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Optimal Synthesis of a Four-Bar Linkage by Method of Controlled Deviation

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Abstract

This paper considers optimal synthesis of a four-bar linkage by method of controlled deviations. The advantage of this approximate method is that it allows control of motion of the coupler in the four-bar linkage so that the path of the coupler is in the prescribed environment around the given path on the segment observed. The Hooke-Jeeves's optimisation algorithm has been used in the optimisation process. Calculation expressions are not used as the method of direct searching, i.e. individual comparison of the calculated value of the objective function is made in each iteration and the moving is done in the direction of decreasing the value of the objective function. This algorithm does not depend on the initial selection of the projected variables. All this is illustrated on an example of synthesis of a four-bar linkage whose coupler point traces a straight line, i.e. passes through sixteen prescribed points lying on one straight line.

Key words: optimal, synthesis, four-bar linkage, Hooke-Jeeves's optimisation algorithm, controlled deviations, objective function.

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1 Introduction

A very important task in a design process is how to make a mechanism which will satisfy desired characteristics of motion of a member, i.e. a mechanism in which one part will surely perform desired (given) motion. There are three common requirements in kinematic synthesis of mechanisms: path generation, function generation and motion generation. In dimensional synthesis there are two approaches: synthesis of precision points and approximate or optimal synthesis.

Precision point synthesis implies that the point at the end member of a mechanism (it is most frequently a coupler in a four-bar linkage) passes through a certain number of desired (exact) points, but without the possibility of controlling a structural error on a path out of those points. Precision point synthesis is restricted by the number of points which must be equal to the number of independent parameters defined by the mechanism. The maximum number of points for a four-bar linkage is nine. If the number of equations generated by the number of exact points is smaller than the number of projected variables, then there is a selection of free variables, so that the problem of synthesis does not have a single-valued solution. When the number of precisison points increases, the problem of precision point synthesis becomes very nonlinear and extremely difficult for solving, and the mechanism obtained by this type of synthesis is in most cases useless: because dimensions of the mechanism members are in disproportion, or the obtained solutions are in the form of complex numbers so there is no mechanism. The maximum number of precision points on the path of the coupler in a four-bar linkage is five in coordinated motion, and nine in uncoordinated motion.

It is seen that the synthesis of mechanisms by "methods of precision points" is restricted by the number of given points, and the increase of precision points to more than nine is practically impossible.

Optimal synthesis of mechanisms is, in fact, a repeated analysis for a random determined mechanism and finding of the best possible one so that it could meet technological requirements, and it is most often used in dimensional synthesis, which implies determination of elements of the given mechanism (lengths, angles, coordinates) necessary for creation of the mechanism in the direction of desired motion. The optimisation algorithm contains the objective function defined by the problem of synthesis and it represents a set of mathematical relations; it must be chosen in such a way that the conditions perform desired tasks presented in a well defined mathematical form. In the optimisation algorithm, the objective function is given a numerical value for every solution and it would be ideal for the objective function to have the result in a minimum (global minimum), which corresponds to the best mechanism possible which should perform a technological procedure, but it is difficult to be achieved because of very complex problems.

The objective function may contain various restrictions, such as: restriction of ratio of lengths of the members, prevention of negative lengths of the members, restrictions regarding transmission angles, etc. In order to achieve this, the so-called penalty functions are introduced and they considerably increase the value of the objective function when the mentioned values go toward undesired direction. Penalty functions included in the objective function during synthesis of mechanisms are also called equilizing functions because they turn the restricted problem of optimisation into the unrestricted one.

In this paper, Hooke-Jeeves's optimisation method is used for synthesis of a four-bar linkage, which, as a method of direct searching, does not use deductions of the objective function, but it compares its values in each iteration and changes mechanism parameters in the direction of decreasing the value of the objective function. The procedure is finished when displacement of the space does not result in a considerable change of the value of the objective function or when the number of iterations reaches the given value. The algorithm enables control of change of the objective function for each projected variable, independent of changes in the other projected variables. This method results in the local minimum of the objective function, but finding more local minimums increases the possibility of providing global convergence within the solution.

Controlled deviations figure in the objective function, i.e. the path of the mechanism point is in the prescribed environment around the given point on the observed segment. Changes of projected variables (parameters) are followed during synthesis by the method of controlled deviations and it can be concluded whether those changes tend to a suitable solution or not. An unfavorable course of synthesis is stopped and it is continued with changed parameters.

