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On injection-ejection fluid influence through different accelerating porous surfaces on unsteady 2D incompressible boundary layer characteristics

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Abstract

Through the porous contour in perpendicular direction, the fluid of the same properties as incompressible fluid in basic flow, has been injected or ejected with velocity who is a function of the contour longitudinal coordinate and time. The corresponding equations of unsteady boundary layer, by introducing the appropriate variable transformations, momentum and energy equations and two similarity parameters sets, are transformed into generalized form. These parameters are expressing the influence of the outer flow velocity, the injection or ejection velocity and the flow history in boundary layer, on the boundary layer characteristics. Obtained generalized solutions are used to calculate the distributions of velocity, and shear stress in laminar-turbulent transition of unsteady incompressible boundary layer on different porous contours: circular cylinder, thin elliptical cylinder and aerofoil, whose centers velocities changes in time as a degree functions. The ejection of fluid postpones the boundary layer separation,

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i.e. laminar-turbulent transition, and vice versa the injection of fluid favours the separation. Boundary layer characteristics are found directly, no further numerical integration of momentum equation.

1 Introduction

The behaviour of a boundary layer in the presence of a positive or negative pressure gradient along the surface is particularly important for the profiles drag calculation as well as for the understanding of the processes which take in a diffuser, because there is transition from laminar to turbulent flow which determines the dividing line between a region with low drag and one where drag is dramatically increased. So one can control laminar-turbulent transition by fluid which has been injected or ejected through the porous contour. Apart from skin friction we are interested in knowing whether the boundary layer will separate under given circumstances and if so, we shall wish to determine the point of separation. The results obtained by means of boundary layer theory dispose, in comparasion with numerical solutions of complete Navier-Stokes equations, significant preference, because they are exact corresponding to the structure of solutions for a great Re numbers, i.e. represent solution which possess the boundary layer character. That is why the numerical method for calculation of Navier-Stokes equations, for a great Re numbers, only is appropriate if algorithm formed for their solving, concerning asymtotic behaviour, gives results which cover up with solutions of boundary layer equations [1, 8]. The generalized similarity method [2, 3, 5] represents one of the ways for improvement of modern analytical methods for the calculation of boundary layers. As a result of these procedure which consists of introducing a conviently chosen set of parameters, quantities characterizing any special problem are elliminated from the governing set of equations and the corresponding boundary conditions. A numerical solution of this equation can be found once for all and then it can be used in any special problem of the boundary layer theory. Thus obtained solutions give a possibility to follow the development of the boundary layer by means of its general characteristics, to determine the optimal flow in desired direction, to control the boundary layer.

2 Mathematical model and generalized similarity equation

Generalized similarity method [2, 3, 5] is exposed to the problem of unsteady incompressible plane boundary layer on the porous surface [9], when the fluid of the same properties as fluid in basic flow has been injected or ejected through the surface in perpendicular direction with velocity v_w. The mathematical model of the noticed problem is described by the following equation:

$$\Psi_{ty} + \Psi_y \Psi_{xy} + (v_w - \Psi_x)\Psi_{yy} = U_t + UU_x + \upsilon \Psi_{yyy} \tag{1}$$

with boundary and initial conditions:

$$\begin{gathered} y = o : \Psi = \Psi_y = 0; \quad y \to \infty : \Psi_y \to U(x,t); \\ t = t_0 : \Psi_y = u_1(x,y); \quad x = x_0 : \Psi_y = u_0(t,y), \end{gathered} \tag{2}$$

where is used: $\Psi(x,y,t)$ – stream function, $U(x,t)$ – free-stream velocity, υ – kinematic viscosity, $u_1(x,y)$ – the streamwise velocity distribution in boundary layer in some determined point of time $t = t_0$, $u_0(t,y)$ – the streamwise velocity distribution in boundary layer in cross-section $x = x_0$, x – streamwise coordinate, y – crosswise coordinate, t – time. Introducing new variables in the form [2, 3, 5]:

$$\begin{gathered} x = x, \quad t = t, \quad \eta = yU^{b_0/2}(a_0\upsilon \int_0^x U^{b_0-1}dx)^{-1/2}, \\ \Phi(x,\eta,t) = \Psi U^{b_0/2-1}(a_0\upsilon \int_0^x U^{b_0-1}dx)^{-1/2}, \end{gathered} \tag{3}$$

