# A form of MHD universal equations of unsteady incompressible fluid flow with variable electroconductivity on heated moving plate

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#### Abstract

This paper deals with laminar, unsteady flow of viscous, incompressible and electro conductive fluid caused by variable motion of flat plate. Fluid electro conductivity is variable. Velocity of the plate is time function. Plate moves in its own plane and in "still" fluid. Present external magnetic filed is perpendicular to the plate. Plate temperature is a function of longitudinal coordinate and time. Viscous dissipation, Joule heat, Hole and polarization effects are neglected.

For obtaining of universal equations system general similarity method is used as well as impulse and energy equation of described problem.

**Keywords**: MHD flow, general similarity method, electroconductive fluid, flat plate.

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#### 1 Introduction

One of the first prospectors who considered natural and forced incompressible viscous fluid flow on the solid plates was Ostrach [1]. Later on Grief with associates [2], Gupta with associates [3] and other scientist investigated fluid flow on inert flat plate. A motion either of flat plate or solid surface causes a change of fluid flow and such a flow has been the exploration subject of Sakiadis [4]. In our previous paper [5], we considered MHD fluid flow caused by variable plate moving. In this paper, we will consider unsteady MHD flow of incompressible fluid with variable electro-conductivity caused by a motion of flat plate with variable velocity. For contemplation of described problem "universalization" method of laminar boundary layer equations has been used, which is developed by L.G.Loicijanskij [6] for steady problems, and Busmarin and Basin [7] for unsteady problems. This method has numerous unsuspected benefits in comparison with other approximated methods. By using this method, which is a very important, universal equations of described problem are obtained. Obtained system of universal equations can be once for all numerically integrated by using a computer and during that only snipping of the system has been considered. The results of universal equations integration can be conveniently saved and then used for general conclusion conveyance about fluid flow and for calculations of particular problems. In this paper our concern is a development of fluid flow universal equations for the described problem.

### 2 Mathematical model

This paper is concerned with the laminar unsteady flow of viscous incompressible electro-conductive fluid caused by variable motion of a flat plate along x-axis (figure 1.). The plate is moving in its own plane and in "undisturbed" fluid. Plate velocity is a function of time t. Applied external magnetic field is perpendicular to the plate whereas external electric field is neglected. All fluid properties are assumed to be constant except the fluid electro-conductivity. Plate temperature is a function of longitudinal coordinate x and time t. Viscous dissipation, Joule heat, Hole and polarization effect are neglected.





The fluid electro-conductivity may be assumed in the following form [8]:

$$\sigma = \sigma_{\infty} \left( 1 - \frac{u}{U} \right); \tag{1}$$

where  $\sigma_{\infty}$  is constant fluid electro-conductivity, u - means longitudinal velocity of the fluid, U - stands for plate velocity.

The mathematical model of such a problem is expressed by the following equations:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + \gamma \frac{\partial^2 u}{\partial y^2} - NU \left(1 - \frac{u}{U}\right);$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2};$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0;$$
(2)

and boundary and initial conditions are given by:

$$u = U(t), v = 0, T = T_w(x, t) \text{ for } y = 0;$$
$$u \to 0, T \to T_\infty \text{ for } y \to \infty;$$
$$u = u_1(x, y), T = T_1(x, y) \text{ for } t = t_1;$$

$$u = u_0(t, y), \ T = T_0(t, y) \text{ for } x = x_0.$$
 (3)

In the system of equations (2) as well as the boundary and initial conditions (3) we employ the notation commonly used in the MHD boundary layer theory: x, y - longitudinal and transverse coordinate respectively; t - time; v - transverse velocity component;  $\gamma$ -coefficient of the kinematics viscosity of fluid.  $N = \sigma_{\infty}B^2/\rho$  where B - magnetic field induction,  $\rho$  - density of fluid,  $\alpha$  - heat conduction coefficient, T fluid temperature,  $T_w$  - plate temperature;  $T_\infty$  - temperature in "undisturbed" fluid;  $u_1(x, y)$  and  $T_1(x, y)$  - values of longitudinal velocity and fluid temperature, respectively, at time instant  $t = t_1$ ;  $u_0(t, y)$  and  $T_0(t, y)$  - longitudinal velocity and fluid temperature in cross section  $x = x_0$ .