Allowed deviations determine the space around the given functions where functions of the mechanism which is being optimized must be found so that the objective function could reach the values approximately equal to zero. Mechanism functions are combined from several straight lines because this is suitable for covering different cases that can be encountered in practice. Three variants of minimization can be applied and then the following is prescribed: the path and the working angle, only the path and only the ratio between the path and the angle.

2 Method of controlled deviations

2.1 Analysis of a four-bar linkage

Figure 1 shows a four-bar linkage as one of the most typical planar linkages. All parameters which could be used for its detailed analysis are presented. Certain equations from the analysis of mechanism are used in the synthesis process, where:

a – the crank (input member)

b – the coupler

c – the rocker

d and r – the parts of the coupler

φ – the input angle (the angle of the crank)

ψ – the output angle (the angle of the rocker)

x_p – the initial position of the point M of the coupler on the path, which corresponds to the initial angle φ_0

x_k – the final position of the point M of the coupler on the path, which corresponds to the final angle φ_k

$\varphi_1 = \varphi_k - \varphi_0$ – the range of change of the working angle necessary for the coupler point to pass through desired positions.

The position of the point A is defined by the expressions:

$$x_A = x_0 + a\cos\varphi \tag{1}$$

$$y_A = y_0 + a\sin\varphi \tag{2}$$

where φ is the angle which defines the current position of the mechanism.

The distance $\overline{EA} = s$ is a variable whose value is

$$s = \sqrt{(x_E - a\cos\varphi)^2 + (y_E - a\sin\varphi)^2} \tag{3}$$

and it creates the angle $\theta\,(-\pi \leqslant \theta \leqslant \pi)$with the positive direction x – axis:

$$\theta = a\tan\left(\frac{y_E - a\sin\varphi}{x_E - a\cos\varphi}\right) \tag{4}$$

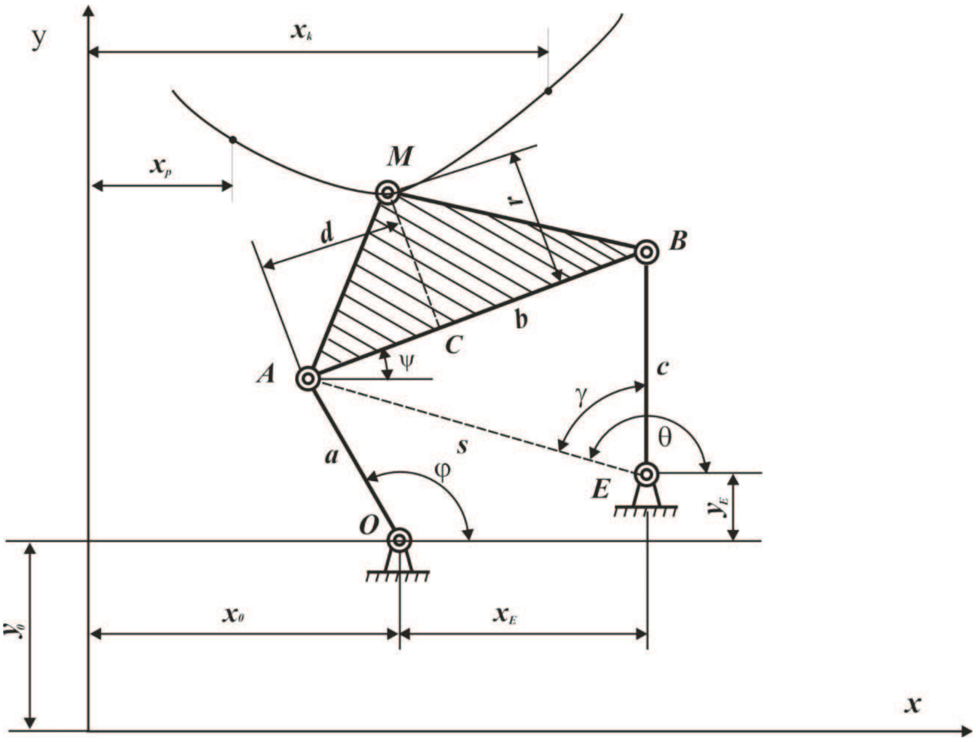

Figure 1: Dimensions of a four-bar linkage

The position of the rocker (the member $\overline{BE} = c$) is double-valued and it depends on whether the point B is below or above the radius vector $\overline{AE}$. The coordinates of the point B are determined by the expressions:

$$x_B = x_0 + x_E + c \cdot \cos(\theta - t\gamma) \quad and \tag{5}$$

$$y_B = y_0 + y_E + c \cdot \sin(\theta - t\gamma), \tag{6}$$

where t – the coefficient whose value is:

