273	13.6000002	1.0000000	11.9928120	0.9600000	0.2511062
281	14.0000002	1.0000000	12.3928121	0.9600000	0.2511062
289	14.4000002	1.0000000	12.7928121	0.9600000	0.2511062
297	14.8000002	1.0000000	13.1928121	0.9600000	0.2511062
305	15.2000002	1.0000000	13.5928121	0.9600000	0.2511062
313	15.6000002	1.0000000	13.9928121	0.9600000	0.2511062
321	16.0000002	1.0000000	14.3928121	0.9600000	0.2511062
329	16.4000002	1.0000000	14.7928121	0.9600000	0.2511062
337	16.8000003	1.0000000	15.1928121	0.9600000	0.2511062
345	17.2000003	1.0000000	15.5928121	0.9600000	0.2511062
353	17.6000003	1.0000000	15.9928121	0.9600000	0.2511062
361	18.0000003	1.0000000	16.3928121	0.9600000	0.2511062
369	18.4000003	1.0000000	16.7928121	0.9600000	0.2511062
377	18.8000003	1.0000000	17.1928121	0.9600000	0.2511062
385	19.2000003	1.0000000	17.5928121	0.9600000	0.2511062
393	19.6000003	1.0000000	17.9928121	0.9600000	0.2511062
401	20.0000003	1.0000000	18.3928121	0.9600000	0.2511062

This solution describes the dissociated gas flow in the laminar boundary layer along a flat plane ($u_e = u_\infty = const,\ f = u_e' Z^{**} = 0,\ \kappa = \kappa_0 = u_\infty^2/2h_1$). Only some, of the obtained numerical solutions are shown in the form of corresponding diagrams. The following figures show: diagram of nondimensional velocity (Fig. 2), nondimensional enthalpy (Fig.3) and (Fig.4), as well as the diagrams of the characteristic values of the boundary layer $B(f)$ (Fig.5), $F_{dp}(f)$ (Fig.6), $\zeta(f)$ (Fig.7) for different values of the porosity parameter. The diagram $\bar{h}(\eta)$ is especially shown for different values of the compressibility parameter at the cross-section of the boundary layer defined with $f = 0.0$ (Fig.8).

5 Discussion of the obtained results and conclusions

Based on the given (and other) diagrams the *general conclusion* that the profiles of the obtained solutions of the boundary layer equations, concerning their behaviour, are the same as with other similar flow problems.

For the considered flow case, the following conclusions can be defined:

- Nondimensional velocity u/u_e at different cross-sections of the boundary layer (different f) converges very fast towards one (Fig.2).

- A significant influence of the compressibility parameter κ on the distribution of the nondimensional enthalpy with respect to the boundary layer cross-section is noticed. The compressibility parameter changes even the general character of behaviour of the enthalpy distribution in the boundary layer. For lower values of κ, the enthalpy $\bar{h}$ reaches the maximum value that equals $1-\kappa$, at the outer edge of the boundary layer (Fig.3). However, for higher values of κ, the enthalpy $\bar{h}$ has a maximum $\bar{h}_{\max} > 1 - \kappa$ within the boundary layer itself (Fig.8).

- The diagram (Fig.7) clearly shows that the porosity parameter Λ has an influence on the nondimensional friction function ζ, and therefore on the boundary layer separation point. We can also notice a great influence of this parameter on other important boundary layer characteristics: the value B (Fig.5) and the function F_{dp} (Fig.6).

- If the transversal velocity of injection v_w increases, based on the definition of the porosity parameter $\Lambda(s)$ (9), it can be concluded that the value of this parameter decreases. The diagram (Fig.7) shows that with the decrease of the porosity parameter, the boundary layer separation point moves downstream.

Generally, there are some difficulties in application of the general similarity method to the problem of ideally dissociated air flow in the boundary layer around the porous contour. They are mainly of mathematic nature. The difficulties concerning physical, i.e., thermochemical problems of the gas flow itself are almost insoluble. This method, however, gives important quality results that enable us to study the behaviour of distributions of physical and characteristic values for different boundary layer cross-sections and different forms of the outer velocity functions.

- In order to obtain the more correct (quantity) results, it is necessary to integrate the system (19) in a three-parametric approximation but without localization with respect to the parameter Λ, and especially without localization with respect to the compressibility parameter $\kappa = f_0$. According to some earlier studies [5], it can be expected that this flow problem would show that the change of the compressibility parameter has a great influence on the change of the enthalpy in the boundary layer.