where is: $a_0 = 0,4408,\ b_0 = 5.714$, the group of parameters:

$$f_{k,n} = U^{k-1}U^{(k+n)}_{x^{(k)}t^{(n)}} z^{**^{k+n}} \quad (k,n = 0,1,2,..; \quad k \vee n \neq 0) \tag{4}$$

$$\lambda_{k,n} = -\upsilon^{-1/2} U^k \upsilon^{(k+n)}_{w_{x^{(k)}t^{(n)}}} z.^{k+n+1/2} \quad (k, n = 0, 1, 2...)$$

as new independent variables, and the momentum and energy equations of the considered problem:

$$(U\delta^*)_t + (U^2\delta^{**})_x + UU_x\delta^* - U\upsilon_w - \frac{\tau_w}{\rho} = 0;$$
$$(U^2\delta^{**})_t + U^3{\delta_1}^{**}{}_x + U^2(\delta^*_t + 3\delta^{**}_1 U_x - 2ve) = 0 \tag{5}$$

where are: $z^{**} = \delta^{**2}/\upsilon$, $\delta^{**} = (a_0\upsilon U^{-b_0} \int\limits_0^x U^{b_0-1}dx)^{1/2}B$, $B = \int\limits_0^\infty \Phi_\eta(1-\Phi_\eta)d\eta$.

$$\delta^* = L^{1/2}\int\limits_0^\infty (1-\Phi_\eta)d\eta; \quad \tau_w = \rho\upsilon U^{b_0/2+1}L^{-1/2}(\Phi_{\eta\eta})_{\eta=0};$$

$$\delta^{**}_1 = L^{1/2}\int\limits_0^\infty \Phi_\eta(1-\Phi^2_\eta)d\eta; \quad e = L^{-1/2}\int\limits_0^\infty \Phi^2_{\eta\eta}d\eta; \tag{6}$$

$$L = a_0\upsilon U^{-b_0}\int\limits_0^x U^{b_0-1}dx.$$

the equation (1) transformed in the new generalized form. The numerical integration of this generalized similarity equation can be performed "once and forever" only for its approximative form. It means, that the solution of universal equation in practice needs limitation of the number of the independent variables. It leads to the necesity of application of the "segment" method, in which all variables from someone have to be equal to zero, because the set of chosen parameters (4) possesses the following two properties: 1. the first parameters are to be enough "strong", so the solution lies close to the exact solution and 2. the following parameters introduce in solution small corrections only, and provide the convergence to be enough fast. In such a way, the approximative universal equation is obtained. Having the above procedure in mind, the parameters $f_{1,0}, f_{0,1}, \lambda_{0,0}$ will be remained, while all others will be let to be equal to zero. Also, the derivative with respect to the first porous parameter $\lambda_{0,0}$ will be considered as equal to zero. The obtained universal equation in these approximation i.e. in three parametric "once localized" enviroment, has the form:

$$
\begin{aligned}
&B^2\Phi_{\eta\eta\eta} + 0.5[a_0B^2 + (2-b_0)f_{1,0}]\Phi\Phi_{\eta\eta} + f_{1,0}(1-\Phi_\eta^2)+\\
&f_{0,1}(1\Phi_\eta) + (0.5\eta T^{**} + B\lambda_{0,0})\Phi_{\eta\eta} =\\
&\eta B^{-1}[T^{**}(f_{1,0}B_{f_{1,0}} + f_{0,1}B_{f_{0,1}}) - f_{0,1}^2 B_{f_{0,1}}]\Phi_{\eta\eta}+ \qquad (7)\\
&[T^{**}(f_{1,0}\Phi_{\eta f_{1,0}} + f_{0,1}\Phi_{\eta f_{0,1}}) - f_{0,1}^2\Phi_{\eta f_{0,1}} + f_{1,0}F^{**}(\Phi_\eta\Phi_{\eta f_{1,0}} -\\
&\Phi_{f_{1,0}}\Phi_{\eta\eta}) + f_{0,1}(F^{**} - f_{1,0})(\Phi_\eta\Phi_{\eta f_{0,1}} - \Phi_{f_{0,1}}\Phi_{\eta\eta})],
\end{aligned}
$$

and the corresponding boundary conditions (2) are reduced to the following:

$$
\begin{aligned}
&\eta = 0 : \Phi = \Phi_\eta = 0; \quad \eta \to \infty : \Phi_\eta \to 1;\\
&f_{1,0} = f_{0,1} = \lambda_{0,0} = 0 : \Phi = \Phi_0(\eta).
\end{aligned} \qquad (8)
$$

The numerical integration of the equation (7) with boundary conditions (8) has been performed by means of the difference schemes and by using Tridiagonal Algorithm method with iterations.The obtained results can be used in the withdrawing of general conclusions of boundary layer development and in calculation of particular problems.

