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Digital practical tracking: algorithms with vector settling time

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Abstract

In this paper nonlinear stationary digital system with separated control is treated. The new definitions of practical tracking with vector settling time are presented. Furthermore, new criteria and control algorithms which ensure digital practical tracking with vector settling time are given and proven. The results are simulated on an example.

Key words: digital systems, practical tracking, algorithms, settling time

Nomenclature

$B \in \mathbb{R}^{n\times m}$**;** a matrix describing transmission of control action on the object internal dynamics

$\mathbf{b} : \mathbb{R} \times \ldots \times \mathbb{R} \to \mathbb{R}^m$**;** a vector nonlinear or linear function that is well defined

$\mathbf{e}[k; \mathbf{e}_0; \mathbf{y}_d(\cdot), \mathbf{u}(\cdot), \mathbf{z}(\cdot)] = \mathbf{e}(k) = \mathbf{e}_k \in \mathbb{R}^r,\ \mathbf{e}_k = \mathbf{y}_d(k) - \mathbf{y}(k)$**;** the time evolutions of output error vector related to initial error vector $\mathbf{e}_0$, control vector $\mathbf{u}(k)$, disturbance vector $\mathbf{z}(k)$ and desired output vector $\mathbf{y}_d(k)$ at the time $k \in Z_n$

$\mathbf{e}_{m(\cdot)} = \min\{\mathbf{e} : \mathbf{e} \in E_{(\cdot)}\}$ i $\mathbf{e}_{M(\cdot)} = \max\{\mathbf{e} : \mathbf{e} \in E_{(\cdot)}\}$, $\quad {}_{(\cdot)} =_{I,A,F}$**;** the minimum and maximum output error vector for any component of the system, respectively, in the sense $\mathbf{e}_{m(\cdot)} = (e_{1m(\cdot)}, e_{2m(\cdot)}, \ldots, e_{rm(\cdot)})^T$ and $\mathbf{e}_{M(\cdot)} = (e_{1M(\cdot)}, e_{2M(\cdot)}, \ldots, e_{rM(\cdot)})^T$

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$\mathbf{e}_{E(\cdot)}(\mathbf{e}_0)$**;** the vector of extreme output error (minimum or maximum) in the sense

$$e_{iE(\cdot)}(e_{i0}) = \begin{cases} e_{im(\cdot)}, & e_{i0} < 0 \\ 0, & e_{i0} = 0 \\ e_{iM(\cdot)}, & e_{i0} > 0 \end{cases} \qquad {}_{(\cdot)} =_{A,F}$$

$E_{(\cdot)} \in \mathbb{R}^r,\ {}_{(\cdot)} =_{I,A,F}$**;** the set of all permitted $\mathbf{e}_k$ (closed connected neighborhood of $\mathbf{0}_e$) with respect to time sets $\{0\}, Z_n$ and Z_s, respectively

$k \in Z_n$**;** the discrete time, the real time is $t = kT$, $T = t_{k+1} - t_k$ is the sample period. At the initial moment $k = k_0 = 0$

$n_p \in]0, \infty]$**;** discrete time on which tracking is realized

$\mathbf{n}_s(\mathbf{e}_0) = [n_1(e_{10}), n_2(e_{20}), \ldots, n_r(e_{r0})]^T$**;** the vector settling time for all components of the system

$M(\cdot) : Z_n \times \mathbb{N}^r \times \mathbb{R}^r \to \mathbb{R}^{r\times r}, \quad M(\cdot) = diag\{\mu_1(\cdot), \mu_2(\cdot), \ldots, \mu_r(\cdot)\}$**;** the matrix function, diagonal matrix of the functions $\mu_i(\cdot),\ \mu_i(\cdot) : Z_n \times \mathbb{N} \times \mathbb{R} \to \mathbb{R}, \quad i = 1, \ldots, r$

$\mathbf{s}(\mathbf{y}) = [sign(y_1), \ldots, sign(y_r)]^T$**;** the vector function whose element are signs of components the output vector $\mathbf{y}(\cdot)$

$\mathcal{S}_{yd}, \mathcal{S}_u\ i\ \mathcal{S}_z$**;** the sets of all admitted vector: desired output $\mathbf{y}_d(\cdot)$, control $\mathbf{u}(\cdot)$ and disturbance $\mathbf{z}(\cdot)$ on the Z_n, respectively

$\mathbf{u}(\cdot) \in \mathbb{R}^m$**;** the control vector

$\mathbf{y}[k; \mathbf{y}_0; \mathbf{y}_d(\cdot), \mathbf{u}(\cdot), \mathbf{z}(\cdot)] = \mathbf{y}(k) = \mathbf{y}_k \in \mathbb{R}^r$**;** the time evolutions of output vector $\mathbf{y}(k)$ related to initial output vector $\mathbf{y}_0$, control action $\mathbf{u}(\cdot)$ and disturbance $\mathbf{z}(\cdot)$, with respect to desired output vector $\mathbf{y}_d(k)$, on the time set Z_n.

