

# Influence of structural backlash and friction in command system on the aircraft flutter

N.Maričić \*

## Abstract

Experience has shown that aircraft structures are generally affected by structural nonlinearities. The focus in this paper is concentrated on backlash and friction described in hysteresis loop of the classical aircraft command systems and their influence on flutter of aircraft. Based on AGARD No. 665 in paper is done nonlinear flutter velocity analysis in function of backlash and friction in the classical command system of aircraft. Unsteady aerodynamic forces are calculated based on well known Doublet-Lattice Method (DLM). Structural input data are taken from AGARD No. 665. Flutter eigenvalues are obtained by modified k-method. The flutter model of nonlinear aircraft structure is developed on base of harmonic linearization. The aim of paper is to achieve useful and relatively reliable tool for critical observations on different recommendations given in the various airworthiness regulations for nonlinear characteristics of hysteresis loops in the classical command systems of aircrafts.

**Key words:** nonlinear flutter, backlash, friction, hysteresis loop

---

\*Faculty of Technical Sciences, University of Priština, e-mail: maricicn@beotel.yu

## 1 Introduction

In a broad sense, flutter is self-excited oscillation caused by structural and aerodynamic forces coupling. The usual linear theory assumes both the structural and the aerodynamic properties to be independent with respect to (w.r.t.) the amplitudes of oscillation. Solution of the linear flutter problem has long since become a routine matter. However, the results of aircraft structure ground vibration tests and in flight flutter test are pointing that certain structural nonlinearities must always exist. Usually, linear theory gives relatively unreliable prediction of flutter speeds when certain amount of structural nonlinearities is incorporated. These structural nonlinearities are caused by: backlash and friction in command systems of classical aircrafts, spring tab nonlinearity, servo-actuator nonlinear characteristics in the command systems of the modern aircrafts, fixation of the external stores by varying tightening torques on military aircrafts, etc.

A lot of investigators have been dealing with these problems. The nonlinear flutter of command systems of classical aircraft were investigated in [1], [2], and [3]. Nonlinear tab flutter problems were analyzed in [10] and [11]. Influences of servo-actuator nonlinear characteristics due to preload in the command systems of the modern aircrafts and fixation of the external stores by varying tightening torques on military aircraft were investigated in [5] and [12]. Flutter analysis of missile control surfaces containing structural nonlinearities was done in [13].

The purpose of this paper is to represent the author's theoretical investigations and software development in connection to the nonlinear flutter of command systems of classical aircraft with some types of structural nonlinearities, using own previously developed tools for analysis of linear flutter problems. Non-stationary aerodynamic forces are calculated using the Doublet-Lattice Method (software UNAD) and flutter speeds are obtained using the k-method (software FLUTTER).

The goal of the considerations is to demonstrate the developed method of approximate inclusion of one mode containing nonlinear structural characteristics into the aircraft flutter calculation. The proposed theoretical and numerical development is based on the researches of a larger number of authors presented in [1], [2], [3], [4], and [5]. The enclosed

procedure is automated and software NELZAZ has been developed.

In the various airworthiness regulations empirically obtained recommendations are given for allowed values of backlash and friction force in classical command systems respective to flutter clearance. As direct application of in paper outlined results any nonlinear command system (with one nonlinear mode) flutter problem can be analyzed, and critical observations to the recommendations given in the various airworthiness regulations can be achieved.

## 2 Problem statement

Let, for sake of easier deriving interpretation, the advent of the lifting surface flutter be analyzed. Due to the simplicity of consideration, let only three modes be enclosed, and one of this modes be command system (surface) rotation mode. Based on the assumption about the structural and aerodynamic model linearity w.r.t. oscillations amplitudes, the problem defines the following equations system of the flutter eigenvalues:

$$\sum_{s=1}^3 \{\delta_{r,s} \mu_r [\omega^2 - \omega_r^2 (1 + ig_r + ig)] + A_{r,s}^*\} q_r = 0; \quad r = 1, (1), 3. \quad (1)$$

In upper expression denote:  $i$  the imaginary unit,  $\delta_{r,s}$  Kronecker's symbol,  $\omega$  the current angular frequency,  $g$  the current damping decrement,  $\omega_r$  the angular eigen frequency and  $g_r$  the structural damping coefficient of the  $r$ -the mode. The generalized mass of  $r$ -the mode ( $\mu_r$ ) and the generalized aerodynamic force  $A_{r,s}^*$  are defined by expressions:

$$\mu_r = \iint_S \rho_m (h_r^*)^2 dS; \quad A_{r,s}^* = \frac{\rho U_0^2}{2} \iint_S \Delta C p_s^* h_r^* dS.$$

In the preceding expressions the following notations are used:  $S$  lifting surface area,  $\rho$  air density,  $U_0$  velocity of the undisturbed air stream,  $\rho_m$  mass density of the lifting surface per unit  $S$ ,  $h_r^*$  displacement of the  $r$ -the mode shape,  $h_s^*$  displacement of the  $s$ -the mode shape and  $\Delta C p_s^*$  the aerodynamic loading of the  $s$ -the mode. In case of considering the command surface rotation mode, where the backlash and friction effects

are introduced, the problem becomes nonlinear. This implies modification of Equations (1).

Backlash and friction in the command system considered are influencing the rotation mode generalized vibration coefficients of the command surface. If only this mode would be considered, i.e., a material system with one degree of freedom (DOF), its nonlinear behavior can be described by means of the following equation in time domain:

$$I_h \ddot{\delta}(t) + M[\delta(t), \dot{\delta}(t)] = M_{aero}(t). \quad (2)$$

In Equation (2)  $\delta$  is the rotation angle and  $I_h$  is the command surface moment of inertia w.r.t. its hinge axis. The magnitude  $M_{aero}(t)$  is a nonlinear function containing the stiffness and damping characteristics of the nonlinear mode. The forcing, i.e., exciting force is the aerodynamic moment  $M_{aero}(t)$ .

If the problem of eigenvalues is considered only, Equation (2) acquires the form:

$$I_h \ddot{\delta}(t) + M[\delta(t), \dot{\delta}(t)] = 0. \quad (3)$$

By generalization of Equation (3) it is passed to the problem, considered in detail in [6] and [7]:

$$Q(p)\delta + R(p)M(\delta, p\delta) = 0; \quad p = \frac{d(\dots)}{dt}, \quad (4)$$

where  $Q(p)$  and  $R(p)$  are polynomials and  $M[\delta(t), \dot{\delta}(t)]$  is a nonlinear function.

If the solution of Equation (3) is a periodical function, the concept of harmonic linearization based on [7] can be used. Applied to the nonlinear Equation (2), if the exciting moment  $M_{aero}(t)$  is harmonic, i.e.,

$$M_{aero}(t) = \bar{M}_{aero} \sin(\omega t + \varepsilon).$$

The solution  $\delta(t)$  of this equation is a linear combination of the basic

and higher harmonics. As the participation of the higher harmonics is small, it can be assumed that:

$$\delta(t) = \bar{\delta} \sin(\omega t). \quad (5)$$

By applying the concept of harmonics linearization, the nonlinear function  $M[\delta(t), \dot{\delta}(t)]$  is approximated by the first Fourier series harmonic. Higher harmonics of this nonlinear function are neglected. The constant, i.e., zero member in the Fourier expansion, according to [7], has to be equal zero, in order that the solution of Equation (4) has the form defined by Equation (5). In this paper has been proven that in this case the nonlinear function  $M[\delta(t), \dot{\delta}(t)]$  should be centrally symmetrical w.r.t. the origin ( $\delta=0$ ). Based on afore stated development follows:

$$M(\delta, \dot{\delta}) = b_1 \sin(\omega t) + b_2 \cos(\omega t). \quad (6)$$

In the preceding expression:

$$b_1 = \frac{1}{\pi} \int_0^{2\pi} M(\delta, \dot{\delta}) \sin(\omega t) d(\omega t); \quad b_2 = \frac{1}{\pi} \int_0^{2\pi} M(\delta, \dot{\delta}) \cos(\omega t) d(\omega t). \quad (7)$$

Let

$$M(\delta, \dot{\delta}) = K_\delta \delta(t) + B_I \dot{\delta}(t). \quad (8)$$

By substituting Equation (5) into Equation (8) follows:

$$M(\delta, \dot{\delta}) = K_\delta \bar{\delta} \sin(\omega t) + B_I \omega \bar{\delta} \cos(\omega t). \quad (9)$$

By comparing Equation (6) with Equation (9) follows:

$$b_1 = K_\delta \bar{\delta}; \quad b_2 = B_I \omega \bar{\delta}. \quad (10)$$

From theory of oscillations, it is known that  $b_1$  is defining the characteristic of torsion stiffness, and  $b_2$  the damping characteristic of the considered amplitude of nonlinear system  $\bar{\delta}$ .