For further consideration of the described problem in every particular case i.e. for prescribed values of  $U, N, T_w, T_\infty, u_1, T_1, u_0, T_0$  the system of equations (2) can be solved with given corresponding boundary and initial conditions (3). Obtained solution for each particular fluid flow problem then can be used for corresponding conclusions.

#### **3** Universal equations

In boundary layer theory Shkadov [9], Loicijanskij [6] and Saljnikov [10] introduced general similarity method in different forms. By means of this method with adequate choice of transformations and appropriate similarity parameters universal equations and universal boundary conditions can be obtained. Such equations and boundary conditions are independent on particular problem treated. Due to such a fact this procedure in literature is very often called "universalization" method.

Taking into account that the general similarity method gives good results not only for simple boundary layer problems but also for very complicated cases (cf. [11,12,13]) we make an attempt here to evolve this method in Loicijanskij version [6] to the described problem.

Accordingly, following the general similarity method [6,9,10] at the beginning we introduce flow function  $\Psi(x, y, t)$  by relations:

$$\frac{\partial\Psi}{\partial x} = -v \ , \ \frac{\partial\Psi}{\partial y} = u, \tag{4}$$

and then new variables in following form:

$$t = t, \ x = x, \ \eta = \frac{y}{h(x,t)}, \ \Phi(x,t,\eta) = \frac{\Psi(x,y,t)}{U(t)h(x,t)},$$
$$\Theta(x,t,\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}$$
(5)

where h(x, t) - some characteristic scale of transverse coordinate.

Further on instead of variables **x** and **t** new set of parameters is introduced as follows:

$$z = \frac{h^2}{\gamma}; \quad f_k = \frac{1}{U} \frac{d^k U}{dt^k} z^k \quad (k = 1, 2, ...),$$

$$g_{k,n} = U^{k-1} z^{k+n} \frac{\partial^{k-1+n} N}{\partial x^{k-1} \partial t^n}, \quad l_{k,n} = \frac{U^k}{q} \frac{\partial^{k+n} q}{\partial x^k \partial t^n} z^{k+n} ,$$
where  $q = T_w - T_\infty, \quad (k, n = 0, 1, 2, ...; \ k \lor n \neq 0)$  (6)

By using introduced flow function (4), the new variables (5) and sets of parameters (6) we transform the system (2) into the equations:

$$\Phi \left[\eta; (f_k), (g_{k,n}), (l_{k,n})\right] + G \left[\eta; (f_k), (g_{k,n}), (l_{k,n})\right] p + II \left[\eta; (f_k), (g_{k,n}), (l_{k,n})\right] F = 0$$

$$\Phi_1 \left[\eta; (f_k), (g_{k,n}), (l_{k,n})\right] + G_1 \left[\eta; (f_k), (g_{k,n}), (l_{k,n})\right] p + II_1 \left[\eta; (f_k), (g_{k,n}), (l_{k,n})\right] F = 0$$
(7)

where, for the sake of brevity, the next notations are introduced:

$$\Phi\left[\eta;\left(f_{k}\right),\left(g_{k,n}\right),\left(l_{k,n}\right)\right] = \frac{\partial^{3}\Phi}{\partial\eta^{3}} + f_{1}\left(1 - \frac{\partial\Phi}{\partial\eta}\right) - g_{1,0}\frac{\partial\Phi}{\partial\eta}\left[1 - \frac{\partial\Phi}{\partial\eta}\right] - \sum_{k=1}^{\infty}P_{k}\frac{\partial^{2}\Phi}{\partial\eta\partial f_{k}} +$$