$\mathcal{Y}_{(\cdot)}(k) = \mathcal{Y}_{(\cdot)}[k; \mathbf{y}_d(k); E_{(\cdot)}] = \{\mathbf{y} : \mathbf{y}(k) = \mathbf{y}_d(k) - \mathbf{e}(k), \mathbf{e}(k) \in E_{(\cdot)}\},\ {}_{(\cdot)} =_{I,A,F}$**;** the set value of the set function of all admitted output vector $\mathbf{y}(k)$ with respect to $\mathbf{y}_{d0}$ and $E_{(\cdot)}$ on the appropriate time sets $\{\mathbf{0}\}, Z_n, Z_s$, respectively

$\mathbf{z}(\cdot) : Z_n \to \mathbb{R}^p$**;** the disturbance vector function, evolutions $\mathbf{z}(t)$ on the time set Z_n.

$Z_n = [0, n_p[,\ n_p \in \mathbb{N}$**;** the discrete time set on which tracking realization is required.

$Z_s = [\mathbf{n}_s, n_p\mathbf{1}[$**;** the discrete time set of settling time of any components, n_{si} is first moment when $i-$th components goes into the set $\mathcal{Y}_F(\cdot)$, $k\mathbf{1} \in Z_s$, in the sense $k \in [n_{si}, n_p[$, $i = 1, 2, \ldots, r$

$\mathbf{1} = (1, 1, \ldots, 1)^T$**;** unity vector of appropriate dimension

1 Introduction

The concept of practical tracking of nonlinear digital automatic control systems was introduced by Grujić [1] in 1985. This concept has further been developed by the same author in [2, 3].

Consideration of dynamical behavior of a technical plant at limited and pre-specified time interval, with pre-specified quality of such a behavior, shows that for most technical plants this is the most adequate concept of tracking and control at all. This concept completely satisfies the practical technical requirements in viewpoint of dynamical behavior at limited time interval and with pre-specified quality of this behavior. The practical approach implies physically possible and realizable system initials, maximal admitted output deviations with respect to desired values (according to desired accuracy) and all of them at different time sets which are of technical interest. In this case, system could be influenced by disturbances, expected or unexpected, which belong to the set of admitted disturbances. Also, the synthesized control belongs to the set of admitted and physically realizable controls.

So far, most of papers of the type are related to practical tracking of continuous automatic control systems. In this paper, practical tracking of nonlinear stationary digital systems with vector settling time is considered.

2 Problem statement

In this paper, the stationary digital automatic control object, whose mathematical model with all actuators and sensors, is given by nonlinear discrete equations

$$\begin{aligned} \mathbf{f}(\mathbf{x}_k, \ldots, \mathbf{x}_{k+\alpha}, \mathbf{z}_k) &= B\mathbf{b}(\mathbf{u}_k), \\ \mathbf{y}_k &= \mathbf{g}(\mathbf{x}_k, \mathbf{z}_k), \end{aligned} \tag{1}$$

is observed, where: $\mathbf{x}_k \in \mathbb{R}^n, \mathbf{z}_k \in \mathbb{R}^p, \mathbf{u}_k \in \mathbb{R}^m, \mathbf{y}_k \in \mathbb{R}^r$ are the state vector, the disturbance vector, the input vector and output vector, respectively, and vector function $\mathbf{f} : \mathbb{R}^n \times \ldots \times \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$, $\mathbf{g} : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^r$ and $\mathbf{b} : \mathbb{R}^m \to \mathbb{R}^m$ are function of system internal dynamics, output and control function including actuators and control final elements nonlinearities, respectively. This model represents large class of plants which are physically continuous but observed only in discrete time instants.

The control matrix B for real technical systems usually has more rows then columns[1], never oppositely and rarely the same number, and satisfies one of next requirements:

- matrix B together with vector function $\mathbf{b}(\mathbf{u}_k)$ could be extended to non singular form such that $det(BB^T) \neq 0$ (see [8]);
- there exists a matrix $C \in \mathbb{R}^{m\times n}$ such that $det(CB) \neq 0$.