Aimed at natural phenomenological explanation of the afore stated, the following analysis can be carried out. Let the linear angular modal oscillations of the body be considered (material points system), moment of inertia  $I_h$ . Let this body be connected to a torsion spring of stiffness  $K_\delta$ . Using Equation (3) and Equation (8), the considered modal oscillations described by Equation (2) can be transformed to the following differential equation:

$$I_h \ddot{\delta}(t) + B_I \dot{\delta}(t) + K_\delta \delta(t) = 0. \quad (11)$$

If the system is oscillating harmonically it can be assumed that the solution of Equation (11) is defined by Equation (5). Substituting Equation (5) into Equation (11) yields:

$$[-\omega^2 I_h \sin(\omega t) + B_I \omega \cos(\omega t) + K_\delta \sin(\omega t)] \bar{\delta} = 0. \quad (12)$$

Same oscillation described via generalized vibration coefficients is defined by the following equation:

$$\mu \ddot{\delta}(t) + B_\mu \dot{\delta}(t) + \gamma \delta(t) = 0,$$

$$[-\omega^2 \mu \sin(\omega t) + B_\mu \omega \cos(\omega t) + \gamma \sin(\omega t)] \bar{\delta} = 0. \quad (13)$$

Based on the definition of the generalized vibration coefficients from [8], the following relations are known:

$$\mu = I_h \bar{\delta}^2; \quad B_\mu = g \omega \mu; \quad \gamma = \omega^2 \mu. \quad (14)$$

By comparing Equation (12) and Equation (13), taking into account the relations in Equation (14) it follows:

$$B_\mu = B_I \bar{\delta}^2; \quad \gamma = K_\delta \bar{\delta}^2. \quad (15)$$

By transforming Equation (14) and Equation (15) it is obtained:

$$B_I = \frac{B_\mu}{\bar{\delta}^2} = \frac{g\omega\mu}{\bar{\delta}^2} = g\omega I_h; \quad K_\delta = \frac{\omega^2\mu}{\bar{\delta}^2} = \omega^2 I_h. \quad (16)$$

Based on the upper expressions, Equation (10) acquires the following form:

$$b_1 = K_\delta \bar{\delta} = \omega^2 I_h \bar{\delta}; \quad b_2 = B_I \omega \bar{\delta} = g\omega^2 I_h \bar{\delta}.$$

From upper relations follows:

$$\omega = \sqrt{\frac{b_1}{I_h \bar{\delta}}}; \quad g = \frac{b_2}{b_1}. \quad (17)$$

Let for lifting surface considered the rotation of its command surface  $\delta$  be the third mode. If for this mode its nonlinear hysteresis loop  $M(\delta, \dot{\delta})$  is known, based on Equation (7) and Equation (8) the corresponding coefficients  $b_1$  and  $b_2$  can be calculated. Substituting into Equation (17) it follows:

$$\omega_3^2 = \frac{b_1}{I_h \bar{\delta}}; \quad g_3 = \frac{b_2}{b_1}.$$

Based on the derived nonlinear problem, Equation (1) reduces by harmonic linearization, to an equivalent linear system:

$$\sum_{s=1}^3 \{\delta_{r,s} \mu_r [\omega^2 - \omega_r^2 (1 + ig_r + ig)] + A_{r,s}^*\} q_r = 0; \quad r = 1, 2; \quad (18)$$

$$A_{3,1}^* q_1 + A_{3,2}^* q_2 + \{I_h \bar{\delta}^2 [\omega^2 - \frac{b_1}{I_h \bar{\delta}} (1 + i \frac{b_2}{b_1} + ig)] + A_{3,3}^*\} q_3 = 0.$$

The command surface performs nonlinear oscillations with amplitude  $\bar{\delta}$ . The “linear” system Equation (18) w.r.t amplitude  $\bar{\delta}$ , via the parameters  $\bar{\delta}$ ,  $b_1$  and  $b_2$  is containing implicit the nonlinear function  $M(\delta, \dot{\delta})$ .

By means of this procedure the influences of backlash and function of the command surface are practically included into the calculation of the critical flutter speed of the considered lifting surface.

### 3 Modification of problem statement

By means of the previously developed procedure, the mathematical model was defined, by means of which the considered problem is being solved. The question posed is how this model can be adapted to the usual calculations of the flutter eigenvalues. The well-known package NASTRAN was available, or the software UNAD [9], developed by the author.

The accent of the analysis in this paper lies on the nonlinear structural effects. Hence generality is not lost, if it is assumed that the command surface is oscillating with small amplitude  $\bar{\delta}^*$ . This implies the application of the linear non-stationary aerodynamics for calculation lifting surface aerodynamic loading. Using previous mentioned conclusion, the need for setting the calculations of the lifting surface aerodynamic loading for various oscillation amplitudes  $\bar{\delta}$  of the command surface is surpassed. This is realized by direct scaling of the elements of the generalized aerodynamic forces matrix, which are including the rotation mode of the command surface, proportionally to the ratio of the current and reference amplitudes of the command surface oscillations.

Previous assumption is also confirmed by the fact that the models of linear aerodynamics are reliable for the calculations of the arrangement of aerodynamic loads for small deflections of the command surfaces ( $\bar{\delta}^* < \pm 6^\circ$ ). Fact is that the flutter is an event linked with the maximal aircraft operation speeds. At these flight conditions, for drag minimization, the command surfaces of aircraft are deflected up to a few degrees. This implies the phenomenological correctness of the assumption introduced.

In order to include the procedure developed in previous Chapter 2. into the concept given in [9], it is necessary to perform corresponding modification.

First, the inertial normalization of the mode shapes should be performed. Let  $\mu_r$  be the generalized mass of the  $r$ -the mode, with the mode shape  $h_r^*$ . Then, after inertial normalization, it follows:



$$\mu_r = 1[\text{kgm}^2] = \iint_S \rho_m (h_r^*/\sqrt{\mu_r})^2 dS = \iint_S \rho_m h_r^2 dS; \quad h_r = h_r^*/\sqrt{\mu_r}.$$

That means, with known mode shapes and their generalized masses, calculation of the generalized aerodynamic forces matrices and the flutter eigenvalues in [9] is realized via the set of inertial normalized mode shapes of the unit generalized masses.

