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Ionized gas boundary layer on a porous wall of the body within the electroconductive fluid

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Abstract

This paper investigates the ionized gas flow in the boundary layer, when the contour of the body within the fluid is porous. Ionized gas is exposed to the influence of the outer magnetic field induction $B_m = B_m(x)$, which is perpendicular to the contour of the body within the fluid. It is presumed that the electroconductivity of the ionized gas is a function only of the longitudinal coordinate, i.e. $\sigma = \sigma(x)$. By means of adequate transformations, the governing boundary layer equations are brought to a generalized form. The obtained generalized equations are solved in a four-parameter localized approximation. Based on the obtained numerical solutions, diagrams of important physical values and characteristics of the boundary layer have been made. Conclusions have also been drawn.

Keywords: boundary layer, ionized gas, magnetic field induction, electroconductivity, porous contour, General Similarity Method, porosity parameter.

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1 Introduction and the governing boundary layer equations

This paper studies the ionized gas i.e. air flow in the boundary layer on a body of arbitrary shape. The flow is two-dimensional (2-D) and the contour of the body within the fluid is POROUS.

The main goal of this investigation is to derive so-called generalized boundary layer equations by application of the General Similarity Method and to solve the obtained equations. Based on the obtained numerical solutions it is necessary to draw conclusions on behavior of certain physical values and characteristics in the boundary layer.

The study presented here represents a continuation of our earlier studies of this very complicated flow case. The studies undertaken before were concerned with the case when the wall of the body within the fluid was NON-POROUS.

As known, ionization of air occurs at high temperatures. These temperatures are a characteristic of the boundary layer when, for example, an aircraft is moving with a supersonic velocity through the Earth atmosphere. In this case there are positively charged ions, electrons and atoms (of oxygen and nitrogen). Thermodynamic equilibrium occurs when velocities of ionization and a reverse process of recombination are high enough.

One of the most important properties of ionized gas is its electroconductivity σ. Under the influence of the outer magnetic field induction $\vec{B}_m$ an electric flow appears in this gas. Due to this flow the so-called Lorentz force and Joule heat generate. These two effects result in appearance of new terms. These terms, therefore, are not present in the boundary layer equations of a homogenous non-ionized gas.

This paper studies the flow when the outer magnetic field is perpendicular to the contour of the body within the fluid [1] and when the so-called inner magnetic field can be neglected. Because of the small thickness of the boundary layer, the field induction is considered to be $B_{my} = B_m = B_m(x)$ [1], and the magnetic Reynolds number is considered to be very low.

Therefore, for the case of the ionized gas flow in the magnetic field, under the conditions of equilibrium ionization, the laminar steady and 2-D boundary layer equations [1] have the following form:

$$\frac{\partial}{\partial x}\,(\rho u) + \frac{\partial}{\partial y}\,(\rho v) = 0\,,$$

$$\rho u\,\frac{\partial u}{\partial x} + \rho v\,\frac{\partial u}{\partial y} = \; -\frac{dp}{dx} + \frac{\partial}{\partial y}\,\left(\mu\,\frac{\partial u}{\partial y}\right) \;-\; \sigma\,B_m^2\,u\,, \tag{1}$$

$$\rho u\,\frac{\partial h}{\partial x} + \rho v\,\frac{\partial h}{\partial y} = u\,\frac{dp}{dx} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\,\left(\frac{\mu}{Pr}\,\frac{\partial h}{\partial y}\right) \;+\; \sigma\,B_m^2\,u^2.$$

In the system (1), the first equation represents the continuity equation, the second - the momentum, and the third - the energy equation of the ionized gas boundary layer.

The corresponding boundary layer conditions for the considered case of the POROUS CONTOUR of the body within the fluid, are:

$$\begin{aligned} &u = 0\,, \qquad \underline{v = v_w(x)}\,, \qquad h = h_w \qquad for \quad y = 0\,;\\ &u \to u_e(x)\,, \qquad\qquad\qquad h \to h_e(x) \quad for \quad y \to \infty\,. \end{aligned} \tag{2}$$

In the given equations (1) and in the boundary conditions (2) the notations common in the boundary layer theory are used for different physical values. Here, $u(x,\ y)$ is the longitudinal projection of the velocity in the boundary layer, $v\,(x,\ y)$- the transversal projection, ρ- the ionized gas density, p-the pressure, h- the enthalpy, μ- the coefficient of dynamic viscosity and Pr- Prandtl number. Subscripts stand for: w- values on the wall of the body within the fluid, e- physical values on the outer edge of the boundary layer.

Note that $v_w(x)$ denotes the given velocity with which the ionized gas flows perpendicularly through the solid POROUS WALL of the body within the fluid. Here, $v_w > 0$, at injection and $v_w < 0$ at gas ejection.