Except previous conditions, system (1) satisfies the next assumptions:

A1: each component of output vector $\mathbf{y}_k$ is measurable at every instant $k \in Z_n$;

A2: each component of vector $\mathbf{x}_k$ either is measurable or could be calculated as $\mathbf{x}_k = \mathbf{g}^I(\mathbf{y}_k, \mathbf{z}_k)$. The components of vector $\mathbf{x}_{k+i}$ are measurable or could be calculated by means of the previous vector $\mathbf{x}_{k+i-1}, i = 1, 2, \ldots, \alpha$;

A3: all components of disturbance vector $\mathbf{z}_k$ are measurable at every instant $k \in Z_n$;

A4: the vector functions: of internal dynamics $\mathbf{f}(\cdot)$, of output $\mathbf{g}(\cdot)$ and of control $\mathbf{b}(\cdot)$ are available;

A5: there exists a solution of vector function $\mathbf{b}(\cdot)$ related to $\mathbf{u}_k$ and it is unique and

A6: there exists a matrix $F \in \mathbb{R}^{r\times m}$ such that $det(FF^T) \neq 0$.

3 Definition

Definition 1 *The plant (1) controlled by digital control* $\mathbf{u}(\cdot) \in \mathcal{S}_u$ *exhibits practical tracking with vector settling time* $\mathbf{n}_s(\mathbf{e}_0)$ *with respect to*

[1]This means that number of states is larger than number of controls.

$\{n_p, \mathcal{Y}_I(\cdot), \mathcal{Y}_A(\cdot), \mathcal{Y}_F(\cdot), \mathcal{S}_{yd}, \mathcal{S}_z\}$ *if and only if, for every* $[\mathbf{y}_d(\cdot), \mathbf{z}(\cdot)] \in \mathcal{S}_{yd} \times \mathcal{S}_z$ *there exists* $\mathbf{u}(\cdot) \in \mathcal{S}_u$ *such that* $\mathbf{y}_0 \in \mathcal{Y}_I(\mathbf{y}_{d0}; E_I)$ *implies*

$$\mathbf{y}[k; \mathbf{y}_0; \mathbf{y}_d(\cdot), \mathbf{u}(\cdot), \mathbf{z}(\cdot)] \in \mathcal{Y}_A(k), \quad \forall k \in Z_n$$

and

$$\mathbf{y}[k; \mathbf{y}_0; \mathbf{y}_d(\cdot), \mathbf{u}(\cdot), \mathbf{z}(\cdot)] \in \mathcal{Y}_F(k), \quad \forall k\mathbf{1} \in [\mathbf{n}_s(\mathbf{e}_0), n_p\mathbf{1}[,$$

(cf. figure 1.) ■

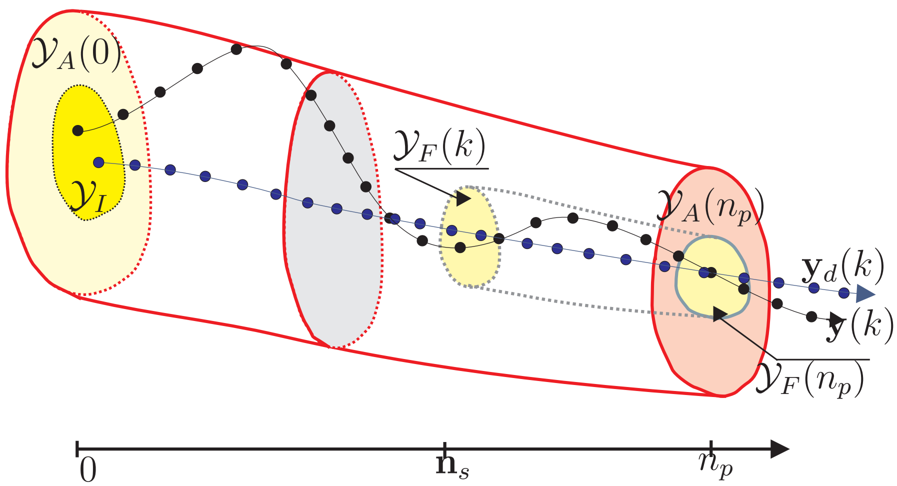

Figure 1: Practical tracking on $\mathcal{R}_y$

The definition of practical tracking with settling time has been given by Grujić, and it has been expressed via the output error vector $\mathbf{e}(\cdot)$ and scalar settling time, but not via output vector $\mathbf{y}(\cdot)$ and vector settling time. This definition determines higher quality of tracking because settling time is elementwised, for each element of output vector $\mathbf{y}(\cdot)$.

Definition of practical tracking with vector settling time for nonlinear continuous automatic control systems has been given in [5] by Lazić.