3 Unsteady boundary layers on different porous contours

Universal solutions of the equation (7) $\Phi''(0)$, B are used to calculate the charasteristic properties of unsteady boundary layer on [9, 10]:

a) Circular cylinder whose center velocity changes with time as a degree function. Substituting nondimensional coordinates: $\tilde{x} = x/R$ and $\tilde{t} = U_\infty t/R$, where R is circular cylinder semidiameter and U_∞ – endlessly velocity, nondimensional potentional external velocity seems:

$$\tilde{U}(\tilde{x},\tilde{t}) = \tilde{U}_1(\tilde{t})\tilde{U}_2(\tilde{x}) = (\tilde{B} + \tilde{A}\tilde{t}^n)\sin\tilde{x} \tag{9}$$

with constant values for $\tilde{A}, \tilde{B}, n$, (Fig.1).

b) Thin elliptical cylinder where the ratio of semi-axes is $a/R = 0.125$ whose center velocity changes with time as a degree function. By substituting the nondimensionsl variables $\tilde{U}, \tilde{x}, \tilde{t}$ (see Fig.4) as:

$$\tilde{U} = U/U_\infty; \quad \tilde{x} = \int_0^\varphi (1 - 0.984\cos^2\varphi)^{1/2} d\varphi;$$

$$\tilde{t} = U_\infty t/R; \quad \tilde{l} = \tilde{x}(\pi),$$

where U_∞ is the velocity at infinity, and φ polar angle, the external velocity just outside the boundary layer in nondimensional form seems:

$$\tilde{U}(\tilde{x},\tilde{t}) = 1.125(\tilde{B} + \tilde{A}\tilde{t}^n)(1 + 0.015ctg^2\varphi)^{-1/2}, \tag{10}$$

with $\tilde{A}, \tilde{B}, n$ as a constant values;

c) Wing aerofoil [9] whose center velocity also changes with time as a degree function. Substituting nondimensional coordinates: $\tilde{x} = x/l$ and $\tilde{t} = Ut/l$, where is l-chord and U_∞ – endlessly velocity, nondimensional potentional external velocity seems

$$\tilde{U}(\tilde{x},\tilde{t}) = \tilde{U}_1(\tilde{t})\tilde{U}_2(\tilde{x}) = (\tilde{B} + \tilde{A}\tilde{t}^n)\tilde{U}_2(\tilde{x}) \tag{11}$$

with constant values for $\tilde{A}, \tilde{B}, n$. The Fig.7 shows potential external velocity $\tilde{U}_2(\tilde{x}) = U/U_\infty$ on wing aerofoil measured by J.Stueper in free flight, where is lift coefficient $c_l = 0.4$, Reynolds number $R = 4.10^6$ and chord $l = 1.8m$. [9], p.670, Fig.22.4.

Now, substituting (9),(10),(11) in (4),(6) yields the following relations for the universal functions for all three cases::

$$\frac{f_{1,0}}{B^2} = a_0\tilde{U}^{-b_0}\tilde{U}_x Q; \quad \frac{f_{0,1}}{B^2} = a_0\tilde{U}^{-(b_0-1)}\tilde{U}_t Q;$$

$$\frac{\lambda_{0,0}}{B} = -v_w (a_0 \frac{Q}{\upsilon} \tilde{U}^{b_0})^{1/2}; \quad Q = \int_0^{\tilde{x}} \tilde{U}^{b_0 - 1} d\tilde{x}. \tag{12}$$