For the inertial normalized mode shapes  $\mu_r = \mu_s = 1$  [kg m<sup>2</sup>], the corresponding elements of the generalized aerodynamic forces matrix can be expressed in the following way:

$$A_{r,s} = \frac{A_{r,s}^*}{\sqrt{\mu_r \mu_s}} = \iint_S \frac{h_r^*}{\sqrt{\mu_r}} \frac{\Delta C p_s^*}{\sqrt{\mu_s}} dS = \iint_S h_r \Delta C p_s dS; \quad \Delta C p_s = \frac{\Delta C p_s^*}{\sqrt{\mu_s}}.$$

In the preceding relations,  $\Delta C p_s$  is the distribution function of the non-stationary aircraft aerodynamic load for the inertial normalized boundary conditions of the  $s$ -the mode.

Equations (18) for the three eigenforms of lifting surface oscillations, where the third mode is the rotation of its command surface, before inertial normalization have the form:

$$\{\mu_1[\omega^2 - \omega_1^2(1 + ig_1 + ig)] + A_{1,1}^*\}q_1 + A_{1,2}^*q_2 + A_{1,\bar{\delta}}^*q_{\bar{\delta}} = 0,$$

$$A_{2,1}^*q_1 + \{\mu_2[\omega^2 - \omega_2^2(1 + ig_2 + ig)] + A_{2,2}^*\}q_2 + A_{2,\bar{\delta}}^*q_{\bar{\delta}} = 0, \quad (19)$$

$$A_{\bar{\delta},1}^*q_1 + A_{\bar{\delta},2}^*q_2 + \{I_h(\bar{\delta}^*)^2[\omega^2 - \omega_{\bar{\delta}}^2(1 + i\frac{b_2}{b_1} + ig)] + A_{\bar{\delta},\bar{\delta}}^*\}q_{\bar{\delta}} = 0.$$

In Equation system (19)  $\bar{\delta}^*$  is the non-normalized, i.e., the real modal oscillation amplitude of the command surface. In the third equation of

system Equation (19), the nonlinear angular frequency of the command surface rotation mode is defined by expression (17)  $\omega_{\bar{\delta}}^2 = b_1 / (I_h \bar{\delta}^*)$ .

Let the corresponding inertial normalizations for all three modes have been performed, with the generalized masses  $\sqrt{\mu_1}$ ,  $\sqrt{\mu_2}$  and  $\sqrt{I_h (\bar{\delta}^*)^2}$ . If  $\bar{\delta}^* = \bar{\delta}_{ref}^*$  for an arbitrary material point on the wing surface, it follows:

$$h_1 = \frac{h_1^*}{\sqrt{\mu_1}}; \quad h_2 = \frac{h_2^*}{\sqrt{\mu_2}}; \quad h_{\bar{\delta}_{ref.}} = \bar{\delta}_{ref.} R_h = \frac{\bar{\delta}_{ref.}^* R_h}{\sqrt{\mu_{\bar{\delta}_{ref.}}^*}},$$

where  $R_h$  is the distance of the material point measured from the hinge axis of the command surface. The generalized mass of the rotation mode, based on the third equality of the upper relation, is:

$$\mu_{\bar{\delta}_{ref.}}^* = I_h (\bar{\delta}_{ref.}^*)^2; \quad \mu_{\bar{\delta}_{ref.}} = 1 [kgm^2].$$

After inertial normalization the Equation system (19), acquires the form:

$$[\omega^2 - \omega_1^2(1 + ig_1 + ig) + A_{1,1}]q_1 + A_{1,2}q_2 + A_{1,\bar{\delta}_{ref.}}q_{\bar{\delta}} = 0,$$

$$A_{2,1}q_1 + [\omega^2 - \omega_2^2(1 + ig_2 + ig) + A_{2,2}]q_2 + A_{2,\bar{\delta}_{ref.}}q_{\bar{\delta}} = 0, \quad (20)$$

$$A_{\bar{\delta}_{ref.},1}q_1 + A_{\bar{\delta}_{ref.},2}q_2 + \left\{ \omega^2 - \frac{(b_1)_{ref.}}{I_h \bar{\delta}_{ref.}^*} \left[ 1 + i \frac{(b_2)_{ref.}}{(b_1)_{ref.}} + ig \right] + A_{\bar{\delta}_{ref.},\bar{\delta}_{ref.}} \right\} q_{\bar{\delta}} = 0.$$

In Equation system (20), for  $r=1,2$  and  $s=1,2$ , it follows:

$$A_{r,s} = \iint_S \frac{h_r^*}{\sqrt{\mu_r}} \frac{\Delta C p_s^*}{\sqrt{\mu_s}} dS = \iint_S h_r \Delta C p_s dS;$$

$$A_{r,\bar{\delta}_{ref.}} = \iint_S \frac{h_r^*}{\sqrt{\mu_s}} \frac{\Delta C p_{\bar{\delta}_{ref.}}^*}{\sqrt{I_h (\bar{\delta}_{ref.}^*)^2}} dS = \iint_S h_r \Delta C p_{\bar{\delta}_{ref.}} dS;$$

$$A_{\bar{\delta}_{ref},s} = \iint_S \frac{h_{\bar{\delta}_{ref}}^*}{\sqrt{I_h(\bar{\delta}_{ref}^*)^2}} \frac{\Delta C p_s^*}{\sqrt{\mu_s}} dS = \iint_S h_{\bar{\delta}_{ref}} \Delta C p_s dS; \quad (21)$$

$$A_{\bar{\delta}_{ref},\bar{\delta}_{ref}} = \iint_S \frac{h_{\bar{\delta}_{ref}}^*}{\sqrt{I_h(\bar{\delta}_{ref}^*)^2}} \frac{\Delta C p_{\bar{\delta}_{ref}}^*}{\sqrt{I_h(\bar{\delta}_{ref}^*)^2}} dS = \iint_S h_{\bar{\delta}_{ref}} \Delta C p_{\bar{\delta}_{ref}} dS.$$

The Equation system (20), obtained by inertial normalization of the Equation system (19), is corresponding directly to the form of the initial expressions in the procedure for calculation of the flutter eigenvalues in [9]. For arbitrarily adopted real reference amplitude  $\bar{\delta}^* = \bar{\delta}_{ref}^*$ , which by means of the corresponding normalization reduces to  $\bar{\delta}_{ref} = 1/\sqrt{I_h}$ , the procedure from [9] can be applied and directly calculated the critical flutter speeds of the lifting surface considered. Based on the presented, it should be kept in mind, that due to the inertial normalization performed  $\mu_{\bar{\delta}_{ref}} = 1$  [kg m<sup>2</sup>]. For some other oscillations amplitude the rotation of the command surface  $\bar{\delta}^* = \bar{\delta}_{nonlin}^*$  the Equation system (19) is of the form:

$$\{\mu_1[\omega^2 - \omega_1^2(1 + ig_1 + ig)] + A_{1,1}^*\}q_1 + A_{1,2}^*q_2 + A_{1,\bar{\delta}_{nonlin}}^*q_{\bar{\delta}} = 0,$$

$$A_{2,1}^*q_1 + \{\mu_2[\omega^2 - \omega_2^2(1 + ig_2 + ig)] + A_{2,2}^*\}q_2 + A_{2,\bar{\delta}_{nonlin}}^*q_{\bar{\delta}} = 0, \quad (22)$$

$$A_{\bar{\delta}_{nonlin},1}^*q_1 + A_{\bar{\delta}_{nonlin},2}^*q_2 + \{I_h(\bar{\delta}_{nonlin}^*)^2[\omega^2 - \frac{(b_1)_{nonlin}}{I_h\bar{\delta}_{nonlin}^*}(1 + i\frac{(b_2)_{nonlin}}{(b_1)_{nonlin}} + ig)] + A_{\bar{\delta}_{nonlin},\bar{\delta}_{nonlin}}^*\}q_{\bar{\delta}} = 0.$$

In Equations (22)  $\bar{\delta}_{nonlin}^*$  is nonlinear and non-normalized, i.e., it is the real modal oscillations amplitude of the command surface rotation.