$$\begin{split} \sum_{\substack{k,n=0\\k\neq n\neq 0}}^{\infty} \left[ g_{k+1,n} X\left(g_{k,n};\eta\right) + F_{k,n} X\left(l_{k,n};\eta\right) - R_{k,n} \frac{\partial^2 \Phi}{\partial \eta \partial g_{k,n}} - T_{k,n} \frac{\partial^2 \Phi}{\partial \eta \partial l_{k,n}} \right]; \\ G\left[\eta; \left(f_k\right), \left(g_{k,n}\right), \left(l_{k,n}\right)\right] &= \frac{1}{2} \eta \frac{\partial^2 \Phi}{\partial \eta^2} - \sum_{k=1}^{\infty} k f_k \frac{\partial^2 \Phi}{\partial \eta \partial f_k} - \sum_{\substack{k,n=0\\k\neq n\neq 0}}^{\infty} \left(k+n\right) \left(g_{k,n} \frac{\partial^2 \Phi}{\partial \eta \partial g_{k,n}} + l_{k,n} \frac{\partial^2 \Phi}{\partial \eta \partial l_{k,n}}\right) II\left[\eta; \left(f_k\right), \left(g_{k,n}\right), \left(l_{k,n}\right)\right] &= \frac{1}{2} \Phi \frac{\partial^2 \Phi}{\partial \eta^2} + \sum_{k=1}^{\infty} k f_k X\left(f_k;\eta\right) + \sum_{\substack{k,n=0\\k\neq n\neq 0}}^{\infty} \left(k+n\right) \left(g_{k,n} X\left(g_{k,n};\eta\right) + l_{k,n} X\left(l_{k,n};\eta\right)\right) \\ \Phi_1\left[\eta; \left(f_k\right), \left(g_{k,n}\right), \left(l_{k,n}\right)\right] &= \frac{1}{\Pr} \frac{\partial^2 \Theta}{\partial \eta^2} - l_{1,0} \Theta \frac{\partial \Phi}{\partial \eta} - l_{0,1} \Theta - \sum_{k=1}^{\infty} P_k \frac{\partial \Theta}{\partial f_k} + \sum_{\substack{k\neq n\neq 0\\k\neq n\neq 0}}^{\infty} \left[g_{k+1,n} Y\left(g_{k,n};\eta\right) + F_{k,n} Y\left(l_{k,n};\eta\right) - R_{k,n} \frac{\partial \Theta}{\partial g_{k,n}} - T_{k,n} \frac{\partial \Theta}{\partial l_{k,n}}\right]; \\ G_1\left[\eta; \left(f_k\right), \left(g_{k,n}\right), \left(l_{k,n}\right)\right] &= \frac{1}{2} \eta \frac{\partial \Theta}{\partial \eta} - \sum_{k=1}^{\infty} k f_k \frac{\partial \Theta}{\partial f_k} - \sum_{\substack{k\neq n\neq 0\\k\neq n\neq 0}}^{\infty} \left(k+n\right) \left(g_{k,n} \frac{\partial \Theta}{\partial g_{k,n}} + l_{k,n} \frac{\partial \Theta}{\partial l_{k,n}}\right) \\ II_1\left[\eta; \left(f_k\right), \left(g_{k,n}\right), \left(l_{k,n}\right)\right] &= \frac{1}{2} \Phi \frac{\partial \Theta}{\partial \eta} + \sum_{k=1}^{\infty} k f_k Y\left(f_k;\eta\right) + \sum_{\substack{k\neq n\neq 0\\k\neq n\neq 0}}^{\infty} \left(k+n\right) \left(g_{k,n} Y\left(g_{k,n};\eta\right) + l_{k,n} Y\left(l_{k,n};\eta\right)\right) \\ F &= U \frac{\partial Z}{\partial x}; \quad p = \frac{\partial Z}{\partial t}; \end{split}$$