After using (3), the expressions for the dimensionless skin frictions $\tilde{\tau}_w$, in all three cases, have the forms:

a)

$$\begin{aligned} \tilde{\tau}_w = \tau_w \frac{R_e^{1/2}}{\rho U_\infty^2} = \tilde{U}^{(b/2)+1} (a_0 \int_0^{\tilde{x}} \tilde{U}^{b_0-1} d\tilde{x})^{-1/2} \Phi''(0) = \\ \left[(B + A\tilde{t}^n)^3 (\sin \tilde{x})^{b_0+2} \right]^{1/2} \left[a_0 \int_0^{\tilde{x}} (\sin \tilde{x})^{b_0-1} d\tilde{x} \right]^{-1/2} \Phi''(0); \end{aligned} \tag{13}$$

b)

$$\begin{aligned} \tilde{\tau}_w = 1.193 [4(\tilde{B} + \tilde{A}\tilde{t}^n)^3 (\sin \varphi)^{b_0+2}]^{1/2} \\ [a_0 (\sin^2 \varphi + 0.015 \cos^2 \varphi)^{(b_0+2)/2} \\ \int_0^{\varphi} (\sin^2 \varphi)^{b_0-1} (\sin^2 \varphi + 0.015 \cos^2 \varphi)^{(2-b_0)/2} d\varphi]^{-1/2} \Phi''(0). \end{aligned} \tag{14}$$

c)

$$\tilde{\tau}_w = 2\tau_w \frac{R^{1/2}}{\rho U_\infty^2} = 2\tilde{U}^{(b/2)+1} (a_0 \int_0^{\tilde{x}} \tilde{U}^{b_0-1} d\tilde{x})^{-1/2} \Phi''(0). \tag{15}$$

Now, we select a given set of the constants $\tilde{A}, \tilde{B}, n$ and for particular point on contour $\tilde{x}_0$ and time $\tilde{t}_0$ searching by (12), the universal functions $(f_{1,0}/B^2)_0, (f_{0,1}/B^2)_0$ have been obtained concerning $[\Phi''(0)]_0$. Afterwards, using (13), (14) and (15) one can determine $\tilde{\tau}_w$ distributions on contour in all three cases.

For determination of the nondimensional streamwise velocity distibution $\tilde{u}(\tilde{x}, \tilde{t})$ in some boundary layer cross-section $\tilde{x}_0$ and for time $\tilde{t}_0$, one can use the relation:

$$\frac{\tilde{u}(\tilde{x}_0, \tilde{t}_0)}{\tilde{U}(\tilde{x}_0, \tilde{t}_0)} = \frac{\partial}{\partial \eta} \Phi \left[\eta; f_{1,0}(\tilde{x}_0, \tilde{t}_0); f_{0,1}(\tilde{x}_0, \tilde{t}_0); \lambda_{0,0}(\tilde{x}_0, \tilde{t}_0) \right], \tag{16}$$

where $\partial\Phi/\partial\eta$ is universal solution obtained from numerical integration of equation (7).

For controling the boundary layer separation point, shear stress and drag, the expression for nondimensional ejection and injection velocity distribution $\tilde{v}_w$ can be applicable in form:

$$\tilde{v}_w = -\lambda_{0,0} B^{-1} \left(v\tilde{U}^{b_0}\right)^{\frac{1}{2}} \left(a_0 \int\limits_0^{\tilde{x}} \tilde{U}^{b_0-1} d\tilde{x}\right)^{-\frac{1}{2}} \tag{17}$$

obtained by (4) for k=n=0, from whom for wanted separation point $\tilde{x}_0$ in time $\tilde{t}_0$, one can determine the needful value for $\tilde{v}_w$. On that way we can control the boundary layer separation

Preliminary calculations of expressions (12)–(17) have been made for a great accelerating flow ($\tilde{A} = +1$; $\tilde{B} = 1; n = 1$); $\tilde{t} = 0.0, 0.1, 0.2$; $\lambda_{0,0} = 0.0, +0.1$ (ejection through the contour), -0.1 (injection).