$t = 1$, for the case when the point B is above the line segment s (Figure 1), and

$t = -1$, for the case when the point B is below the line segment s (crossed mechanism).

$\gamma-$ is the angles created between the rocker and the line segment$s$. It takes the value $(0 \leqslant \gamma \leqslant \pi)$ and is determined by the expression:

$$\gamma = \arccos\left(\frac{s^2 + c^2 - b^2}{2sc}\right). \tag{7}$$

ψ is the angle created between the coupler $\overline{AB}$ and the positive part of the x – axis. Its value is determined by the expressions:

$$\psi = a\tan\left(\frac{y_B - y_A}{x_B - x_A}\right). \tag{8}$$

Finally, the position of the point M of the coupler, i.e. the point moving along the desired path, is determined by the expression:

$$x_M = x_0 + a\cos\varphi + d\cos\psi + r\cos\left(\frac{\pi}{2} + \psi\right) \quad and \tag{9}$$

$$y_M = y_0 + a\sin\varphi + d\sin\psi + r\sin\left(\frac{\pi}{2} + \psi\right). \tag{10}$$

2.2 Formulation of the method of controlled deviations

In order for a mechanism within a device to perform its operation ideally, it is necessary that at a certain section of the path the observed point should move according to the exactly defined law. The desired law of motion in a general case can be represented by the shape function and the position function.

In a large number of cases the mechanism will perform its operation successfully even when the working part of its path does not coincide with the ideal one. It is enough that the real path is in the "controlled" space around the ideal path and that deviations are prescribed in advance.

For the four bar linkage whose optimisation we want to perform, motion of the coupler point on a segment is defined by the expressions (9) and (10), within which all projected variables figure, either directly or indirectly.

Solution of the problem is reduced to forming of the objective function where deviations are bigger than those allowed and to bringing of the objective function within the allowed deviations around the given (desired) ones. Then the values of real deviations do not figure in the synthesis process, so it can be concluded that if there is one solution then there is an infinite number of similar solutions. Let:

$$\xi = |y_M - f_y| - dy \quad and \tag{11}$$

$$\eta = |x_M - f_x| - dx \tag{12}$$

where:

x_M and y_M – the real coordinates of the point$M$,

f_x and f_y – the desired coordinates of the point$M$, i.e. the coordinates which are on the desired path,

dx i dy – the allowed (given) deviations of the point.

Then:

$$\xi = \begin{cases} \xi & \xi > 0 \\ 0 & \xi \leqslant 0 \end{cases}, \tag{13}$$

$$\eta = \begin{cases} \eta & \eta > 0 \\ 0 & \eta \leqslant 0 \end{cases}, \tag{14}$$

By minimising these deviations, i.e. by minimising the objective function (Chapter 2.3), the parameters of the mechanism obtain optimal values. Figure 2 and 3 show the given functions in the field of allowed deviations. Shaded parts represent undesired deviations which should be minimised.

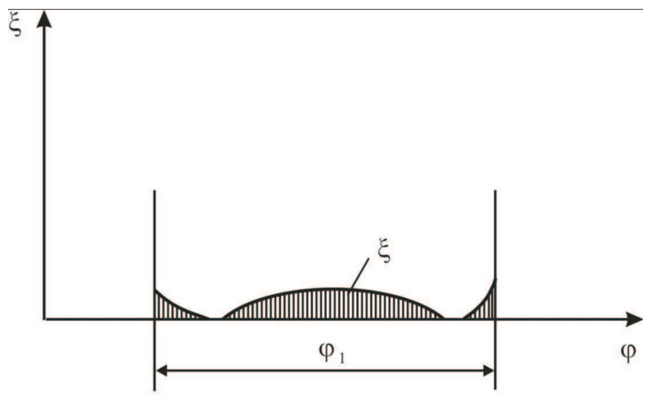

Figure 2:

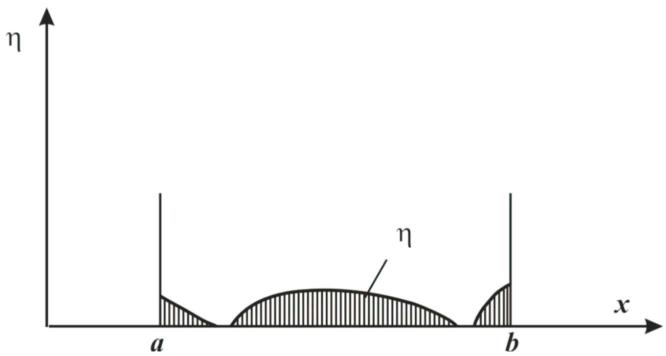

Figure 3:

2.3 Objective function

The objective function contains two parts. One part represents the sum of squares of deviations of the given functions from the path described by the coupler point, and the other part refers to the restrictions:

$$f(x) = \sum_{i=1}^{n} \left(\xi_i^2 + \eta_i^2\right) + \sum_{j=1}^{m} k_j g_j \tag{15}$$

where:

n – the number of parameters participating in the optimisation

m – the number of restrictions

g_j – the restrictions

k_j – the penalty functions taking different values depending on the restrictions.

The restrictions which provide that there are no negative lengths of the members are:

$$g_1 = -a < 0,$$
$$g_2 = -b < 0$$
$$g_3 = -c < 0$$
$$g_4 = -d < 0$$

It is very important that there is no disproportion of the mechanism members, so we should make sure that the mechanism obtained by the synthesis procedure should be feasible. It means that a big ratio between the longest member of the mechanism and the shortest one cannot exist. Therefore, the restrictions which give dependence between the longest member of the mechanism and the shortest one are introduced:

$$g_5 = \frac{l_{\max}}{l_{\min}} - 5 < 0$$

And, finally, we must take care about the range of change of the transmission angle, i.e. Grashof's conditions must be satisfied, i.e. penalty functions should be introduced in order to check the safety of coupling the resulting mechanism with any desired position so that incorrect joints could be avoided when the optimal solution is searched.

$$(g_6 = \cos\gamma - 0.96 < 0), \quad g_i = |\cos\gamma| - 0.96 < 0,$$

This penalty function prevents the transmitting angle from approaching, i.e. 180 degrees, because then there is self-locking in the mechanism. The most optimal point in the synthesis is that the value of the transmitting angle ranges from 15 and 165 degrees, and this is why there is the value of 0.96, i.e. $\cos 15^0 \approx 0.96$.

3 Hooke-Jeeves's optimisation algorithm

In the optimisation algorithm, we should first determine the initial point or the initial vector $X^0 = X^0\,(x_1, x_2, ..., x_n)^T$ where n – is equal to the number of projected variables. Then the values of steps for each projected variable (Δx) and the minimum size of the step for the algorithm ($\Delta x_{\min}$) are determined. When searching in the vicinity of the initial point is finished, the objective function is again calculated at the new point and compared to the previous value. If the value is improved, the new point is saved as the basic point, otherwise the negative step ($-\Delta x$) is taken and the objective function is calculated again. If the value of the objective function at the new point is improved, then it is stopped becoming a new basic point. If there is no improvement in searching of the projected space and the value of the step decreases, searching of the space starts again. The value of the step is reduced by half in relation to the previous one, and the increase of the step is performed by one quarter of the previous one. These changes in the value of the step still lead to a stable optimization procedure for the mentioned example. The searching procedure is performed for each projected parameter, which results in the local behavior of the objective function.

In the next step, searching in all projected variables in relation to the recently kept starting point is repeated. The new point keeps that value of the objective function as the improved one and compares it with the previous value. Moving in the space is repeated until the value of the objective function is improved.

Accordingly, this method makes a combination in searching and moving of the space, moving until the value decreases below the tolerance step($\Delta x_{\min}$). When the optimization procedure is finished, the local minimum is achieved, but it is not a guarantee that convergence has been performed and an acceptable solution has been obtained. Therefore, the optimization Hooke-Jeeves's method with different values of the initial projected vectors is applied, which gives a set of solutions which can be calculated and compared for the purpose of obtaining the best solution possible.