4 Control algorithm

Theorem 1 *Let the plant (1) satisfy assumptions A1–A6 and let* $\mathcal{S}_u = \{\mathbf{u}(\cdot)\}$ *with control function* $\mathbf{b}[\mathbf{u}(\cdot)]$ *determined by*

$$\mathbf{b}(\mathbf{u}_k) = F^T(FF^T)^{-1}\{F(CB)^{-1}C\mathbf{f}(\mathbf{x}_k, \cdots, \mathbf{x}_{k+\alpha}, \mathbf{z}_k) + \Delta(\mathbf{e}_{k-1}) + M(k-1; \mathbf{n}_s; \mathbf{e}_0)\mathbf{s}(\mathbf{e}_{k-1})\}, \tag{2}$$

$$\forall[k, \mathbf{e}_0, \mathbf{y}_d(\cdot), \mathbf{z}(\cdot)] \in Z_n \times E_I \times \mathcal{S}_{yd} \times \mathcal{S}_z.$$

The plant (1) controlled by digital control $\mathbf{u}(\cdot) \in \mathcal{S}_u$ *exhibits practical tracking with vector settling time* $\mathbf{n}_s(\mathbf{e}_0) \in [0, n_p\mathbf{1}[$ *in space* $\mathcal{R}_y$*, with respect to* $\{n_p, \mathcal{Y}_I(\cdot), \mathcal{Y}_A(\cdot), \mathcal{Y}_F(\cdot), \mathcal{S}_{yd}, \mathcal{S}_z\}$ *if for every* $i = 1, 2, \ldots, r$ *the following holds*

$$\mu_{i1}(n_{si}; e_{i0}) \in \begin{cases} \left[\dfrac{(|e_{i0}| - |e_{iEF}(e_{i0})|)}{n_{si}}, \dfrac{|e_{i0}|}{n_{si}}\right], & \forall e_{i0} \in E_{Ii} \backslash E_{Fi}, \\ \left[0, \dfrac{|e_{i0}|}{n_{si}}\right], & \forall e_{i0} \in E_{Fi}, \end{cases} \tag{3}$$

$$\mu_{i2}(n_{si}; e_{i0}) = 0, \tag{4}$$

$$\mu_i(k; n_{si}; e_{i0}) = \begin{cases} \mu_{i1}(n_{si}; e_{i0}), & k \in [0, n_{si}(e_{i0})[, \\ \mu_{i2}(n_{si}; e_{i0}), & k \in [n_{si}(e_{i0}), n_p[, \end{cases} \tag{5}$$

where coefficient $\mu_i(k; n_{si}; e_{i0})$ *is defined for all* k *in time intervals* $[0,\ n_{si}(e_{i0})[$ *and* $[n_{si}(e_{i0}),\ n_p[$.

Proof: Multiplying the first equation of system (1) by matrix $F(CB)^{-1}C$ from the left side we have

$$F\mathbf{b}(\mathbf{u}_k) = F(CB)^{-1}C\mathbf{f}(\mathbf{x}_k, \ldots, \mathbf{x}_{k+\alpha}, \mathbf{z}_k). \tag{6}$$

On the other hand, the same type of multiplication of (2) by matrix F gives

$$F\mathbf{b}(\mathbf{u}_k) = F(CB)^{-1}C\mathbf{f}(\mathbf{x}_k, \ldots, \mathbf{x}_{k+\alpha}, \mathbf{z}_k) + \Delta(\mathbf{e}_{k-1}) + M(k-1; \mathbf{n}_s; \mathbf{e}_0)\mathbf{s}(\mathbf{e}_{k-1}). \tag{7}$$

Now, comparing of (6) and (7) we acquire

$$\Delta(\mathbf{e}_{k-1}) = -M(k-1;\mathbf{n}_s;\mathbf{e}_0)\mathbf{s}(\mathbf{e}_{k-1}). \tag{8}$$

Since the previous system is stationary inside both time sets Z_s and Z_n, the shift by one sampling period gives

$$\Delta\mathbf{e}[k;\mathbf{e}_0;\mathbf{y}_d(\cdot),\mathbf{u}(\cdot),\mathbf{z}(\cdot)] = -M(k;\mathbf{n}_s;\mathbf{e}_0)\mathbf{s}[\mathbf{e}(k)]. \tag{9}$$

Let us observe behavior of the system (9) for arbitrary values: of desired output vector $\mathbf{y}_d(\cdot) \in \mathcal{S}_{yd}$, of disturbance vector $\mathbf{z}(\cdot) \in \mathcal{S}_z$, as well as arbitrary vector of initial error $\mathbf{e}_0 \in E_I$ and all of them for any component $i = 1, 2, \ldots, r$ of the considered system (9). Now, the vector equation (9) can be expressed in scalar form, for any $i-$th component

$$\Delta e_i[k;e_{i0};\mathbf{y}_d(\cdot),\mathbf{u}(\cdot),\mathbf{z}(\cdot)] = -\mu_i(n_{si};e_{i0})sign(e_k). \tag{10}$$