The generalized mass of the command surface rotation mode in this case is:

$$\mu_{\bar{\delta}_{nonlin.}}^* = I_h (\bar{\delta}_{nonlin.}^*)^2.$$

For the dynamic model of the lifting surface considered, by means of Equations (21) the elements of the generalized aerodynamic forces matrix for the referent amplitude of the command surface rotation  $\bar{\delta} = \bar{\delta}_{ref.}$  are calculated. For this calculation the procedure from [9] can be applied directly, i.e., the corresponding method of non-stationary linear aerodynamics. Let the rotation mode of the command surface with the new real modal amplitude  $\bar{\delta}^* = \bar{\delta}_{nonlin.}^*$  was normalized by the corresponding new generalized mass, i.e. by the factor  $\sqrt{\mu_{\bar{\delta}_{nonlin.}}^*}$ . Transferring the distribution of the non-stationary aerodynamic loads of the command surface rotation mode from the old  $\bar{\delta}_{ref.}$  to the new amplitude  $\bar{\delta}_{nonlin.}$ , the following relations can be obtained:

$$\begin{aligned} \Delta C p_{\bar{\delta}_{nonlin.}}^* &= \frac{\partial C p_{\bar{\delta}}^*}{\partial \bar{\delta}^*} \bar{\delta}_{nonlin.}^* \Leftrightarrow \frac{\Delta C p_{\bar{\delta}_{nonlin.}}^*}{\sqrt{\mu_{\bar{\delta}_{nonlin.}}^*}} = \frac{\partial C p_{\bar{\delta}}^*}{\partial \bar{\delta}^*} \frac{1}{\sqrt{I_h}}, \\ \Delta C p_{\bar{\delta}_{ref.}}^* &= \frac{\partial C p_{\bar{\delta}}^*}{\partial \bar{\delta}^*} \bar{\delta}_{ref.}^* \Leftrightarrow \frac{\Delta C p_{\bar{\delta}_{ref.}}^*}{\sqrt{\mu_{\bar{\delta}_{ref.}}^*}} = \frac{\partial C p_{\bar{\delta}}^*}{\partial \bar{\delta}^*} \frac{1}{\sqrt{I_h}}. \end{aligned}$$

From upper relations it follows:

$$\frac{\Delta C p_{\bar{\delta}_{nonlin.}}^*}{\sqrt{\mu_{\bar{\delta}_{nonlin.}}^*}} = \frac{\Delta C p_{\bar{\delta}_{ref.}}^*}{\sqrt{\mu_{\bar{\delta}_{ref.}}^*}} \Leftrightarrow \Delta C p_{\bar{\delta}_{nonlin.}} = \Delta C p_{\bar{\delta}_{ref.}}. \quad (23)$$

The first two equations of Equation system (22), by using Equations (21) and Equations (23), can be modified into the following form:

$$[\omega^2 - \omega_1^2(1 + ig_1 + ig) + A_{1,1}]q_1 + A_{1,2}q_2 + A_{1,\bar{\delta}_{ref.}}q_{\bar{\delta}} = 0, \quad (24)$$

$$A_{2,1}q_1 + [\omega^2 - \omega_2^2(1 + ig_2 + ig) + A_{2,2}]q_2 + A_{2,\bar{\delta}_{ref.}}q_{\bar{\delta}} = 0.$$

By inertial normalization of the command surface rotation mode with  $\sqrt{\mu_{\bar{\delta}_{nonlin.}}^*}$ , i.e.:

$$h_{\bar{\delta}_{nonlin.}} = \frac{h_{\bar{\delta}_{nonlin.}}^*}{\sqrt{\mu_{\bar{\delta}_{nonlin.}}^*}} = \frac{R_h}{\sqrt{I_h}} = \frac{h_{\bar{\delta}_{ref.}}^*}{\sqrt{\mu_{\bar{\delta}_{ref.}}^*}} = h_{\bar{\delta}_{ref.}}.$$

It is obtained that the modal surface of this mode of normalized amplitude  $\bar{\delta}_{nonlin.}$  is being transformed in the same way as in Equation (23). Due to that, the third equation of Equation system (22) can be modified in the following manner:

$$A_{\bar{\delta}_{ref.},1}q_1 + A_{\bar{\delta}_{ref.},2}q_2 + \{\omega^2 - \omega_{nonlin.}^2[1 + ig_{nonlin.} + ig] + A_{\bar{\delta}_{ref.},\bar{\delta}_{ref.}}\}q_{\bar{\delta}} = 0, \quad (25)$$

where

$$\omega_{nonlin.}^2 = \frac{(b_1)_{nonlin.}}{I_h \bar{\delta}_{nonlin.}}; \quad g_{nonlin.} = \frac{(b_2)_{nonlin.}}{(b_1)_{nonlin.}}. \quad (26)$$

Based on Equation (24), Equation (25) and Equation (26), the Equation system (22) reduces to the system:

$$[\omega^2 - \omega_1^2(1 + ig_1 + ig) + A_{1,1}]q_1 + A_{1,2}q_2 + A_{1,\bar{\delta}_{ref.}}q_{\bar{\delta}} = 0,$$

$$A_{2,1}q_1 + [\omega^2 - \omega_2^2(1 + ig_2 + ig) + A_{2,2}]q_2 + A_{2,\bar{\delta}_{ref.}}q_{\bar{\delta}} = 0, \quad (27)$$

$$A_{\bar{\delta}_{ref.},1}q_1 + A_{\bar{\delta}_{ref.},2}q_2 + \{\omega^2 - \omega_{nonlin.}^2[1 + ig_{nonlin.} + ig] + A_{\bar{\delta}_{ref.},\bar{\delta}_{ref.}}\}q_{\bar{\delta}} = 0.$$

By means of the developed procedure, it is proven that the theoretical approach, defined by Equation system (18), can be modified into the equivalent Equation system (27). It is shown that the non-stationary aerodynamic forces can be calculated for one amplitude of the command surface rotation. Then, by simple scaling, using inertial normalization, new values of these forces for some other oscillations amplitude can be obtained. By means of this the tedious repeating of the lifting surface generalized aerodynamic forces matrices calculations for various oscillations amplitudes of its command surface, has been avoided.

The enclosed deriving was given for the lifting surface with three oscillation modes, for the sake of interpretation simplicity. As it can be seen from the deriving, there are no limitations whatsoever that the same procedure be generalized and applied to the aircraft dynamic model, with the real existing modes. The choice of the mode containing the structural nonlinearities is also arbitrary. Limitation in the analysis is that only one mode with nonlinear characteristics can be considered.

## 4 Fourier expansion of centrally symmetrical hysteresis loop $M[\delta(t), \dot{\delta}(t)]$

### 4.1 Zero member of Fourier expansion of $M[\delta(t), \dot{\delta}(t)]$

The nonlinear function  $M[\delta, \dot{\delta}]$  implicitly contains the nonlinear structural characteristics of the mode analyzed. Depending on the development phase and availability of the aircraft, this nonlinear function can be known from different sources: from measurement on the real object, from tests on the test bench, or it can be assumed and parametrically varied based on previous experience.

Experimental determining of the nonlinear function  $M[\delta, \dot{\delta}]$  can be realized by means of quasi-stationary measurements of the hysteresis loop of one full cycle of this structural event. For instance, let the command loop be considered, together with the corresponding command surface. The quasi-stationary measurement is realized by means of very slow excitation of this command surface rotation mode. Then the inertial moment in Equation (2) becomes negligibly small, and it follows:

$$M[\delta(t), \dot{\delta}(t)] = M_{aero}(t).$$

If the excitation moment  $M_{aero}(t)$  is recorded as function of the quasi-stationary harmonic oscillation  $\delta(t)$  in domain of its full period, the function  $M[\delta, \dot{\delta}]$ , i.e., the structural hysteresis loop of the object considered, is obtained on the plotter as in Figure 1(a).