 $P_k = -f_1 f_k + f_{k+1};$   $R_{k,n} = (k-1) f_1 g_{k,n} + g_{k,n+1};$ 

$$F_{k,n} = -l_{1,0}l_{k,n} + l_{k+1,n}; \quad T_{k,n} = (kf_1 - l_{0,1}) l_{k,n} + l_{k,n+1};$$
$$X(x_1; x_2) = \frac{\partial \Phi}{\partial x_1} \frac{\partial^2 \Phi}{\partial \eta \partial x_2} - \frac{\partial \Phi}{\partial x_2} \frac{\partial^2 \Phi}{\partial x_1 \partial \eta};$$
$$Y(x; \eta) = \frac{\partial \Phi}{\partial x} \frac{\partial \Theta}{\partial \eta} - \frac{\partial \Theta}{\partial x} \frac{\partial \Phi}{\partial \eta}.$$
(8)

Obtained system of equations (7) is not "universal" because it contains multipliers  $F = U \frac{\partial z}{\partial x}$ ,  $p = \frac{\partial z}{\partial t}$ , which are functions of coordinates x, t. If we express multipliers F and p in function of variables which explicitly depend only on the above introduced parameters system of equations (7) will be universal. In other words next equalities must be fulfilled:

$$U\frac{\partial z}{\partial x} = F\left[\left(f_{k}\right), \left(g_{k,n}\right), \left(l_{k,n}\right)\right]$$
$$\frac{\partial z}{\partial t} = p\left[\left(f_{k}\right), \left(g_{k,n}\right), \left(l_{k,n}\right)\right].$$
(9)

Functions F and p are obtained by making use of the impulse equation

$$\frac{\partial}{\partial t} \left( U\delta^* \right) + \frac{\partial}{\partial x} \left( U^2 \delta^{**} \right) - NU\delta^{**} = -\gamma \left( \frac{\partial u}{\partial y} \right)_{\infty}, \tag{10}$$

and energy equation

$$\frac{\partial}{\partial t} \left( U^2 \delta^{**} \right) + U^2 \frac{\partial \delta^*}{\partial t} - 3U^2 \varphi - 2NU^2 \delta_1^{**} - 2\gamma U^2 e = 0 \qquad (11)$$

of described problem. In the above two equations the following notations are introduced:

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy, \quad \delta^{**} = \int_0^\infty \frac{u}{U} \left(1 - \frac{u}{U}\right) dy,$$
$$\delta_1^{**} = \int_0^\infty \left(\frac{u}{U}\right)^2 \left(1 - \frac{u}{U}\right) dy, \quad e = \int_0^\infty \left[\frac{\partial \left(u/U\right)}{\partial y}\right] dy,$$

$$\varphi = \int_{0}^{\infty} \left(\frac{u}{U}\right)^2 \frac{\partial u}{\partial x} dy.$$
 (12)

However, before determination of function F and p, we must define scale h(x,t) which is present in equations (5). This scale is chosen in such a way to satisfy following equations:

$$\frac{\partial}{\partial x} \left( U^2 h \right) = \frac{\partial}{\partial x} \left( U^2 \delta^{**} \right) + \frac{\partial}{\partial t} \left( U \delta^* \right)$$
$$\frac{\partial}{\partial t} \left( U^2 h \right) = \frac{\partial}{\partial t} \left( U^2 \delta^{**} \right) - 3U^2 \varphi - 2NU^2 \delta_1^{**} - 2\gamma U^2 e \qquad (13)$$

The equations of impulse (10) and energy (11) now take the form:

$$\frac{\partial}{\partial x} \left( U^2 h \right) - NU \delta^{**} + \gamma \left( \frac{\partial u}{\partial y} \right)_{\infty} = 0,$$
$$\frac{\partial}{\partial t} \left( U^2 h \right) + U^2 \frac{\partial \delta^*}{\partial t} = 0.$$
(14)

From the first of them we obtain:

$$U\frac{\partial z}{\partial x} = 2\left(g_{1,0}H - \zeta\right),\tag{15}$$

where the following marks are introduced:

$$H = \frac{\delta^{**}}{h} = \int_{0}^{\infty} \frac{\partial \Phi}{\partial \eta} \left( 1 - \frac{\partial \Phi}{\partial \eta} \right) d\eta;$$
  
$$\zeta = \left. \frac{\partial \left( u/U \right)}{\partial \left( y/h \right)} \right|_{y \to \infty} = \left. \frac{\partial^2 \Phi}{\partial \eta^2} \right|_{\eta \to \infty}.$$
 (16)