Solution of system (10), for $k_0 = 0$, is given in Lemma 1 (see Appendix A) as

$$e_i[k;e_{i0};\mathbf{y}_d(\cdot),\mathbf{u}(\cdot),\mathbf{z}(\cdot)] = e_{i0} - \mu_i(n_{si};e_{i0})ksign(e_{i0}). \tag{11}$$

From this solution, for $e_{i0} \neq 0$, multiplying by $sign(e_{i0})$ and taking into account Lemma 3 as well as properties of function $sign(\cdot)$ we get

$$|e_i[k;e_{i0};\mathbf{y}_d(\cdot),\mathbf{u}(\cdot),\mathbf{z}(\cdot)]| = |e_{i0}| - \mu_i(n_{si};e_{i0})k. \tag{12}$$

The equations (3), (4) and (5) ensure nonnegative value of coefficient $\mu_i(n_{si};e_{i0})$ for any $e_{i0} \in E_I$, which together with solution (12) provide that absolute value of output error $|e_i(\cdot)|$ from (12) does not increase but decreases or remains at initial error value, i.e. $|e_i(k)| \leq |e_i(0)|$. In the case when the initial output error is $e_{i0} = 0$, from the equation (10) it follows that the error value does not change, but it remains at its initial value. Since also, $E_F \subset E_I \subseteq E_A$ and since the error sets E_F, E_I and E_A are closed connected neighborhoods of zero error vector $\mathbf{0}_e$, the consequence is (because of definitions of sets $\mathcal{Y}_I, \mathcal{Y}_A(\cdot)$ and $\mathcal{Y}_F(\cdot)$ and their interrelationships $\mathcal{Y}_F(\cdot) \subset \mathcal{Y}_I \subseteq \mathcal{Y}_A(\cdot)$) that for each component of the system in the vector form

$$\mathbf{y}[k;\mathbf{y}_0;\mathbf{y}_d(\cdot),\mathbf{u}(\cdot),\mathbf{z}(\cdot)] \in \mathcal{Y}_A(k), \quad \forall k \in Z_n \tag{13}$$

holds. Thus, the first condition of definition 1 is satisfied.

Further, let us observe behavior of system (1) on both time intervals $k \in [0, n_{si}[$ and $k \in [n_{si}, n_p[$ and for any $e_{i0} \in E_{Ii}$. Let, firstly, $e_{i0} \in E_{Ii} \backslash E_{Fi}$, consequently the initial error is out of set E_{Fi}. Then, according to (3), value of coefficient $\mu_1(\cdot)$ which is on the lower boundary in (12) and after n_{si} of sampling periods, is obtained.

In the case when the initial error is inside set E_{Fi} and, moreover, on lower boundary of interval (3), then according to (12), there is no change of the output error, but it remains identical as the initial error. From previous discussion it follows that criteria (3), i.e. function $\mu_1(\cdot)$, for any initial error value, cause that the error after n_{si} sampling instant, on time interval $k \in [0, n_{si}[$, enters the set E_{Fi}.

In another time interval $k \in [n_{si}, n_p[$ criteria (5),i.e. coefficient $\mu_2(\cdot)$ due to continuity of motion, imply that there is no any error change. Therefore the previous achieved value is kept staying inside set E_{Fi}. Keeping the error inside the set E_{Fi} and recalling the definition of set $\mathcal{Y}_{Fi}(\cdot)$ it follows that outputs $y_i(\cdot)$ remain in the $\mathcal{Y}_{Fi}(\cdot)$ for all $k \in [n_{si}, n_p[$, which is in vector form, for all components of system, could be written as

$$\mathbf{y}[k; \mathbf{y}_0; \mathbf{y}_d(\cdot), \mathbf{u}(\cdot), \mathbf{z}(\cdot)] \in \mathcal{Y}_F(k), \quad \forall k\mathbf{1} \in Z_s. \tag{14}$$

From the preceding considerations it is found that equations (13) and (14) are valid for arbitrary values $[\mathbf{e}_{i0}, i, \mathbf{y}_d(\cdot), \mathbf{z}(\cdot)] \in E_I \times \{1, 2, \ldots, r\} \times \mathcal{S}_{yd} \times \mathcal{S}_z$, consequently for each mentioned value. Accordingly, we may finally conclude that the plant (1) exhibits practical tracking with vector settling time in sense of the definition 1. Therefore, the stated theorem is proved. ■