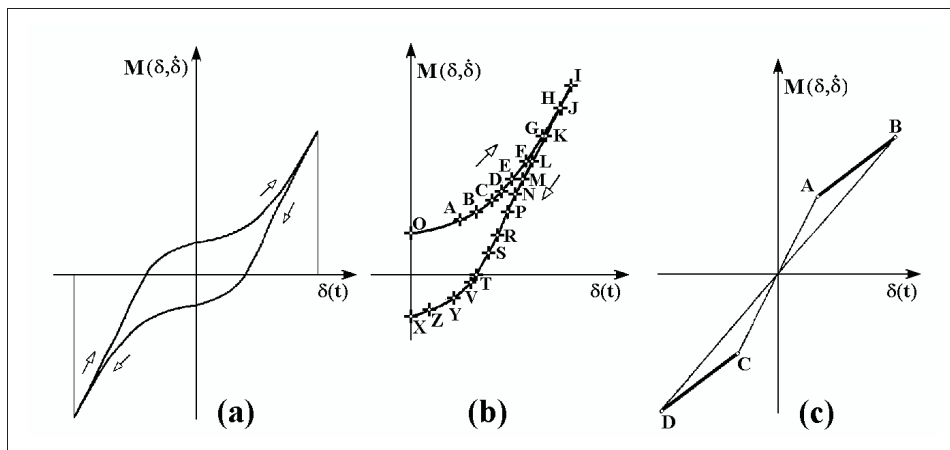


Figure 1: Hysteresis loops

Based on [7], in order to be able to apply the procedure displayed in this paper, the nonlinear function  $M[\delta, \dot{\delta}]$  expanded into a Fourier series should have a constant (zero) expansion member equal zero, i.e.,  $b_0=0$ . At this point of paper, it will be proven that all the centrally symmetrical (w.r.t. point  $\delta(t) = 0$ ) points of the nonlinear function  $M[\delta, \dot{\delta}]$  are satisfying the stated condition. The central symmetry w.r.t. the coordinate system origin is synonymous odd function. For any odd function per definition it holds  $f(\delta) = -f(-\delta)$ .

The arbitrary, continuous, centrally, symmetrical function  $M[\delta, \dot{\delta}]$ , is presented in Figure 1(a) and can be substituted in the nonnegative domain of the first half-period of the variable  $\delta(t)$ , with desired accuracy, by a set of polygonal rectilinear lines (segments), as in Figure 1(b).

The original of the polygonal line form Figure 1(b) and its centrally symmetrical image, for the symmetry center  $[\delta(t)=0 ; M=0]$ , are presenting the approximation of function  $M[\delta(t), \dot{\delta}(t)]$  in the domain of the first

full period of the variable  $\delta(t) = 0$ . For each next period of the variable  $\delta(t)$ , the same approximation of the nonlinear function holds, because the variable  $\delta(t)$  based on Equation (5) is a pure harmonic function. By the stated, it was achieved that for any oscillations period of the variable in Equation (5), the nonlinear function  $M[\delta, \dot{\delta}]$  is approximated with the desired accuracy by arranged pairs of centrally symmetrical segments.

The condition from [7] "if  $M[\delta, \dot{\delta}]$  is an odd function, then the constant member  $b_0$  in the Fourier's development of this function equals zero" can be proven based on the previously displayed approximation of this function by a set of arranged pairs of centrally symmetrical segments. Let the arranged pair of centrally symmetrical segments be given as in Figure 1(c).

The points designated in Figure 1(c) were determined by the following coordinates in the amplitude and time domains:

$$\begin{aligned} A[\delta_A, M_A] &\equiv A[\omega t_A, M_A]; & C[-\delta_A, -M_A] &\equiv C[\omega t_A + \pi, -M_A], \\ B[\delta_B, M_B] &\equiv B[\omega t_B, M_B]; & D[-\delta_B, -M_B] &\equiv D[\omega t_B + \pi, -M_B]. \end{aligned}$$

Let the segments  $\overline{AB}$  and  $\overline{CD}$  be the  $j$ -the arranged pair of the centrally symmetrical segments. Then, in the amplitude domain is:

$$\overline{AB} : M = k(\delta - \delta_A) + M_A; \quad \overline{CD} : M = k(\delta - \delta_A) - M_A,$$

$$\delta = \bar{\delta} \sin(\omega t); \quad k = \frac{M_B - M_A}{\delta_B - \delta_A}.$$

Contribution of the  $j$ -the pair of centrally symmetrical segments to member  $b_0$  of the Fourier expansion of function  $M[\delta, \dot{\delta}]$  is:

$$\begin{aligned} b_0^j &= (b_0^j)_p + (b_0^j)_m, \\ (b_0^j)_p &= \frac{1}{\pi} \int_{\omega t_A}^{\omega t_B} \{k[\bar{\delta} \sin(\omega t) - \delta_A] + M_A\} d(\omega t), \end{aligned} \quad (28)$$



$$(b_0^j)_m = \frac{1}{\pi} \int_{\omega t_A + \pi}^{\omega t_B + \pi} \{k[\bar{\delta} \sin(\omega t) + \delta_A] - M_A\} d(\omega t).$$

By integration of the upper expressions, after a few steps, it shows that it is:

$$(b_0^j)_p = \frac{1}{\pi} \{k\bar{\delta}[\cos(\omega_B t) - \cos(\omega_A t)] - (k\delta_A - M_A)\omega(t_B - t_A)\},$$

$$(b_0^j)_m = \frac{1}{\pi} \{-k\bar{\delta}[\cos(\omega_B t) - \cos(\omega_A t)] + (k\delta_A - M_A)\omega(t_B - t_A)\}.$$

By substituting the upper relations into (28), it follows:

$$b_0^j = (b_0^j)_p + (b_0^j)_m = 0.$$

If  $n$  is the number of the arranged pairs, by which the function  $M[\delta, \dot{\delta}]$  is approximated, then it follows:

$$b_0 = \sum_{j=1}^n b_0^j = 0. \tag{29}$$

By that it was proven that for an arbitrary, continuous, centrally symmetrical, nonlinear function  $M[\delta, \dot{\delta}]$  the condition described by Equation (29) is fulfilled, i.e., that the procedure proposed in this paper holds. It should be noted that the function  $M[\delta, \dot{\delta}]$  has not to be continuous from the first and higher derivatives in the amplitude domain.

#### 4.2 First member of Fourier expansion of $M[\delta(t), \dot{\delta}(t)]$

By applying the concept outlined in the Chapter 4.1., for the  $j$ -the pair of centrally symmetrical segments, defined in Figure 1(c),  $b_1^j$  and  $b_2^j$  can be calculated. According to that figure, it holds that:

$$A[\delta_A, M_A] \equiv A[\delta_{1,j}, M_{1,j}] \Leftrightarrow A[\omega t_{1,j}, M_{1,j}] \equiv A[\omega t_A, M_A],$$

$$B[\delta_B, M_B] \equiv B[\delta_{2,j}, M_{2,j}] \Leftrightarrow B[\omega t, M] \equiv B[\omega t_B, M_B],$$

$$C[-\delta_A, -M_A] \equiv C[-\delta_{1,j}, -M_{1,j}] \Leftrightarrow C[\omega t_{1,j} + \pi, -M_{1,j}] \equiv C[\omega t_A + \pi, -M_A],$$

$$D[-\delta_B, -M_B] \equiv D[-\delta_{2,j}, -M_{2,j}] \Leftrightarrow D[\omega t + \pi, -M] \equiv D[\omega t_B + \pi, -M_B],$$

$$\overline{AB} : M = k_j(\delta - \delta_{1,j}) + M_{1,j}; \quad \overline{CD} : M = k_j(\delta + \delta_{1,j}) - M_{1,j},$$

$$\delta = \bar{\delta} \sin(\omega t); \quad k_j = \frac{M_{2,j} - M_{1,j}}{\delta_{2,j} - \delta_{1,j}},$$

$$\psi_{1,j} = \omega t_{1,j} = \arcsin\left(\frac{\delta_{1,j}}{\bar{\delta}}\right); \quad \psi_{1,j} \in \{0, \pi\},$$

$$\psi_{2,j} = \omega t_{2,j} = \arcsin\left(\frac{\delta_{2,j}}{\bar{\delta}}\right); \quad \psi_{2,j} \in \{0, \pi\}.$$