The optimization algorithm contains two programs. The first one is Hooke-Jeeves's algorithm for searching and moving of the space, and the other one calculates the actual value of the objective function which includes controlled deviations. The objective function performs complete kinematic analysis, checks locking of the joint, checks disproportion of the members, etc. and returns numerical values into Hooke-Jeeves's algorithm, which uses those values for determination of the optimum direction in the step and the size of the step for moving of the space. The total number of variables that participate in the optimisation can, in principle, be any one, and in the example mentioned below it is eleven.

4 Numerical example

The method is illustrated on the example of synthesis of a four bar linkage (Figure 5), in which the coupler point passes through 16 given points lying on a straight line. The initial values of the variables are given in Table 1, and the optimal values of the variables, (dimensions of the mechanism members, coordinates of the support of the crank and the rocker as well as the values of the initial angle and the working angle) are presented in Table 2. Synthesis has been performed for the case when all 16 projected points lie on the x – axis being regularly distributed on the radius vector l=980. mm.

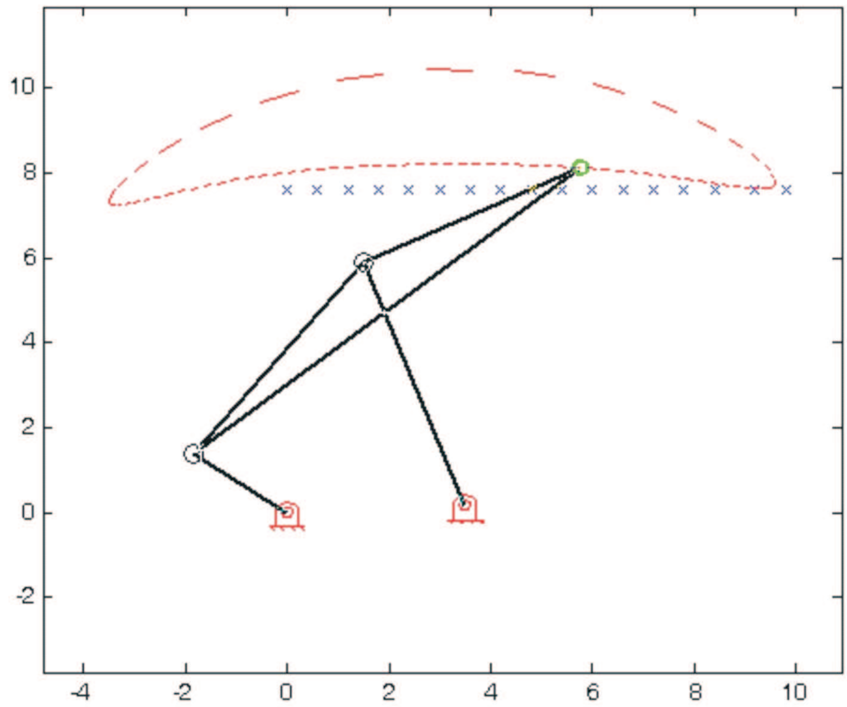

Figure 4: Initial mechanism. The point of support of the crank is placed at the coordinate beginning: **x** – denotes the projected points, **- - - -** the path described by the coupler point before the beginning of optimisation.

Table 1: Initial values of the projected variables. The lengths and coordinates of the points are given in [mm], and the initial angle and the working angle in [rad].

a	**b**	**c**	**d**	**r**	**x0**	**y0**	**xe**	**ye**	**f0**	**f1**
235	562	607	994	-209	102	-780	384	20	-1.2	-3.5