5 Simulation results

For simulation of control algorithms described in theorem 1 we use the plant - manipulator with two rotational joints described by differential equations (17) (see figure 3) specified by means of next values:

$m_1 = 8kg,\ m_{2n} = 6kg;$

the masses of the elements $\overline{OA}$ and $\overline{AB}$, respectively;

$l_1 = 0.9m,\ l_2 = 0.6m;$
the lengths of the elements $\overline{OA}$ and $\overline{AB}$, respectively;

$t = 2s,\ T = 10^{-3}s = 1ms,\ n_p = 2000$; the tracking time, the sampler period and the discrete time of practical tracking, respectively;

$$\mathbf{y}_d(k) = \begin{cases} 0.8 - 10^{-4}k - 0.2(1 - e^{(-\frac{k}{200})}), \\ 0.4 + 10^{-4}k - 0.15\sin(\frac{k}{200})(1 - e^{-\frac{k}{125}}), \end{cases}$$
vector of desired outputs $\forall k \in Z_n$;

$\mathbf{n}_s = (\frac{n_p}{2}\ \ \frac{n_p}{2.5})^T = (1000\ \ 800)^T$; the vector of time settling;

$E_I = \{\mathbf{e} : (-0.10\ \ -0.08)^T \leq \mathbf{e} \leq (0.10\ \ 0.08)^T\}$;
the set of initial errors[2] $\mathbf{e}_0, k = 0$;

$E_A = \{\mathbf{e} : (-0.10\ \ -0.08)^T \leq \mathbf{e} \leq (0.10\ \ 0.08)^T\}$;
the set of actual errors $\mathbf{e}_k, k \in Z_n$;

$E_F = \{\mathbf{e} : (-0.02\ \ -0.01)^T \leq \mathbf{e} \leq (0.02\ \ 0.01)^T\}$;
the set of final errors $\mathbf{e}_k, k \in Z_s$;

$B = I_2$; the control matrix being unity matrix and

F; the subsidiary matrix, $F(\mathbf{q}) = [J(\mathbf{q})A(\mathbf{q})^{-1}B]^{-1} = A(\mathbf{q})J(\mathbf{q})^{-1}$
according to [4].

On the time set $Z_n \backslash Z_s$, sine change of error is chosen. This error satisfies conditions of theorem 1, and its sine change enables soft pass from one error change law to the another one at the vector moment $\mathbf{n}_s$. According to the algorithm above, the *min* and the *max* values for the actual values from example (17), are calculated. Based on them, values of elements of a matrix $M(k, \mathbf{n}_s; \mathbf{e}_{x0})$ are adopted with

$$M\left(k, \mathbf{n}_s; \mathbf{e}_{x0}\right) = \begin{cases} diag\left\{0.045\sin(\frac{\pi}{2n_{s1}}k), 0.072\sin(\frac{\pi}{2n_{s2}}k)\right\}, \forall k \in Z_n \backslash Z_s, \\ \qquad\qquad\qquad\qquad\qquad\qquad 0, \quad \forall k \in Z_s. \end{cases}$$

[2]Dimensions of all errors are in $[m]$.

6 Conclusions

Conditions (3) to (5) of the theorem 1 are designed in the way that, the error, on the time set $Z_n \backslash Z_s$, is brought into the set E_F, if it was outside the set E_F. Otherwise, if the error is in the set E_F at the initial moment, the conditions from (3) to (5) make error getting smaller towards zero or keep its default value. In the time set Z_s the above conditions keep the error value at the level, which was achieved until the instant $\mathbf{n}_s$. From this it follows that the total error change is happened in the time set $Z_n \backslash Z_s$, while on the time set Z_s control is synthesized keeping the error value on the previously obtained level.