Curves note:

a) Circular cylinder - Fig.1, Fig.2, Fig.3

b) Thin elliptical cylinder - Fig.4, Fig.5, Fig.6

c) Aerofoil - Fig.7, Fig.8, Fig.9

4 Conclusions

It's found that for both in confuser and in diffuser contour regions the fluid ejection ($\lambda_{0,0} = 0.1$) increases the shear stress and postpones the separation of boundary layer i.e. laminar-turbulent transition section, and vice versa the fluid injection ($\lambda_{0,0} = -0.1$) reduces the shear stress and favors the flow separation, see Fig.3,6,9. In a favorable pressure gradient, where flow separation can never occur, the velocities profiles are very rounded, there is no point of inflection and there can be no separation. It is case when is ejection and when is very strong acceleration fluid flow (Fig.5,8). On this Figures one can see that laminar profiles of this type are very resistant to a transition to turbulence [4,9]. In an adverse pressure gradient, epecially for a circular cylinder (Fig.2), a point of inflection occurs in the boundary layer, its distance from the

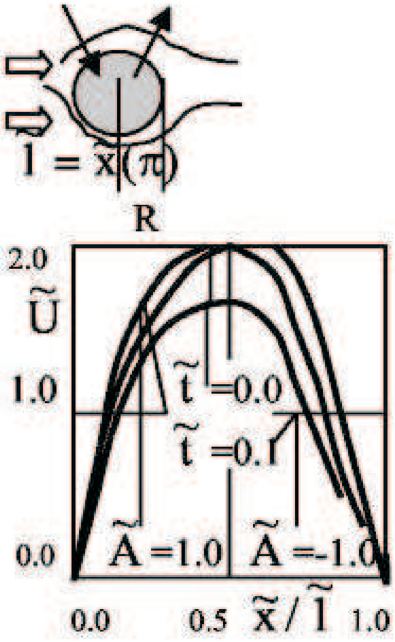

Figure 1: Potential velocity on circular cylinder. Shear stress distributions on circular cylinder