Then it is:

$$\begin{aligned} b_1^j &= \frac{1}{\pi} \int_{\omega t_A}^{\omega t_B} \{k[\bar{\delta} \sin(\omega t) - \delta_A] + M_A\} \sin(\omega t) d(\omega t) + \\ &\quad \frac{1}{\pi} \int_{\omega t_A + \pi}^{\omega t_B + \pi} \{k[\bar{\delta} \sin(\omega t) + \delta_A] - M_A\} \sin(\omega t) d(\omega t), \\ b_2^j &= \frac{1}{\pi} \int_{\omega t_A}^{\omega t_B} \{k[\bar{\delta} \sin(\omega t) - \delta_A] + M_A\} \cos(\omega t) d(\omega t) + \\ &\quad \frac{1}{\pi} \int_{\omega t_A + \pi}^{\omega t_B + \pi} \{k[\bar{\delta} \sin(\omega t) + \delta_A] - M_A\} \cos(\omega t) d(\omega t). \end{aligned}$$

By integration of the upper expressions, it follows:

$$b_1^j = \frac{1}{\pi} \left\{ k_j \bar{\delta} (\psi_{2,j} - \psi_{1,j}) + \frac{k_j \bar{\delta}}{2} [\sin(2\psi_{1,j}) - \sin(2\psi_{2,j})] + 2(k_j \delta_{1,j} - M_{1,j})(\cos\psi_{2,j} - \cos\psi_{1,j}) \right\}, \tag{30}$$

$$b_2^j = \frac{1}{\pi} \left\{ \frac{k_j \bar{\delta}}{2} [\cos(2\psi_{1,j}) - \cos(2\psi_{2,j})] - 2(k_j \delta_{1,j} - M_{1,j})(\sin\psi_{2,j} - \sin\psi_{1,j}) \right\}.$$

By summing Equation (30) over all n pairs of the centrally symmetrical segments, the first members of the development into the Fourier series the nonlinear function  $M[\delta, \dot{\delta}]$  are obtained:

$$b_1 = \sum_{j=1}^n b_1^j; \quad b_2 = \sum_{j=1}^n b_2^j. \tag{31}$$

The presented procedure is adapted for simple programming on computer.

### 4.3 Examples

The procedure from Chapters 4.1. and 4.2. are used in two examples to demonstrate the development of the nonlinear centrally symmetrical functions  $M[\delta, \dot{\delta}]$  into the corresponding Fourier series. The examples are taken from [3] and [5] and are using for the verification of the procedure proposed.

The first example, illustrated in Figure 2(a), is representing the simplest case of the  $M[\delta, \dot{\delta}]$  function, i.e., the hysteresis loop of the centrally symmetrical backlash without friction.

Let, in the amplitude and time domains, the following points are defined:

$$O[0, 0] \equiv O[0, 0]; \quad A[\delta_A, 0] \equiv A[\omega t_A, 0]; \quad B[\bar{\delta}, M_B] \equiv B[\pi/2, M_B],$$

$$C[\delta_A, 0] \equiv C[\pi - \omega t_A, 0]; \quad D[0, 0] \equiv D[\pi, 0].$$

The corresponding straight lines in analytical form are:

$$\begin{aligned} \overline{OA} \equiv \overline{CD} : M = 0; \quad \overline{AB} \equiv \overline{BC} : M = k(\delta - \delta_A); \\ \delta = \bar{\delta} \sin(\omega t); \quad k = \frac{M_B}{\bar{\delta} - \delta_A}. \end{aligned}$$

By applying Equation (30) and Equation (31), it follows:

$$b_1 = \frac{1}{\pi} \{k\bar{\delta}[\pi - 2\psi_A + \sin(2\psi_A)] - 4k\delta_A \cos\psi_A\}; \quad b_2 = 0, \quad (32)$$

$$\psi_A = \omega t_A = \arcsin\left(\frac{\delta_A}{\bar{\delta}}\right); \quad \bar{\delta} = \delta_B.$$

The obtained relations in Equations (32) are the same as the results in [3] and [4]. As  $M[\delta, \dot{\delta}]$  is describing a centrally symmetrical frictionless case, it must be  $b_0=0$  and  $b_2=0$ .

Second example was taken from [5]. On Figure 2(b) the function  $M[\delta, \dot{\delta}]$  is given.

Let according to Figure 2(b) the following points are defined:

$$\begin{aligned} O[0, M_A] \equiv O[0, M_A]; \quad A[\delta_A, M_A] \equiv A[\omega t_A, M_A]; \\ B[\bar{\delta}, M_B] \equiv B[\pi/2, M_B], \end{aligned}$$

$$\begin{aligned} C[\delta_C, M_C] \equiv C[\omega t_C, M_C]; \quad D[\delta_D, -M_A] \equiv D[\omega t_D, -M_A]; \\ E[0, -M_A] \equiv E[\pi, -M_A]. \end{aligned}$$

Based on these points the following functions can be defined:

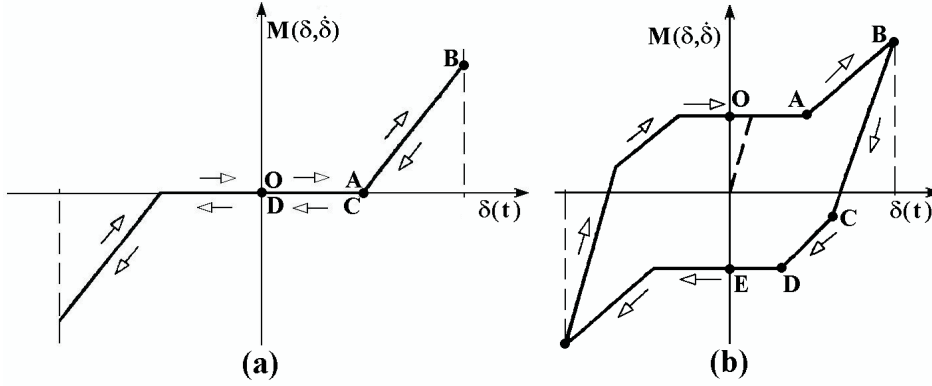


Figure 2: Hysteresis loops for examples

$$\overline{OA}: M = M_A$$

$$\overline{AB}: M = k_1(\delta - \delta_A) + M_A$$

$$\overline{BC}: M = k_2(\delta - \bar{\delta}) + M_B = k_2(\delta - \bar{\delta}) + k_1(\bar{\delta} - \delta_A) + M_A$$

$$\overline{CD}: M = k_1(\delta - \delta_C) + M_C = k_1(\delta - \delta_C + \bar{\delta} - \delta_A) + k_2(\delta_C - \bar{\delta}) + M_A$$

$$\overline{DE}: M = -M_A$$

$$\delta = \bar{\delta} \sin(\omega t); \quad k_1 = \frac{M_B - M_A}{\bar{\delta} - \delta_A}; \quad k_2 = \frac{M_B - M_C}{\bar{\delta} - \delta_C}.$$