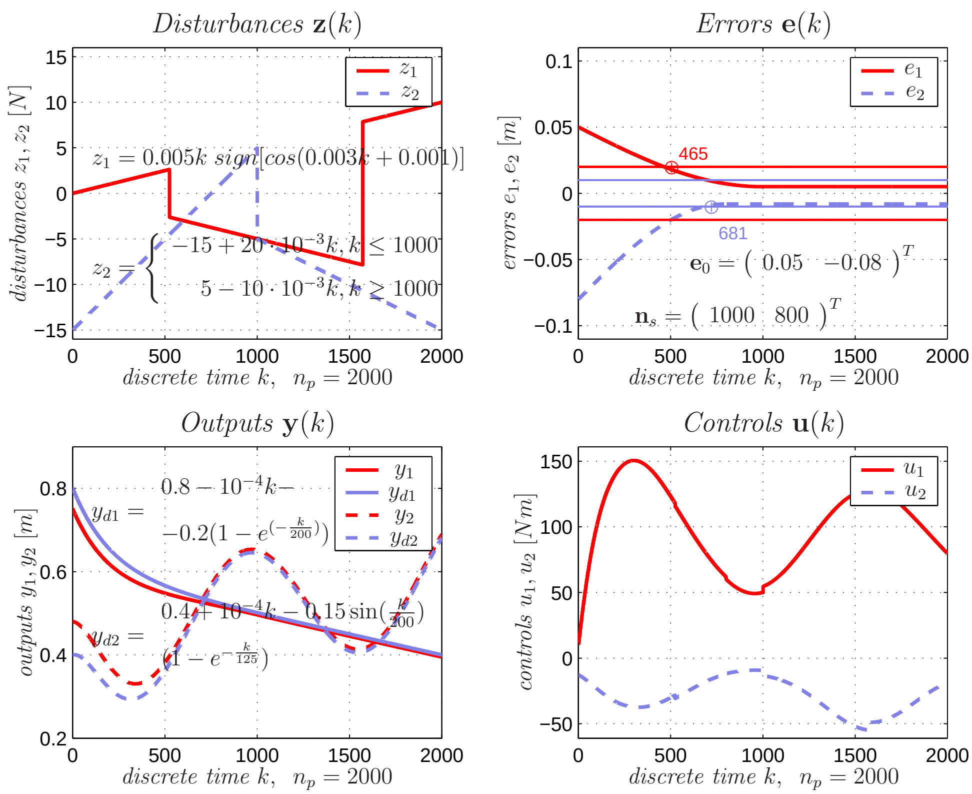

Figure 2: Results of simulations by the algorithms from theorem 1 applied to example (17)

The simulation results, of the algorithm from theorem 1, which is applied to the robot (17), are given in figure 2. From the mentioned simulation, it could be realized that the control based on this algorithm forces the foregoing plant to exhibits tracking with the vector settling time, which confirms thesis of the theorem 1. In the diagram on the figure 2, which deals with the output error change, mark "$\oplus$" denotes the first moments of the error's arrival into the set E_F.

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A Appendix - Lemma

Lemma 1 *Let the discrete time system be given by a scalar difference equation*

$$x_{k+1} = x_k - \mu(n;x_0)sign(x_k),\ x_k \in \mathbb{R},\ k \in Z_n, \mu : \mathbb{N} \times \mathbb{R} \to \mathbb{R}, \quad (15)$$

where real function $\mu(\cdot)$ *satisfies* $\mu(n;x_0) = \frac{|x_0|}{n}$. *Then the motion* $x(\cdot;k_0;x_0)$ *of system (15) is unique and continuous through every* $(k_0, x_0) \in Z_n \times \mathbb{R}$ *being determined by*

$$x(k;k_0;x_0) = \begin{cases} x_0 - \mu(n;x_0)(k-k_0)sign(x_0),\ k \in [k_0, k_0+n[, \\ 0,\ k \in [k_0+n, \infty[, \end{cases} \quad (16)$$

while the state $x(k;k_0;0_x)$ *is equilibrium state of the system.* ■

Lemma 2 *The zero equilibrium state* $x(k;0;0_x)$, *of system (15) is global asymptotic stable for* $\mu(k;n;x_0) \in \mathbb{R}^+$ *and for* $\forall k \in Z_n$. ■

Lemma 3 *The motion* $x(\cdot;0;x_0)$ *of system (15) for any* $\mu(k;n;x_0) \leq \frac{|x_0|}{n}$ *does not change sign, that is, sign of the motion keeps the same value as at initial moment or the motion becomes zero.* ■

For the proofs of previous lemmas see [9].

B Appendix - Mathematical model

Mathematical model of manipulator is gives by the system equation

$$A[\mathbf{q}(t), m(t)]\,\ddot{\mathbf{q}}(t) + \mathbf{h}[\mathbf{q}(t), \dot{\mathbf{q}}(t), m(t)] + \mathbf{G}[\mathbf{q}(t), m(t)] + \mathbf{J}^T(t)\,\mathbf{F}_{sp}(t) = \mathbf{M}(t), \qquad (17)$$
$$\mathbf{y}(t) = \mathbf{g}[\mathbf{q}(t)],$$

where $m_2 = m_{2n} + m(t); \quad l_{2p} = l_2; \quad l_{C1} = \frac{l_1}{2}; \quad l_{C2} = \frac{l_{2p}m(t) + \frac{l_2}{2}m_{2n}}{m_2}$