Variables H and  $\zeta$  depend only on the chosen parameters, so it is proved that quantity F which is expressed by the equation (15) has the same dependence. From the second equation (14) we obtain:

$$\frac{\partial z}{\partial t} = \frac{-2\left[2f_1 + \sum_{k=1}^{\infty} P_k \frac{\partial H_1}{\partial f_k} + \sum_{\substack{k,n=0\\k \lor n \neq 0}}^{\infty} \left(R_{k,n} \frac{\partial H_1}{\partial g_{k,n}} + T_{k,n} \frac{\partial H_1}{\partial l_{k,n}}\right)\right]}{1 + H_1 + 2\sum_{k=1}^{\infty} kf_k \frac{\partial H_1}{\partial f_k} + 2\sum_{\substack{k,n=0\\k \lor n \neq 0}}^{\infty} (k+n) \left(g_{k,n} \frac{\partial H_1}{\partial g_{k,n}} + l_{k,n} \frac{\partial H_1}{\partial l_{k,n}}\right)},$$
(17)

where next notation is introduced:

$$H_1 = \frac{\delta^*}{h} = \int_0^\infty \left(1 - \frac{\partial \Phi}{\partial \eta}\right) d\eta, \qquad (18)$$

and it is noticed that quantity  $\frac{\partial z}{\partial t}$  also depends only on the introduced parameters. Existence of equation (9) is now proved and explicit form of the same equation is determined. Now we may claim that the system of equations (7) presents an universal system of equations of observed problem and it can be presented in more convenient form:

$$\frac{\partial^{3}\Phi}{\partial\eta^{3}} + f_{1}\left(1 - \frac{\partial\Phi}{\partial\eta}\right) - g_{1,0}\frac{\partial\Phi}{\partial\eta}\left(1 - \frac{\partial\Phi}{\partial\eta}\right) + \frac{1}{2}F\Phi\frac{\partial^{2}\Phi}{\partial\eta^{2}} + \frac{1}{2}\eta p\frac{\partial^{2}\Phi}{\partial\eta^{2}} = \sum_{k=1}^{\infty} \left[E_{k}\frac{\partial^{2}\Phi}{\partial\eta\partial f_{k}} + D_{k}X\left(\eta;f_{k}\right)\right] + \sum_{\substack{k,n=0\\k\vee n\neq0}}^{\infty} \left[L_{k,n}\frac{\partial^{2}\Phi}{\partial\eta\partial g_{k,n}} + J_{k,n}\frac{\partial^{2}\Phi}{\partial\eta\partial l_{k,n}} + K_{k,n}X\left(\eta;g_{k,n}\right) + S_{k,n}X\left(\eta;l_{k,n}\right)\right] \frac{1}{\Pr}\frac{\partial^{2}\Theta}{\partial\eta^{2}} + \frac{1}{2}F\Phi\frac{\partial\Theta}{\partial\eta} - l_{1,0}\Theta\frac{\partial\Phi}{\partial\eta} - l_{0,1}\Theta + \frac{1}{2}\eta p\frac{\partial\Theta}{\partial\eta} = \sum_{\substack{k=1\\k\vee n\neq0}}^{\infty} \left[E_{k}\frac{\partial\Theta}{\partial f_{k}} + D_{k}Y\left(\eta;f_{k}\right)\right] + \sum_{\substack{k=1\\k\vee n\neq0}}^{\infty} \left[L_{k,n}\frac{\partial\Theta}{\partial g_{k,n}} + J_{k,n}\frac{\partial\Theta}{\partial l_{k,n}} + K_{k,n}Y\left(\eta;g_{k,n}\right) + S_{k,n}Y\left(\eta;l_{k,n}\right)\right]$$
(19)

where, for the sake of brevity, next notations are introduced and applied above:

$$D_{k} = kFf_{k}; \quad E_{k} = P_{k} + kpf_{k}$$
$$K_{k,n} = (k+n) Fg_{k,n} + g_{k+1,n}; \quad L_{k,n} = R_{k,n} + (k+n) pg_{k,n}$$
$$S_{k,n} = F_{k,n} + (k+n) Fl_{k,n}; \quad J_{k,n} = T_{k,n} + (k+n) pl_{k,n}$$