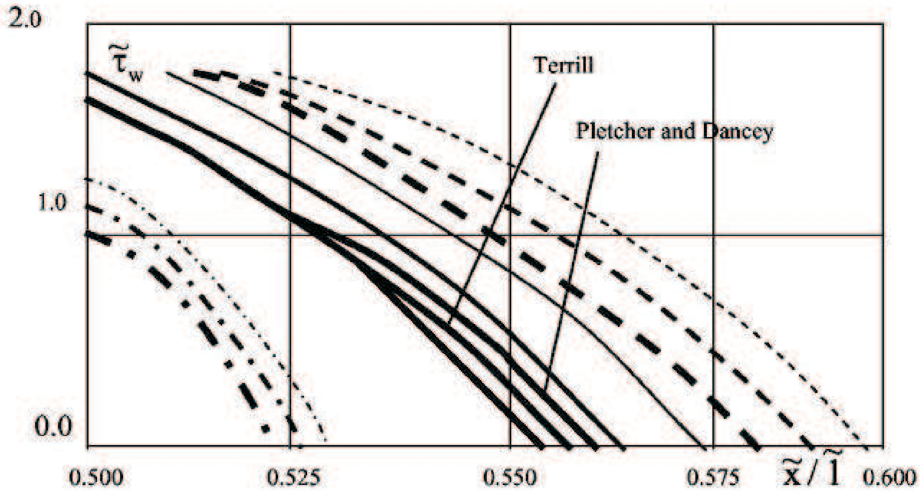

Figure 2: Shear stress distributions on circular cylinder

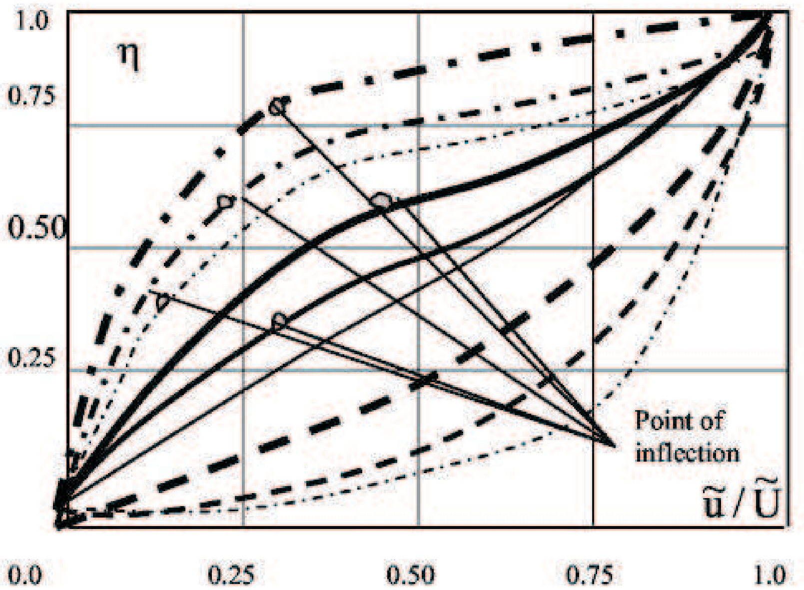

Figure 3: Velocity distributions on 51% of circular cylinder

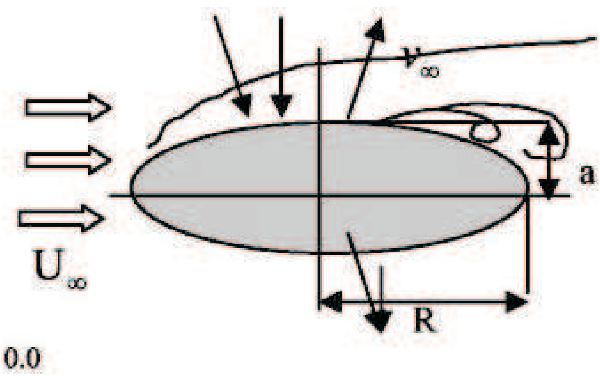

Figure 4: Thin elliptical cylinder

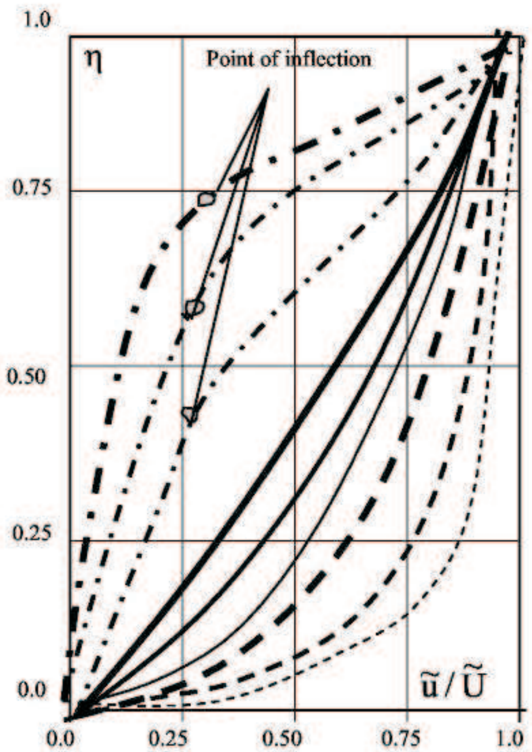

Figure 5: Velocity distributions on 84% thin elliptical c

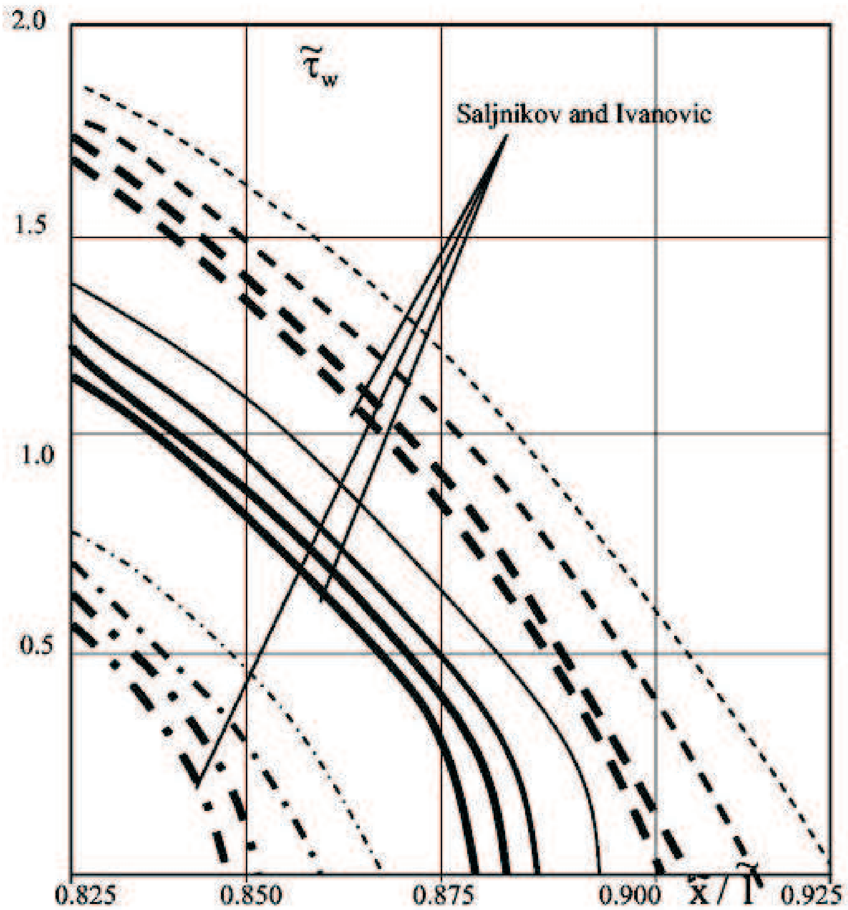

Figure 6: Shear stress distributions on thin elliptical cylinder

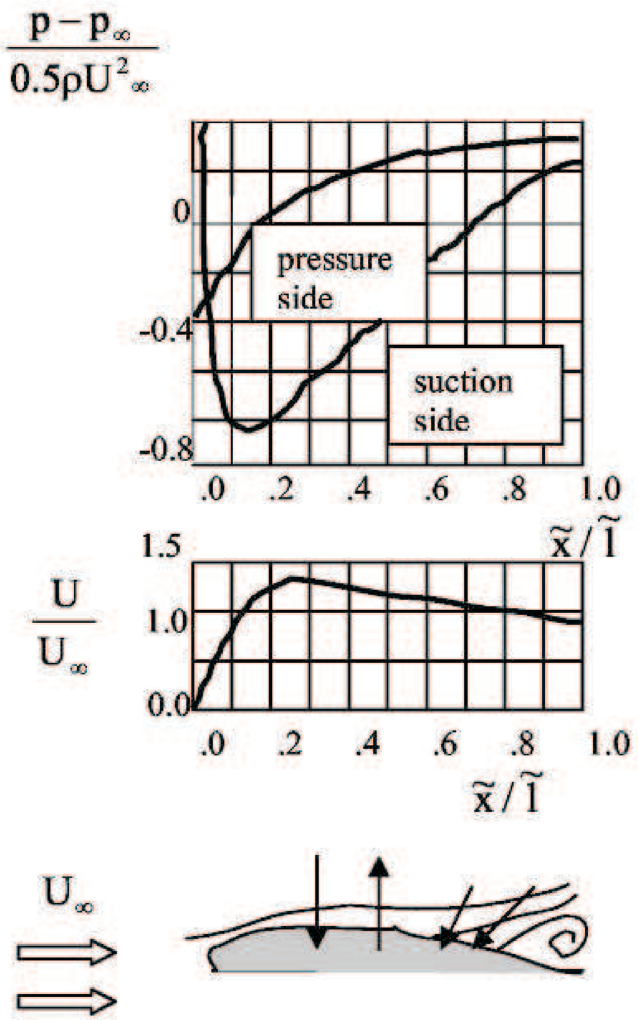

Figure 7: Potential external velocity on aerofoil

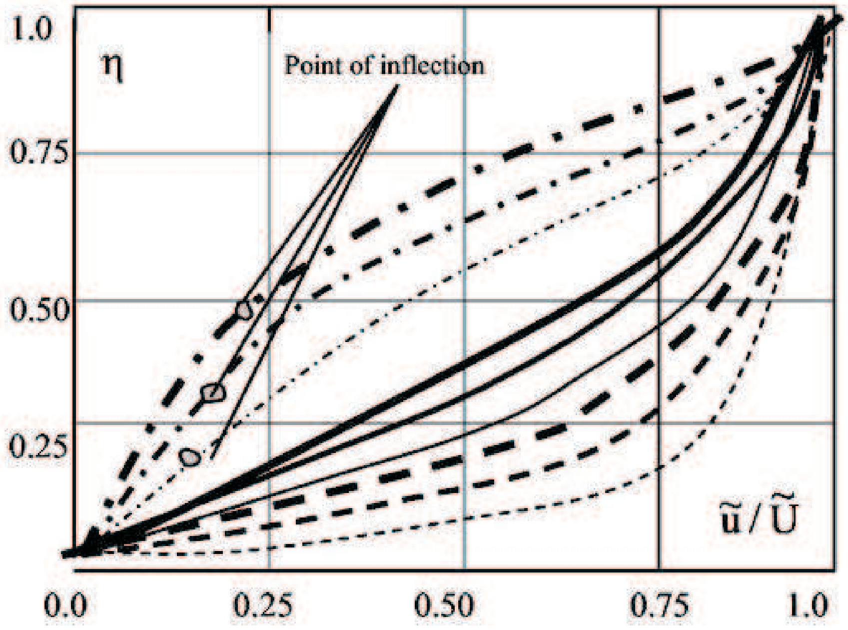

Figure 8: Velocity distributions on 77% of aerofoil contour

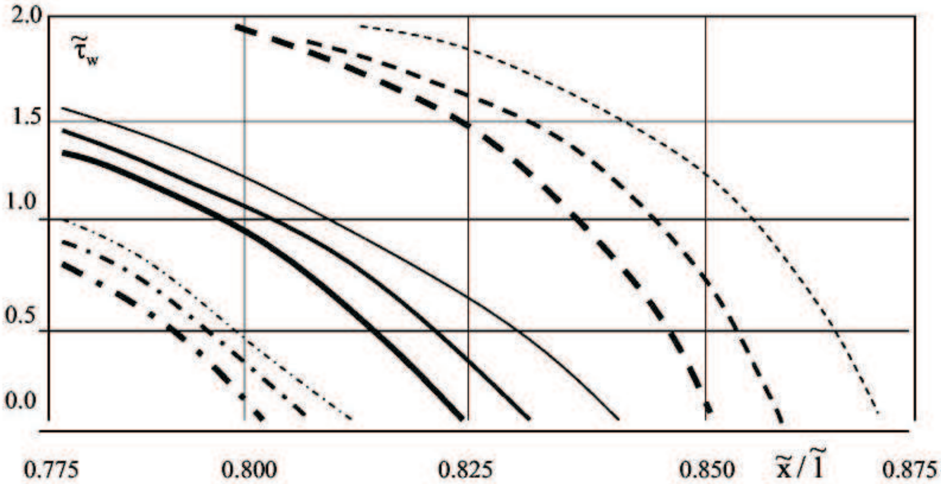

Figure 9: Shear stress distributions on aerofoil

wall increasing with the strength of the adverse gradient. For a weak gradient, when there are the elliptical cylinder (Fig.5) or aerofoil (Fig.8), the flow does not actually separate, but it is vulnerable