

Dynamics of a rigid rotor in the elastic bearings

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Abstract

As a rule in the studies of a rigid rotor in the elastic bearings the authors consider the linear system corresponding to the plane-parallel motion and the effect of self-centring under unlimited growth of the rotation frequency. In the present paper rotor is considered as a mechanical system with four degrees of freedom. Different motions of a statically and dynamically unbalanced vertical rotor supported in the non-linear bearings are studied.

Keywords: unbalanced rotor, precession motion, self-induced vibrations.

1 Introduction

The absolute rigid rotor of mass M and length L is supported vertically in two immovable non-linear bearings in such a way that the center of mass of the rotor is placed symmetrically with respect to the bearings (Fig. 1). The rotor is dynamically symmetric; A is the moment of inertia about the axis of symmetry and B is the equatorial moment of inertia. The distance between the center of mass of the rotor and the axis of revolution (static eccentricity) is equal to e , the angle between

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the axis of dynamical symmetry and the axis of revolution (dynamical eccentricity) is equal to δ . The angle between the plane containing the axis of revolution and the mass center and the plane containing the angle δ is denoted by ϵ .

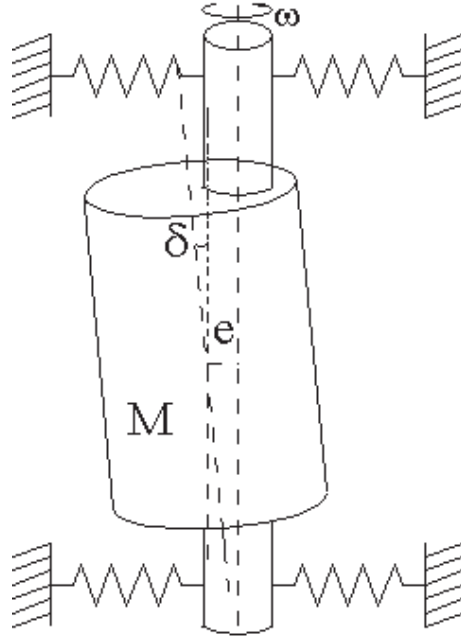


Figure 1: Unbalanced rotor in elastic bearings

It is assumed that the elastic bearings are centrally symmetric and the reactions in the bearings have only radial components. We only consider the case of the hard characteristic of the restoring forces with cubic non-linearity:

$$\mathbf{P}_j = -\mathbf{S}_j (a_0 + a_1 |S_j|^2), \quad j = 1, 2. \quad (1)$$

Here \mathbf{S}_j is the vector describing the displacement of the center of a bearing from the equilibrium position, a_0 and a_1 are the positive real constants, characterising the elasticity of the bearings.

The rotor rotates with an ideal engine (an engine with unlimited power), so that the angular velocity ω is supposed to be constant. The

rotor is under external friction forces \mathbf{R}_j^e , which are proportional to the absolute velocity of the bearing centers:

$$\mathbf{R}_j^e = -\tilde{\mu}_e \dot{\mathbf{S}}_j, \quad j = 1, 2. \quad (2)$$

If one neglects the displacement along the axis of rotation, then the position of the rotor is determined by the co-ordinates of the center of bearings and the rotor is a system with four degrees of freedom. The equations of the motion can be derived by using the theorem of the motion of the center of mass and the theorem of the moments.

We consider vibrations near the equilibrium (vertical) position, when the co-ordinates of the bearings and their velocities and eccentricities e and δ are considered to be small. The system of differential equations in complex variables S_1 and S_2 is the following:

$$\begin{aligned} & \frac{M}{2}(\ddot{S}_1 + \ddot{S}_2) + \tilde{\mu}_e(\dot{S}_1 + \dot{S}_2) + a_0(S_1 + S_2) + \\ & a_1(|S_1|^2 S_1 + |S_2|^2 S_2) = M e \omega^2 \exp(I \omega t), \\ & B(\ddot{S}_2 - \ddot{S}_1) - I \omega A(\dot{S}_2 - \dot{S}_1) + \tilde{\mu}_e \frac{L^2}{2}(\dot{S}_2 - \dot{S}_1) + \\ & \frac{L^2 a_0}{2}(S_2 - S_1 + \frac{a_1}{a_0}(|S_2|^2 S_2 - |S_1|^2 S_1)) = \\ & (B - A)\omega^2 L \delta \exp(I(\omega t - \varepsilon)). \end{aligned} \quad (3)$$

This is the linear system, but with the non-linear cubic terms in the restoring forces.