At the end of this paper, it is stated that with this case of compressible fluid flow, we encountered some problems while solving the equation system (23). They are reflected in interruptions in work of the program for some input values in the diffuser area of the boundary layer.

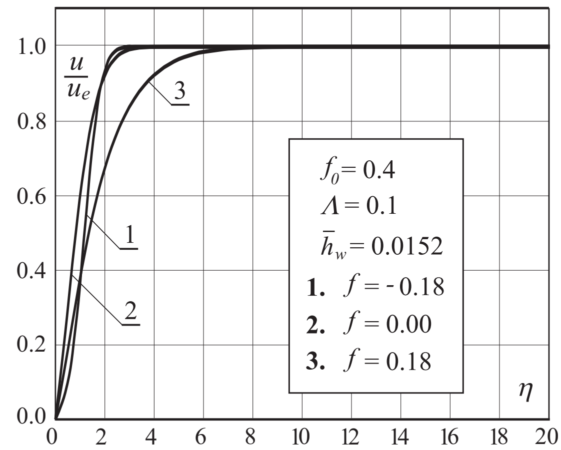

Figure 2: Diagram of nondimensional velocity u/u_e

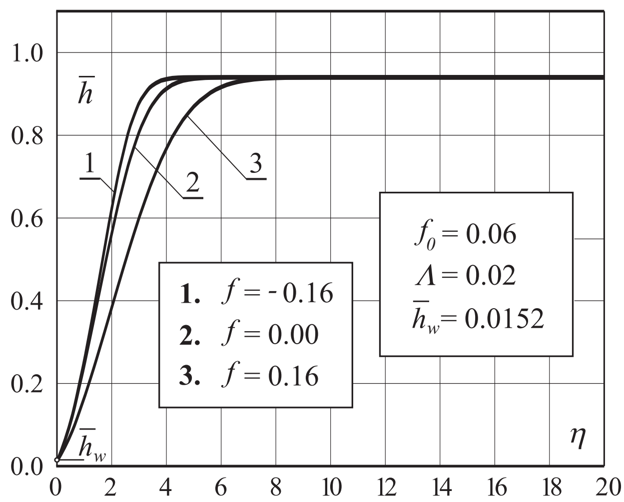

Figure 3: Diagram of nondimensional enthalpy ($\kappa = f_0 = 0.06$)

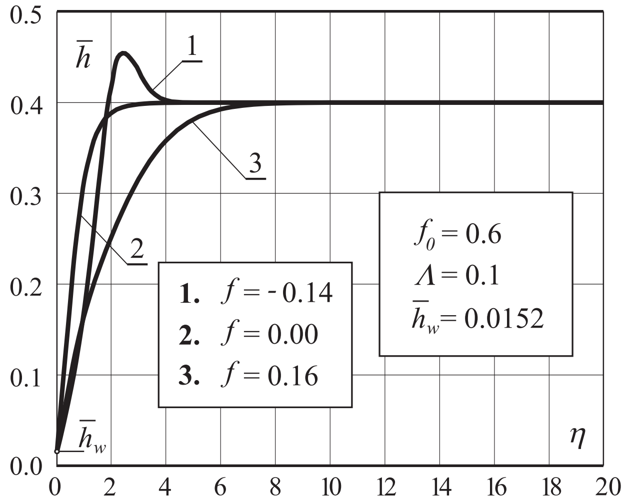

Figure 4: Diagram of nondimensional enthalpy ($\kappa = f_0 = 0.6$)