By applying Equation (30) and Equation (31), for  $(\bar{\delta} > \delta_A)$  and  $(\delta_C \geq \delta_D)$ , it follows:

$$b_1 = \frac{1}{\pi} \left\{ \frac{k_1 \bar{\delta}}{2} [\pi - 2\omega(t_A - t_D + t_C) + \sin(2\omega t_A) - \sin(2\omega t_D) + \sin(2\omega t_C)] + \frac{k_2 \bar{\delta}}{2} [2\omega t_C - \pi - \sin(2\omega t_C)] - 2k_1 \delta_A \cos(\omega t_A) + 2(k_2 - k_1) \delta_C \cos(\omega t_C) + 2k_1 \delta_D \cos(\omega t_D) \right\}, \quad (33)$$

$$b_2 = \frac{1}{\pi} \left\{ \frac{k_1 \bar{\delta}}{2} [1 + \cos(2\omega t_A) - \cos(2\omega t_D) + \cos(2\omega t_C)] - \frac{k_2 \bar{\delta}}{2} [1 + \cos(2\omega t_C)] - \right.$$

$$2k_1\delta_A \sin(\omega t_A) + 4M_A + 2(k_2 - k_1)\delta_C \sin(\omega t_C) + 2k_1\delta_D [1 - \sin(\omega t_D)] \}, \quad (34)$$

$$\omega t_A = \arcsin \left( \frac{\delta_A}{\bar{\delta}} \right); \quad \omega t_B = \frac{\pi}{2};$$

$$\omega t_C = \arcsin \left( \frac{\delta_C}{\bar{\delta}} \right); \quad \omega t_D = \arcsin \left( \frac{\delta_D}{\bar{\delta}} \right)$$

By means of elementary transformations, the coincidence of relations obtained in Equation (33) and Equation (34) with the results in [5] can be proven.

Based on the examples enclosed the validity of the proposed procedure is verified. This procedure is generally applicable for arbitrary, continuous, centrally symmetrical functions  $M[\delta, \bar{\delta}]$ . Examples of centrally asymmetrical hysteresis loops will be analyzed in next papers.

## 5 Numerical example

Described problem is well known in flutter analysis, but on the other hand it is difficult to find appropriate data in aeroelastic literature for verification of in the paper proposed procedure. That's the why AGARD has announced report [1].

Results from testing of nonlinear flutter of half span wing model with aileron in wind tunnel are given in [1]. Mode of the first rotation of aileron contains structural nonlinearity respective to amplitude of aileron's rotation. Based on [1] it was impossible to replicate all necessary input data, but obtained calculation results from proposed method given in this paper are very similar to the results in [1].

Dimensions of half span wing aileron model are given on Figure 3.

Ground vibration test was performed on model and obtained measured modal characteristics  $\mu_j$ (generalized mass),  $f_j$ (eigen frequency) and  $g_j$ (modal structural damping) are given on Figure 4 with normal mode shapes of significant three modes. Only the first mode was nonlinear.

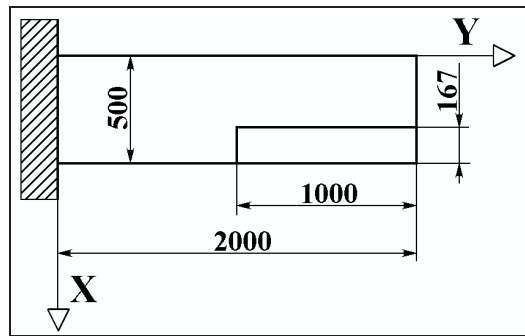


Figure 3: Dimensions of semi wing model

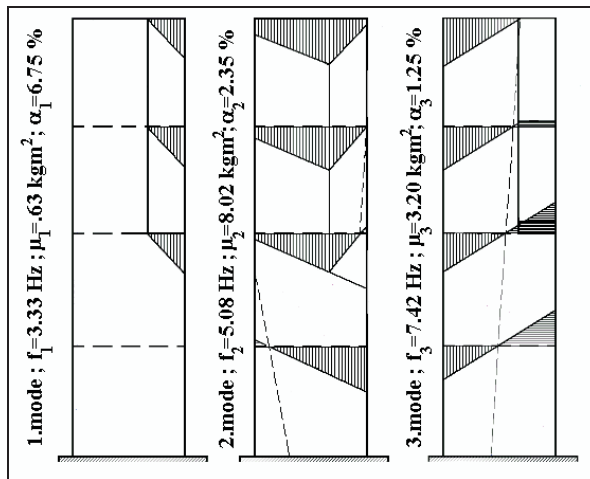


Figure 4: Normal mode characteristics

Unsteady aerodynamic forces of model are calculated by Doublet-lattice method (DLM) using UNAD software, developed by author. Semi wing is divided on 8 equal strips. Each strip is divided on 5 panels.

Three equal panes are on semi wing's strip in front of aileron and two equal panels on aileron part of strip.

Nonlinear function  $M[\delta, \delta]$  of aileron rotation is given on Figure 5. Aileron's rotation backlash is  $\pm 3$  mm ( $\pm 1.02934^\circ$ ) and hinge moment is  $I_h = 0.0176$  kg m<sup>2</sup>. Based on proposed procedure software NELZAZ was developed. Using this program, coefficients  $b_1$  and  $b_2$  of Fourier expansion of any central symmetric function  $M[\delta, \delta]$  can be obtained. In

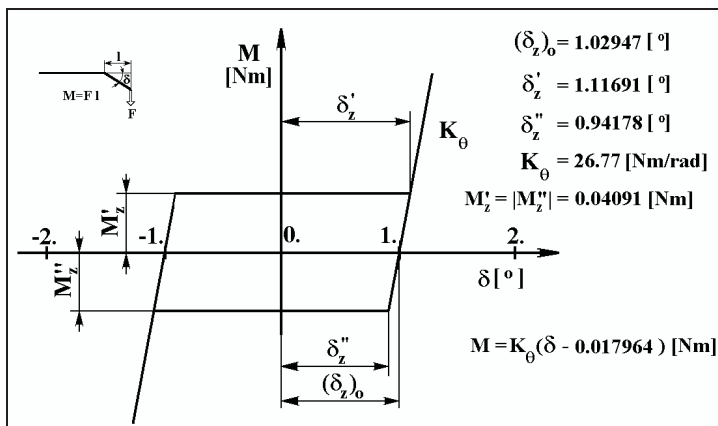


Figure 5: Aileron hysteresis loop

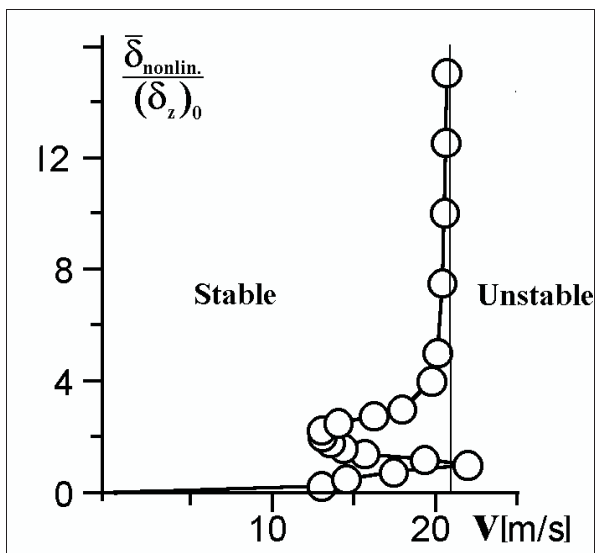


Figure 6: Flutter boundary of nonlinear model

closed loop, program is executing for large number of amplitudes of selected structural nonlinear mode. For each nonlinear selected amplitude  $\delta_{nonlin.}$  appropriate nonlinear eigen frequency  $f(\delta_{nonlin.})$  and structural damping  $g(\delta_{nonlin.})$  are calculated.