Forced vibrations due to either static or dynamical eccentricity or both of them have the form of the direct synchronous precession:

$$S_j = \tilde{R}_j \exp(I \omega t) \exp(I \varphi_j), \quad j = 1, 2, \quad (4)$$

where \tilde{R}_j and φ_j are the real constants characterizing the amplitudes and phases of the bearings displacements. Depending on a type of the surface traced by the axis of a rotor, the rotor motion could be either

cylindrical, or conical or hyperboloidal precession. The characteristic feature for a cylindrical precession is the equality of the amplitudes and phases. For a conical precession the phases are either equal or differed by π . In the first case the axis of the rotor traces the truncated cone and in the second case the rotor axis traces the cone with the cone apex between the bearings. In the case of a hyperboloidal precession the amplitudes and phases relate arbitrarily. We call the precessions as symmetric if both bearings in their plane motions make the circles of equal radii.

2 Symmetric hyperboloidal precessions of completely unbalanced rotor

Here we analyze hyperboloidal precessions for a rotor with two disbalances ($e \neq 0$, $\delta \neq 0$). Symmetric precessions of statically and dynamically unbalanced rotor may only occur for a system without external friction forces \mathbf{R}^e and when $\epsilon = \pi/2$. Otherwise non-symmetric hyperboloidal precessions take place. Consider symmetric hyperboloidal precessions defined as

$$\sqrt{y} = \frac{x}{2} \left(\frac{1}{(1+cy-x)^2} + \frac{d^2}{(k(1+cy)-x)^2} \right)^{1/2}, \quad (5)$$

$$\tan \phi_1 = d \frac{1+cy-x}{k(1+cy)-x}, \quad \phi_1 = -\phi_2.$$

Here for convenience we introduce the amplitude-frequency response in the plane (x, y) ($x = \Omega^2$, $y = R^2$), where the dimensionless variables and parameters are:

$$R = \tilde{R}/(2e), \quad \Omega^2 = \omega^2/(2a_0/M),$$

$$c = 4e^2 \frac{a_1}{a_0}, \quad k = \frac{ML^2}{4B(1-A/B)}, \quad d = \frac{L\delta}{2e}. \quad (6)$$

For the limiting values R_∞ and ϕ_∞ , when x is large enough we obtain

$$\sqrt{y_\infty} = R_\infty = 1/2\sqrt{1+d^2}, \quad \tan \phi_{1,\infty} = d. \quad (7)$$

If we introduce a minimal radius of the hyperboloid of rotation $r = R \cos \phi_1$ and the angle of the deflection of the rotor axis from the vertical $\beta = 2R \sin \phi_1 / \tilde{L}$ ($\tilde{L} = L/(2e)$), then the limiting values for these quantities are $r_\infty = 1/2$ and $\beta_\infty = \delta$ respectively. The last result means that the self-centring regime takes place. The rotor rotates in such a way that the center of mass remains stationary and the axis of dynamic symmetry takes the equilibrium position.

If the rotor is a dynamically prolate body ($\lambda < 1$ and $k > 0$) then the resonance set consists of the lines $1+cy-x=0$ and $k(1+cy)-x=0$, which are the skeleton lines for the modified amplitude-frequency response curve (AFR) in the plane (x, y) (Fig. 2). The cylindrical and conical precessions resonate near the first and the second lines, respectively.