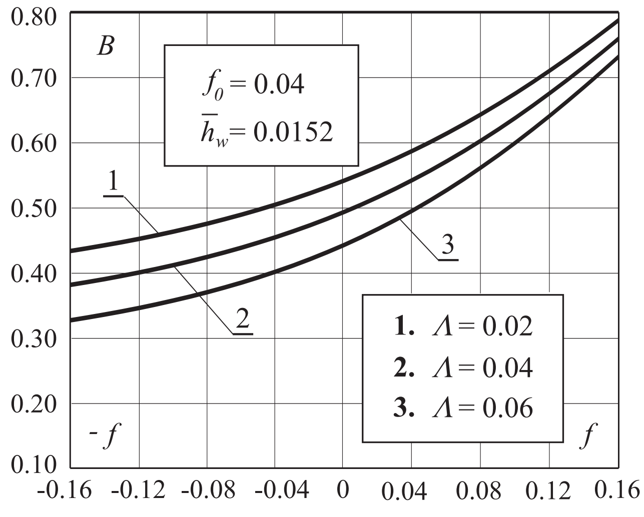

Figure 5: Diagram of values $B(\Lambda)$

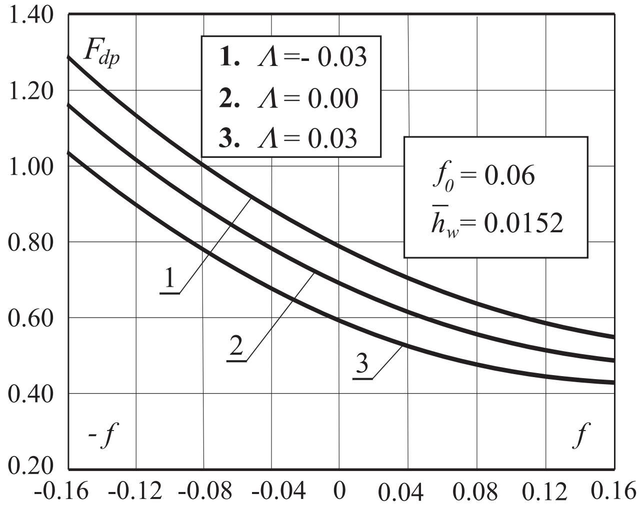

Figure 6: Diagram of characteristic function $F_{dp}(\Lambda)$

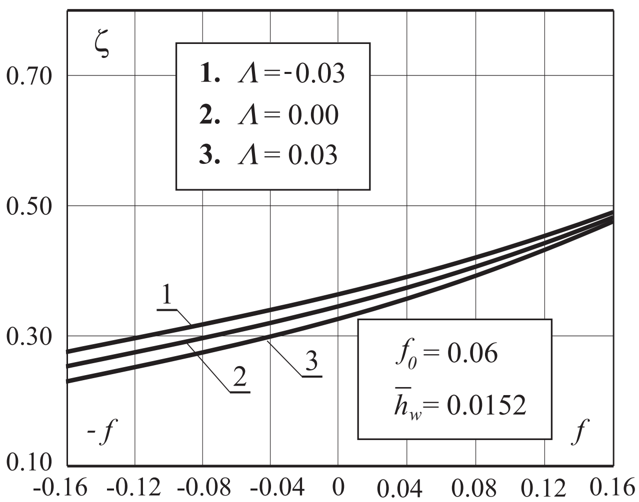

Figure 7: Diagram of nondimensional friction function $\zeta(\Lambda)$

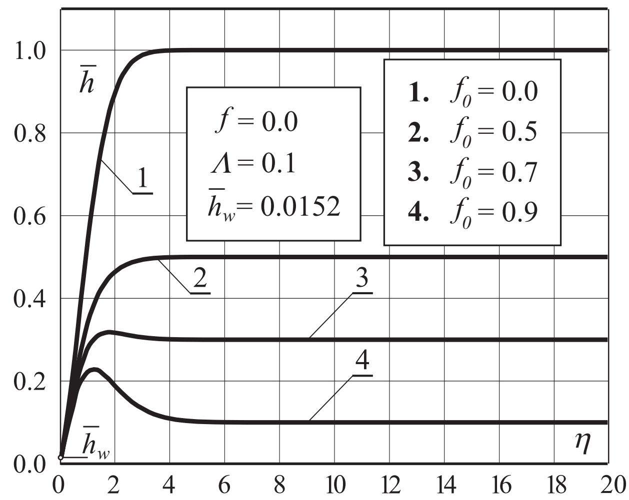

Figure 8: Diagram of nondimensional enthalpy in the cross-section of the boundary layer when $f = 0.0$ for different values of parameter $\kappa = f_0$

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Submitted on July 2005.

Granični sloj strujanja disociranog gasa preko poroznog zida u uslovima ravnotežne disocijacije

UDK 532.526

U radu je istraživano strujanje idealno disociranog vazduha u graničnom sloju u uslovima ravnotežne disocijacije. Pri tome je kontura zida opstrujavanog tela porozna. Pomoću pogodnih transformacija, polazne jednačine graničnog sloja razmatranog problema dovedene su na uopšteni oblik. Dobijene jednačine su numerički rešene u troparametarskom lokalizovanom približenju. Na bazi dobijenih rezultata izvedeni su odgovarajući zaključci o ponašanju pojedinih fizičkih veličina i karakteristika graničnog sloja razmatranog problema strujanja fluida.