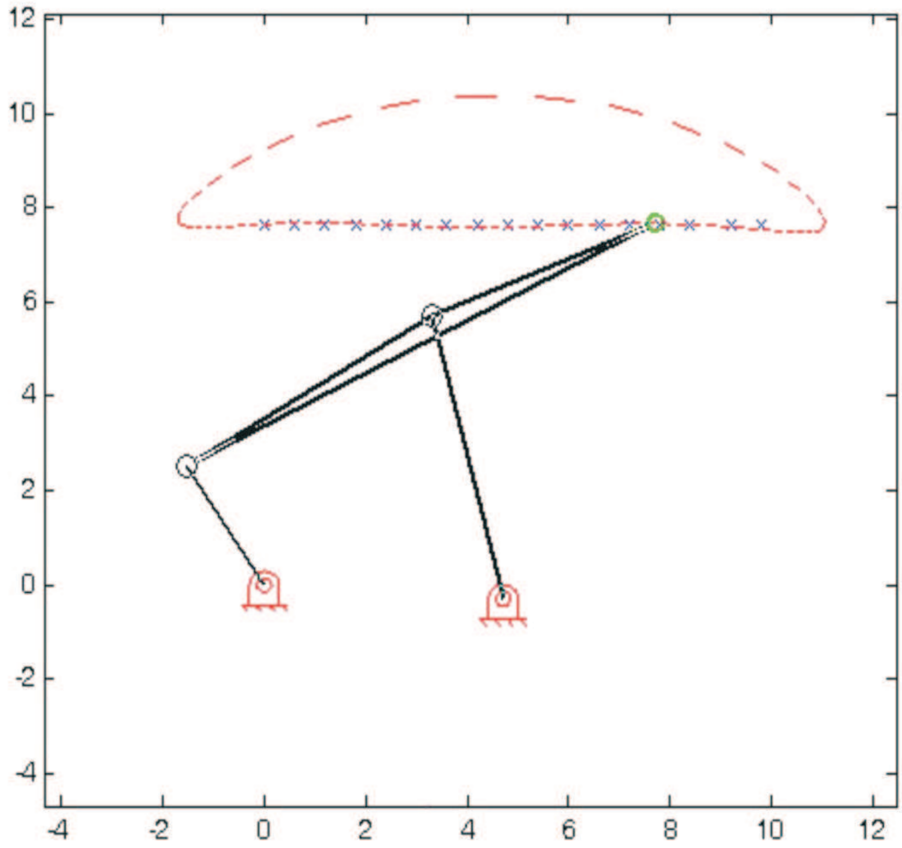

Figure 5: Results of synthesis. The point of support of the crank is placed at the coordinate beginning: **x** – denotes the projected points, **- - - -** the path described by the coupler point upon completion of optimisation.

Table 2: Optimal values of the projected variables. The lengths and coordinates of the points are given in [mm], and the initial angle and the working angle in [rad].

a	**b**	**c**	**d**	**r**	**x0**
294.6	580.3	616.5	1055.3	-79.2	151
y0	**xe**	**ye**	**f0**	**f1**	
-762.8	471	-29.5	-1.22697	-3.56571	

5 Conclusion

This paper describes the procedure of optimal synthesis of a four-bar linkage by the method of controlled deviations, with the application of Hooke-Jeeves's optimisation algorithm. The method of controlled deviations is suitable because deviations of motions of the coupler point in relation to the projected path can be followed at any moment. Also, deviations are controlled by giving values of allowed deviations within which the motion of the coupler point is followed. The method is illustrated on the example of rectilinear motion of the coupler point, although the method can also be very efficiently applied when the path of the point is any algebraic curve.

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Optimalna sinteza cetvorougaonog mehanizma metodom kontrolisanog odstupanja.

UDK 531.01

U radu je razmatrana optimalna sinteza zglobnog četvorougaonog mehanizma metodom kontrolisanih odstupanja. Pogodnost ove aproksimativne metode, je što se može kontrolisati kretanje tačke spojke zglobnog četvorougla tako da se putanja tačke nalazi u propisanoj okolini oko zadate putanje na posmatranom segmentu. U procesu optimizacije korišćen je Hooke-Jeeves optimizacioni algoritam. Kao metod direktnog pretrazivanja ne koriste se izvodi u proracunu, odnosno pojedinacno se uporeduju izracunate vrednosti funkcije cilja u svakoj iteraciji i vrši

se pomeranje u pravcu smanjenja vrednosti funkcije cilja. Ovaj algoritam ne zavisi od početnog izbora projektovanih promenljivih. Sve je ovo ilustrovano na primeru sinteze zglobnog četvorougaonog mehanizma čija tačka spojka trasira pravu liniju, odnosno prolazi kroz šesnaest propisanih tačaka koje leže na jednoj pravoj.