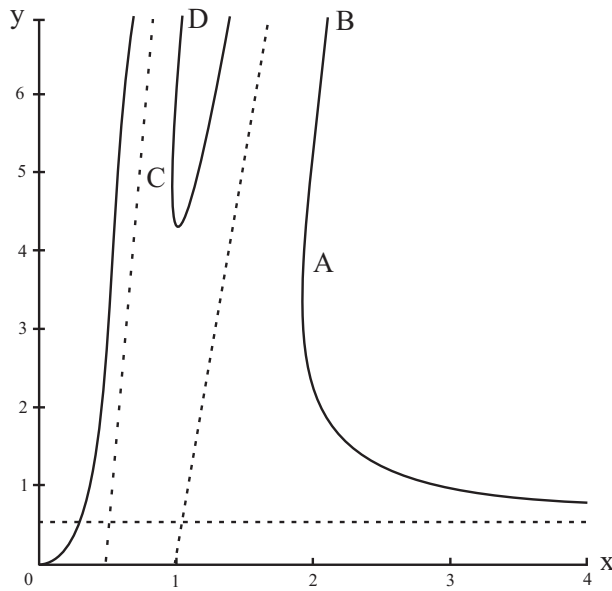


Figure 2: AFR of symmetric hyperboloidal precession

One can see in Fig. 2 that for one frequency there can be either one, three or five different regimes of symmetric hyperboloidal precessions.

It can be shown that in the first approximation the segments AB and CD of AFR for the values $k < 1/3$ and $k > 3$ correspond to the unstable symmetric hyperboloidal precessions.

If rotor is dynamically oblate ($\lambda > 1$ and $k < 0$) then the resonance set is a sole line $1 + cy - x = 0$, near which the cylindrical precession resonates. For that case the segment of AFR with the intermediate value of amplitude corresponds to the unstable motions for any value of the parameter k .

3 Precessions of rotor with one disbalance

If the rotor has only dynamical disbalance ($e = 0$, $\delta \neq 0$) among the steady-state direct synchronous precession motions of the dynamically unbalanced rotor there may be either symmetric or non-symmetric precessions of either conical or hyperboloidal type.

The symmetric conical precessions, i.e. the regime when the center of mass of the rotor is motionless ($s_1 = -s_2 = -s$), are described by the following equations

$$\sqrt{y}\sqrt{(k(1+cy)-x)^2 + \mu_e^2 k^2 x} = \frac{1}{4}x, \quad \tan \phi = -\frac{\mu_e k \sqrt{x}}{k(1+cy)-x}. \quad (8)$$

AFR for the dynamically prolate rotor ($\lambda < 1$ and $k > 0$) in the plane (x, y) has the skeleton line $k(1+cy) - x = 0$ (Fig. 3). For sufficiently large x the limiting value for the amplitude is $R_\infty = 1/4$ and the axis of dynamical symmetry of the rotor tends to take the equilibrium position. This motion corresponds to the self-centring regime of the rotor.

Depending on the parameters of the system there may be one or three regimes of symmetric conical precessions. To study stability of these regimes in the linear approximation the characteristic equation may be represented as a product of two fourth order polynomials in characteristic number p ($M N = 0$). The stability conditions have been reduced to the inequalities $m_4 > 0$, $n_4 > 0$, where m_4 and n_4 are the absolute values of the polynomial M and N .