Matrix $A[\mathbf{q}(t), m(t)] = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ is the matrix of inertia with components

$$a_{11} = m_1 l_{C1}^2 + I_1 + m_2(l_1^2 + l_{C2}^2 + 2l_1 l_{C2}\cos q_2) + I_2,$$
$$a_{12} = m_2 l_1 l_{C2}\cos q_2 + m_2 l_{C2}^2 + I_2,$$
$$a_{21} = a_{12},$$
$$a_{22} = m_2 l_{C2}^2 + I_2.$$

Vector of Coriolis as well as centrifugal force

$$\mathbf{h}\left[\mathbf{q}, \dot{\mathbf{q}}, m(t)\right] = \begin{pmatrix} -\dot{q}_2^2 m_2 l_1 l_{C2}\sin q_2 - 2\dot{q}_1\dot{q}_2 m_2 l_1 l_{C2}\sin q_2 \\ \dot{q}_1^2 m_2 l_1 l_{C2}\sin q_2 \end{pmatrix}$$

while vector of gravitational force is given by the matrix

$$\mathbf{G}\left[\mathbf{q}, m(t)\right] = \begin{pmatrix} m_1 l_{C1} g\cos q_1 + m_2 g\left[l_1\cos q_1 + l_{C2}\cos\left(q_1 + q_2\right)\right] \\ m_2 l_{C2} g\cos\left(q_1 + q_2\right) \end{pmatrix}.$$

Vector of output is given by $\mathbf{y} = \mathbf{g}\left(\mathbf{q}\right) = \begin{pmatrix} l_1\cos q_1 + l_2\cos\left(q_1 + q_2\right) \\ l_1\sin q_1 + l_2\sin\left(q_1 + q_2\right) \end{pmatrix}.$

The masses of the both elements l_1, l_2 are constant while at the end point B of manipulator a variable mass $m(t)$ is situated whose temporal change is given by $m(t) = 2.5 + 2sin(5t)$.

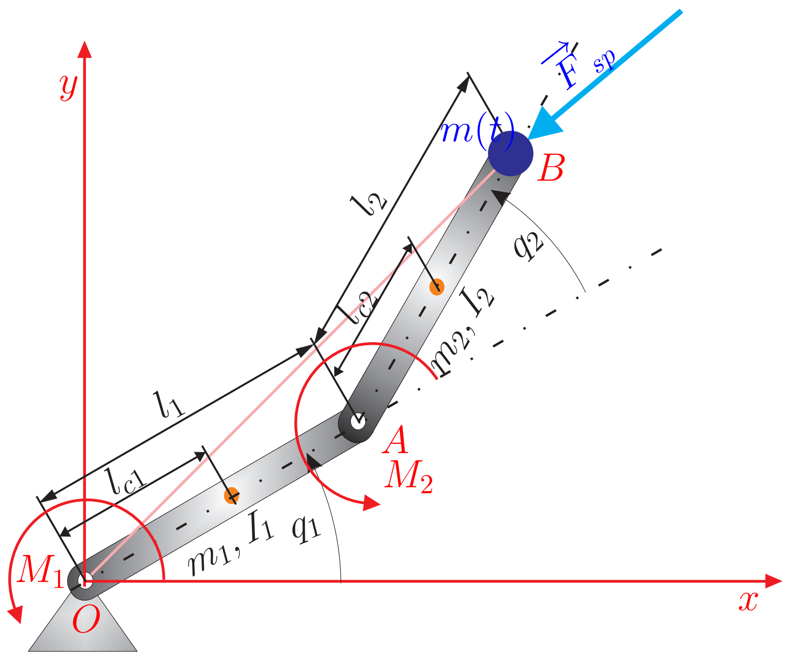

Figure 3: A manipulator with two rotations joints

Also, at the same point the external time varying force $\mathbf{F}_{sp}(t)$ acts, which in this case, has a role of disturbance. Its time change is given by

$$z_1 = 5t\ sign[cos(3t + 0.001)],$$
$$z_2 = \begin{cases} -15 + 20t, t \leq 1, \\ \quad 5 - 10t, t \geq 1. \end{cases} \tag{18}$$

Submitted on June 2005.

Digitalno praktično praćenje: algoritmi sa vektorskim vremenom smirenja

UDK 681.5

U ovom radu posmatra se stacionarni nelinearni digitalni sistem sa razdvojenim upravljanjem. Date su nove definicije praktičnog praćenja sa vektorskim vremenom smirenja. Takodje dati su i dokazani novi kriteriji i upravljački algoritmi koji obezbjedjuju praktično praćenje sa vektorskim vremenom smirenja. Dobijeni rezultati simulirani su na praktičnom primjeru.