to transition to turbulence. Also in the separation point where the wall shear stress is exactly zero, any stronger pressure gradient will actually cause backflow at the wall. Than the boundary layer thickens greatly and the main flow breaks away, or separates from the wall (Fig.3,6,9).

An important advantage of the generalized similarity method demonstrated in this paper is that the velocity profiles, skin friction and laminar-turbulent transition with separation point are found directly, no further numerical integration of momentum equation being involved.

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O uticaju usisavanja-isisavanja fluida kroz različite ubrzavane porozne konture na karakteristike 2D nestacionarnog nestišljivog graničnog sloja

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Kroz poroznu konturu u normalnom pravcu se ubrizgava ili isisava fluid istih osobina kao i osnovni fluid, brzinom koja je funkcija vremena i podužne koordinate. Odgovarajuće jednačine graničnog sloja, uvodjenjem svsishodnih transformacija promenljivih, impulsne i energijske jednačine kao i dva skupa parametara sličnosti, dobijaju uopšteni oblik. Uvedeni parametri predstavljaju uticaj brzine spoljašnjeg strujanja, brzine kojom se fluid kroz poroznu konturu ubrizgava u granični sloj ili isisava iz sloja kao i predistorijeu, na karakteristike graničnog sloja. Dobijena rešenja uopštene sličnosti su upotrijebljena za sračunavanje brzinskih raspodjela i trenja u laminarno-turbulentnim tranzitnim zonama nestacionarnog nestišljivog graničnog sloja na različitim poroznim konturama: kružni cilindar, tanki eliptični cilindar i aeroprofil, čiji se centri kreću vremenski promenljivim brzinama. Isisavanje fluida iz sloja odlaže njegovo odvajanje sa porozne konture, i obrnuto urizgavanje fluida u sloj favorizuje njegovu separaciju. Karakteristike graničnog sloja su nadjene direktno, bez dalje integracije impulsne jednačine za svaku konkretnu konturu.