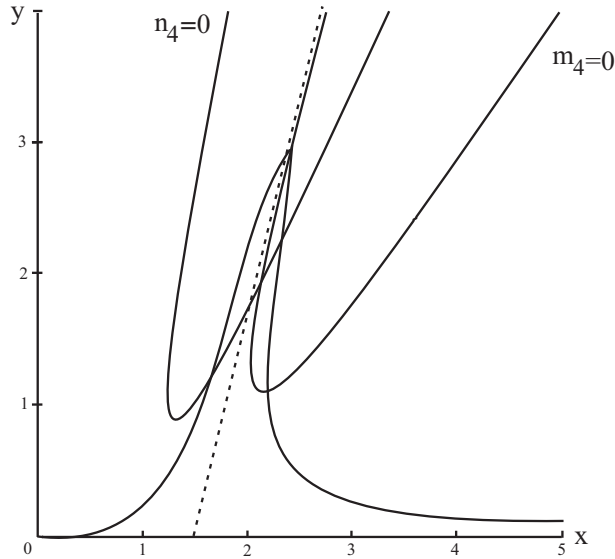


Figure 3: AFR of symmetric conical precession

The sets $m_4 = 0$ and $n_4 = 0$ are the bifurcation sets determining the bounds of the stability domains for the symmetric conical precessions, which have the form of hyperbolas in the plane (x, y) (Fig. 3). In the bifurcation points on the bound, where $m_4 = 0$, the regime of the symmetric conical precessions (when the center of mass of the rotor is motionless) may appear or disappear. Instability of the segment, where inequality $m_4 < 0$ is satisfied is quite common for the theory of non-linear vibrations. In the bifurcation points on the bound, where $n_4 = 0$, the regime of the nonsymmetric hyperboloidal precessions due to the motion of the center of mass may appear or disappear. That means that the symmetric conical precessions are unstable in 3D and the domain of instability in 3D appears near the resonance set for the cylindrical precessions of the rotor with two disbalances. Such instability may exist only for the mechanical system with three and more degrees of freedom. In domain $n_4 < 0$ the symmetric conical precessions may transform to the hyperboloidal precessions or some other regimes (for example, strange attractor).

Among the steady-state motions of statically unbalanced rotor there

may be cylindrical precessions, non-symmetric conical precessions and symmetric and non-symmetric hyperboloidal precessions. For the cylindrical precession the effect of self-centring is typical; the dynamically prolate rotor may be unstable in 3D and for the resulting regimes the axis of rotation of the rotor swings.

4 Non-linear equations of the rotor

In the analysis of the rotor motion with the second order terms the equations are nonsymmetric and can not be represented in the complex variables as we did it in the previous sections. The coefficients at the second derivatives of the co-ordinates of the bearings in their plane of motion depend on the same co-ordinates.

If for a linear system the main motion is the symmetric precession, then for a nonlinear system the symmetric motions for the rotor with either static or dynamic disbalance or both of them can only be symmetric cylindrical precessions for a statically unbalanced rotor. Symmetric cylindrical precessions are described by the same formulas as in the linear approximation.

5 The influence of the internal friction force on the rotor motion

Consider the motion of the rotor under external friction forces \mathbf{R}_j^e and internal friction forces \mathbf{R}_j^i . Internal friction forces are proportional to the relative velocity of motion of the center of bearings and may exist in the bearings due to the oil film that is partly entrained by the rotor:

$$\mathbf{R}_j^i = -\tilde{\mu}_i \left(\dot{\mathbf{S}}_j - I\omega \mathbf{S}_j \right), \quad j = 1, 2. \quad (9)$$

The forces \mathbf{R}^i do not affect the parameters of the direct synchronous precessions of rotor, but make the domain of the stable regimes significantly more narrow and prevent the self-centring effect. For symmetric conical precession of the dynamically unbalanced rotor and cylindrical precessions of statically unbalanced rotor the regimes induced are the self-induced vibration which can be represented in the form

$$s_j = R_j \exp(I\Omega\tau) \exp(I\varphi_j) + r_j \exp(I\Omega_1\tau), \quad (10)$$

where R_j and φ_j are the amplitudes and phases of forced vibrations with the frequency Ω , r_j and Ω_1 are the amplitudes and the frequency of self-induced vibrations [3].

For dynamically unbalanced rotor self-induced vibrations are associated with the motion of the center of mass. When the mass center outlines the circle the resulting motion can be represented as a superposition of symmetric conical precessions ($R_1 = R_2 = R$ and $\exp(I\varphi_2) = -\exp(I\varphi_1) = \exp(I\varphi)$) and symmetric self-induced vibrations ($r_1 = r_2 = r$).