Corresponding universal boundary conditions are now obtained from equations (3) as follows:

$$\Phi = 0, \frac{\partial \Phi}{\partial \eta} = 1, \ \Theta = 1 \quad \text{for} \quad \eta = 0;$$
$$\frac{\partial \Phi}{\partial \eta} \to 0, \ \Theta \to 0 \quad \text{for} \quad \eta \to \infty; \tag{20}$$

$$\Phi = \Phi_0, \quad \Theta = \Theta_0 \quad \text{for} \quad f_k = 0, \quad g_{k,n} = 0, \quad l_{k,n} = 0$$

where  $\Phi_0$  and  $\Theta_0$  represent solutions of system of equations:

$$\frac{\partial^3 \Phi}{\partial \eta^3} + \frac{1}{2} F \Phi \frac{\partial^2 \Phi}{\partial \eta^2} = 0,$$
  
$$\frac{1}{\Pr} \frac{\partial^2 \Theta}{\partial \eta^2} + \frac{1}{2} F \Phi \frac{\partial \Theta}{\partial \eta} = 0.$$
 (21)

System of equations (19) with boundary conditions (20), for exact values of Pr can be integrated numerically by means of computer while during such a process only "snipping" of equations is performed. Obtained results can be conveniently saved and then used for drawing general conclusion about fluid flow and for numerical calculations in other particular cases.

#### 4 Approximated universal equations

Actual solving of equations (19) requires a limitation of number of independent variables cutting some of them. This leads us to indispensable application of sector method, which consists of neglecting of all parameters starting with some index. This brings us to approximated universal equations of described problem. For example, if we retain influence of parameters  $f_1$ ;  $g_{1,0}$ ;  $l_{1,0}$  and neglect influence of the rest of parameters and their derivatives, then the system of universal equations in the three-parameter approximation is obtained.

$$\Im_{1} = Ff_{1}X(\eta; f_{1}) + f_{1}(p - f_{1})\frac{\partial^{2}\Phi}{\partial\eta\partial f_{1}} + Fg_{1,0}X(\eta; g_{1,0}) + l_{1,0}(F - l_{1,0})X(\eta; l_{1,0}) + pg_{1,0}\frac{\partial^{2}\Phi}{\partial\eta\partial g_{1,0}} + (f_{1} + p)l_{1,0}\frac{\partial^{2}\Phi}{\partial\eta\partial l_{1,0}};$$
  

$$\Im_{2} = Ff_{1}Y(\eta; f_{1}) + f_{1}(p - f_{1})\frac{\partial\Theta}{\partial f_{1}} + Fg_{1,0}Y(\eta; g_{1,0}) + (F - l_{1,0})l_{1,0}Y(\eta; l_{1,0}) + pg_{1,0}\frac{\partial\Theta}{\partial g_{1,0}} + (f_{1} + p)l_{1,0}\frac{\partial\Theta}{\partial l_{1,0}}, \quad (22)$$

where  $\Im_1, \Im_2$  are left side of first and second equations of system (19) respectively, and they are introduced to prevent long writing.

Corresponding boundary conditions for equations (22) read:

$$\Phi = 0, \ \frac{\partial \Phi}{\partial \eta} = 1, \ \Theta = 1 \quad \text{for} \quad \eta = 0;$$
$$\frac{\partial \Phi}{\partial \eta} \to 0, \ \Theta \to 0 \quad \text{for} \quad \eta \to \infty;$$
$$\Phi = \Phi_0, \ \Theta = \Theta_0 \quad \text{for} \quad f_1 = 0, \ g_{1,0} = 0, \ l_{1,0} = 0,$$
(23)

where  $\Phi_0$  and  $\Theta_0$  represent solution of system of equations (21).