Critical flutter speeds of semi wing model were calculated for large



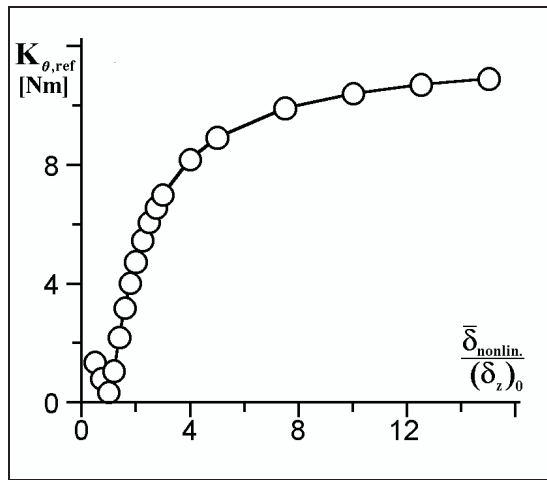


Figure 7: Equivalent aileron hinge stiffness vs.

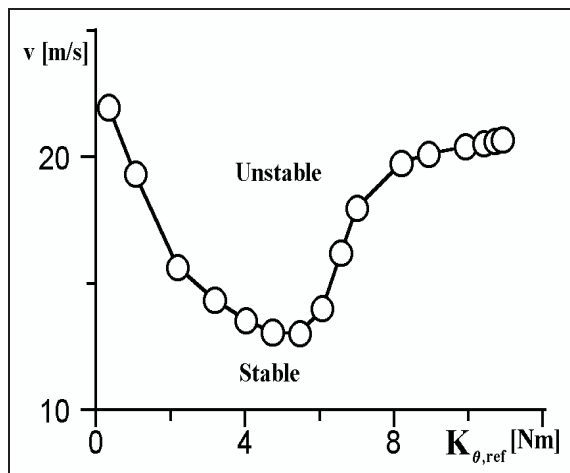


Figure 8: Critical flutter speed vs. equivalent nonlinear amplitude ratio aileron hinge stiffness

number of amplitudes for selected nonlinear mode and other two linear modes using software FLUTTER. Obtained results are given on Figure 6.

The equivalent aileron hinge stiffness of the nonlinear model as a function of the amplitude ratio  $\bar{\delta}_{nonlin.}/(\delta_z)_0$  is plotted on Figure 7. Flut-

ter boundary of the model vs. the equivalent aileron hinge stiffness is given on Figure 8.

Calculated results given in this paper agree very well respective to results in [1].

## 6 Conclusion

The form of the nonlinearity encountered on actual aircraft structures is in general not very well known and is an area worthy of further research. In absence of more definite information, two relatively simple characteristic types of structural nonlinearities are studied: backlash and centrally symmetric hysteresis loop.

The theoretical approach based on harmonic linearization is developed in details. One nonlinear mode is incorporated into classic flutter equations. For any amplitude of oscillation of the nonlinear mode classic calculation of critical flutter speed can be done. Calculation of nonlinear characteristics of selected nonlinear mode is automated by the developed software NELZAZ.

The presented procedure is tested on example [1]. Good coincidence to experimentally obtained results is achieved. The results of this investigation (Figure 6) show that nonlinear effects can influence the flutter speed significantly.

The results of presented investigation can be used in engineering practice for incorporation in flutter analysis a great number of nonlinear cases such as: nonlinear rotation of classic command system surface and it's tab, nonlinear characteristics of servo-actuator, nonlinear modal motion of external store, etc. Per example, nonlinear analysis of external store oscillation influences directly to the value of tightening torque between the store and it's pod.

Using the developed procedure critical analysis of different recommendations given in various airworthiness regulations can be done for distinguished hysteresis loops of classical command systems of aircrafts. Direct application of the outlined results is that any nonlinear command system (with one nonlinear mode) flutter problem can be analyzed.

The further investigations will be focused to the incorporation into flutter analysis effects of asymmetric hysteresis loops, nonlinear charac-

teristics of servo-actuators due to preload and nonlinear modal motions of external store. The main problem in these investigations will be the lack of experimental results.

## References

- [1] E.Breitbach: Effects of Structural Non-Linearities on Aircraft Vibration and Flutter, AGARD No. 665, 1977.
- [2] D.S.Woolston, H.L.Runyan, R.E.Andrew : An Investigation of Certain Types of Structural Nonlinearities on Wing and Control Surface Flutter, Journal of Aerospace Sciences, January 1957.
- [3] S.F.Shen : An Approximate Analysis of Nonlinear Flutter Problems, Journal of Aerospace Sciences, January 1957.
- [4] M.Vukobratovich M.: Contribution to the nonlinear flutter analysis, (in Serbian), Report VS-203, VTI-Zharkovo, 1962.
- [5] O.Sensburg, B.Schoen: Vibration and Flutter Investigations of Aircraft with Special Nonlinear Structural Properties, ZFW, Heft 6, February 1978.
- [6] N.Kryloff, N.Bogoliuboff: Introduction to Nonlinear Mechanic, Princeton University Press, Princeton, 1947.
- [7] E.P.Popov: On the Use of the Harmonic Linearization Method in the Automatic Control Theory, NASA TM 1406, 1957.
- [8] C.A.Beatrix: Experimental Determination of the Vibratory Characteristics of Structures, ONERA Note No. 212 E, Paris, 1974.
- [9] N.Marichich: Contributions to Subsonic Aircraft Flutter Calculation, (in Serbian), Doctoral dissertation, Faculty of Mechanical Engineering, Belgrade, 1989.
- [10] B.Emslie, A.Goldman: The effects of backlash and trailing-edge strips on the flutter speed of a two-dimensional model of a tailplane with tab, Aeronautical Journal, November 1982.

- [11] D.L.Birdsall: A Non-Linear Solution to a Tab-Aileron Flutter Problem, The Aeronautical Journal of the Royal Aeronautical Society, June 1970.
- [12] R.Freymann: A method of designing active flutter suppression systems, ZFW, Heft 7, 1983.
- [13] R.M.Laurenson, R.M.Trn: Flutter Analysis of Missile Control Surfaces Containing Structural Nonlinearities, AIAA Journal, No. 10, October 1980.

Submitted on November 2004, revised on January 2005.

## **Uticaj zazora i trenja u komandnim kolima na flater aviona**

UDK 534.13

Praksa je pokazala da struktura aviona sadrži različite tipove strukturalnih nelinearnosti. Paznja u ovom radu je posvećena zazoru i trenju, koji su obuhvaćeni histreznom petljom komandnog kola klasičnog aviona i njihovom uticaju na flater aviona. Na bazi AGARD-a No.665 u radu je izvršena nelinearna analiza flatera aviona u funkciji zazora i trenja njegovog komandnog kola. Nestacionarne aerodinamičke sile aviona određene su poznatom metodom resetke dubleta. Ulazni podaci, koji definišu karakteristike strukture aviona, uzeti su iz AGARD-a No.665. Sopstvene vrednosti flatera određene su k-metodom. Model jednačina flatera nelinearne strukture aviona razvijen je na bazi metode harmonijske linearizacije. Cilj rada se sastoji u dobijanju upotrebljivog i relativno pouzdanog postupka za kritičku analizu različitih preporuka, datih u brojnim svetskim vazduhoplovnim propisima, s aspekta dozvoljenih vrednosti nelinearnih karakteristika histereznih petlji klasičnih komandnih kola aviona.