Using the harmonic balance method we obtain the following approximate equations for R , r , Ω_1 and φ :

$$r(1 + c(r^2 + R^2) - \Omega_1^2) = 0,$$

$$r(\Omega_1(\mu_e + \mu_i) - \Omega\mu_i) = 0,$$

$$R(1 - \lambda)((k(1 + c(R^2 + 2r^2)) - \Omega^2) \sin \varphi + \mu_e k \Omega \cos \varphi) = 0,$$

$$(1 - \lambda)(R(k(1 + c(R^2 + 2r^2)) - \Omega^2) \cos \varphi - \mu_e k \Omega R \sin \varphi - \frac{1}{4}\Omega^2) = 0. \quad (11)$$

Equations (10) permit to find the amplitudes of self-induced vibrations and forced vibrations as the functions of the rotation frequency and the frequency bound of the soft inducing for the self-induced vibrations:

$$\Omega_s = (1 + \mu_e/\mu_i) \sqrt{1 + 2cR^2}. \quad (12)$$

In Fig. 4 AFR curves for the amplitudes squares ($Y = R^2$, $y = r^2$) vs. frequency square ($x = \Omega^2$) are represented. The dotted line corresponds to the self-induced vibrations. Note that if the angular velocity Ω increases then the amplitude of the self-induced vibrations also goes up. That is a cause of the bearings destruction and rotor collapse.

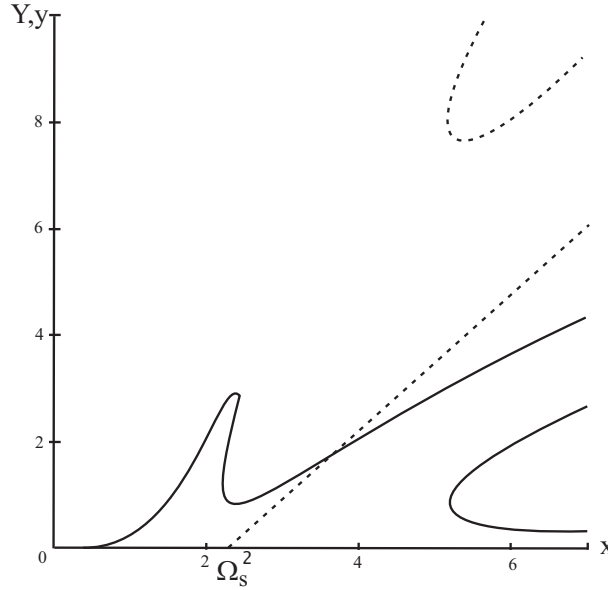


Figure 4: AFR of forced vibrations and self-induced vibrations

If the rotor has only two degrees of freedom then one should seek the modes for self-induced vibrations in the form (9), where $s_1 = -s_2 = -s$. Such regimes may exist only for a dynamically prolate rotor, when $\lambda < 1$ and $k > 0$. Although such type of solutions satisfy formally the system of differential equations of dynamically unbalanced rotor with four degrees of freedom, numerical integration results in the modes related to the motion of the mass center.

For a statically unbalanced rotor ($e \neq 0$, $\delta = 0$) with four degrees of freedom the developing self-induced vibrations form a planar motion.

The resistance force $\mathbf{R}^m = -\tilde{\mu}_m \dot{\mathbf{S}} \frac{\mathbf{S}}{S}$ proportional to the velocity of the radial displacements of the centers of bearings, which appears in a rigid rotor due to the deformation of the balls in the rolling bearing [4], has the same destabilizing effect on the symmetric precessions and generates the self-induced vibrations.

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Submitted on May 2004, revised on July 2004.

Dinamika krutog rotora na elastičnim osloncima

UDK 534.16

Po pravilu se u proučavanju krutog rotora na elastičnim osloncima autori služe linearnim sistemom koji odgovara ravanskom kretanju i efektu samocentriranja pri neograničenom rastu obrtne učestanosti. U ovom radu se rotor posmatra kao sistem sa četiri stepena slobode. Pritom se proučavaju različita kretanja neuravnoteženog rotora oslonjenog na nelinearne oslonce.