In the same way, we could acquire other approximated equations of the described problem.

#### 5 Conclusion

This paper is concerned with unsteady MHD flow of incompressible fluid caused by a motion of flat plate with variable velocity. The fluid electro-conductivity is a linear function of velocity ratio. For a consideration of described problem, multiparameter method is used and universal equations are obtained in assistance with impulse and energy equations of described problem.

## References

- Ostrach, S., Combined natural and forced convection laminar flow and heat transfer of fluids with and without heat sources in channels with linearly varying wall temperatures, *National advisory committee for aeronautics*, pp.1-74,1954.
- [2] Grief, R.; Habib, I.S. and Lin, J.C, J. Fluid Mech., Vol.46, pp-513-523, 1971.
- [3] Gupta P.S., Gupta A.S, Int. J. Heat Mass Transfer, Vol. 17, pp.14-37, 1974.
- [4] Sakiadis, B.C, AICHE, 7-26, pp. 467-476, 1961.
- [5] Boricic Z., Nikodijevic D., Milenkovic D., Stamenkovic Ž., Universal equations of instationary MHD flow of an incompressible fluid over a heated moving plate, Proc. HIPNEF 2002, Vrnjacka Banja, Serbia, (in Serbian), 2002.
- [6] Loicianskij, L. G., Universal equations and parametric approximations in the theory of laminar boundary layer, *PMM – Appl. Math. Mech.*, Vol. 29, No. 1, pp. 70–87, (in Russian), 1967.
- [7] Bushmarin, O. N., Basin, B. Ja, A parametrical method of calculation for a laminar instationary boundary layer, *J. Engng. Phys.*, Vol. 22, No. 2, pp.282-292, (in Russian) 1972.
- [8] Rossow J.V., On flow of electrically conducting fluids over a flat plate in the presence of a tranverse magnetic filed, *NACA*, RP No 1358, 1958.
- [9] Shkadov, V., Boundary layer with pressure gradient within compressible fluid flow, Izv. AN SSSR, OTN, Mechanyka i mashinostroenie, No 2, pp. 28–32, (in Russian) 1963.

- [10] Saljnikov V. N., A Contribution to Universal Solutions of the Boundary Layer Theory, *Theoret. Appl. Mech.*, Vol. 4, pp. 139 – 163, 1978.
- [11] Obrovic B., Equations of ionised gas boundary layer and their parametric solutions, Facta Universitatis, Vol.1,No 5, pp.619-629, 1995.
- [12] Saljnikov V., Boricic Z., Nikodijevic D., Generalised similarity solutions for 3-D laminar compressible boundary layer flows on swept profiled cylinders, Acta Mech., Suppl. 4, pp. 389-399, 1994.
- [13] Ivanovic D., On usteady laminar two-dimensional boundary layer of incopmressible flow for airfoils, Proc. Yuctam, B8-B13, Nis, 1995.

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#### Jedan oblik univerzalnih jednačina nestacionarnog MHD strujanja nestišljivog fluida promenljive elektroprovodnosti na zagrejanoj pokretnoj ploči

UDK 532.526; 533.15

U radu se razmatra laminarno nestacionarno strujanje viskoznog, nestisljivog elektroprovodnog fluida izazvano promenljivim kretanjem ravne ploče. Elektroprovodnost fluida je promenljiva. Brzina ploče je funkcija vremena. Ploča se kreće u sopstvenoj ravni u "mirnom" fluidu. Prisutno spoljašnje magnetno polje je upravno na ploču. Temperatura ploče je funkcija uzdužne koordinate i vremena. Viskozna disipacija, Džulova toplota i Holov efekat su zanemareni. Za dobijanje univerzalnih jednačina korišćena je metoda uopštene sličnosti, kao i energijska i impulsna jednačina posmatranog problema.