The electroconductivity of ionized gas is generally a changeable value which depends on the temperature [1], i.e. the enthalpy. In this paper, it is presumed, by analogy with the magnetic field induction B_m, that the electroconductivity is a function on the longitudinal coordinate x only, i.e.

$$\sigma = \sigma\,(x)\,. \tag{3}$$

Taking into account the boundary condition for the velocity on the outer edge of the boundary layer, the pressure $p(x)$ can be eliminated from the equation system (1). Then the governing equation system of the ionized gas boundary layer becomes:

$$\begin{aligned}
&\frac{\partial}{\partial x}\,(\rho u) + \frac{\partial}{\partial y}\,(\rho v) = 0\,,\\
&\rho u\,\frac{\partial u}{\partial x} + \rho v\,\frac{\partial u}{\partial y} = \rho_e u_e \frac{du_e}{dx} + \frac{\partial}{\partial y}\left(\mu\,\frac{\partial u}{\partial y}\right) + \sigma B_m^2\,(u_e - u)\,,\\
&\rho u\,\frac{\partial h}{\partial x} + \rho v\,\frac{\partial h}{\partial y} = -u\;\rho_e u_e \frac{du_e}{dx} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \\
&\qquad\qquad \frac{\partial}{\partial y}\left(\frac{\mu}{Pr}\,\frac{\partial h}{\partial y}\right) + \sigma B_m^2\,(u^2 - u\,u_e)\;;
\end{aligned} \tag{4}$$

with unchanged boundary conditions (2).

2 Transformations of the boundary layer equations

In order to apply the General Similarity Method to this flow problem, by analogy with other already solved fluid flow problems [2, 3], first, we introduce new transformations in the form of:

$$s(x) = \int_0^x \frac{\rho_w \mu_w}{\rho_0\,\mu_0}\;dx\,, \qquad z(x,\,y) = \int_0^y \frac{\rho}{\rho_0}\;dy. \tag{5}$$

In the transformations (5) and further on in this paper the values ρ_0 and $\mu_0 = \rho_0 \nu_0$ stand for the known values of the density and dynamic viscosity of ionized gas, respectively. Here, ρ_w and μ_w denote the known distributions of these values on the wall of the body within the fluid.

Applying the variables in the form of transformations (5), the momentum equation of the compressible fluid boundary layer becomes formally the same as the corresponding equation of incompressible fluid.

After the stream function $\psi(s,\, z)$has been introduced, by means of the relations

$$u = \frac{\partial \psi}{\partial z}\,, \qquad \tilde{v} = \frac{\rho_0\,\mu_0}{\rho_w \mu_w}\left(u\,\frac{\partial z}{\partial x} + v\,\frac{\rho}{\rho_0}\right) = -\,\frac{\partial \psi}{\partial s}\,, \tag{6}$$

the continuity equation is identically satisfied, and the equation system (4) transforms into:

$$\begin{gathered}
\frac{\partial \psi}{\partial z}\,\frac{\partial^2 \psi}{\partial s \partial z} - \frac{\partial \psi}{\partial s}\,\frac{\partial^2 \psi}{\partial z^2} = \frac{\rho_e}{\rho} u_e \frac{du_e}{ds} + \nu_0\,\frac{\partial}{\partial z}\left(Q\,\frac{\partial^2 \psi}{\partial z^2}\right) + \\
\frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\,\frac{\sigma B_m^2}{\rho}\left(u_e - \frac{\partial \psi}{\partial z}\right)\,, \\
\frac{\partial \psi}{\partial z}\,\frac{\partial h}{\partial s} - \frac{\partial \psi}{\partial s}\,\frac{\partial h}{\partial z} = -\,\frac{\rho_e}{\rho} u_e \frac{du_e}{ds}\,\frac{\partial \psi}{\partial z} + \nu_0\,Q\,\left(\frac{\partial^2 \psi}{\partial z^2}\right)^2 + \\
\nu_0\,\frac{\partial}{\partial z}\left(\frac{Q}{Pr}\,\frac{\partial h}{\partial z}\right) + \frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\,\frac{\sigma B_m^2}{\rho}\left[\frac{\partial \psi}{\partial z}\left(\frac{\partial \psi}{\partial z} - u_e\right)\right]; \\
\frac{\partial \psi}{\partial z} = 0\,, \quad \underline{\frac{\partial \psi}{\partial s} = -\frac{\mu_0}{\mu_w}\,v_w = -\,\tilde{v}_w}\,, \quad h = h_w \quad for \quad z = 0\,, \\
\frac{\partial \psi}{\partial z} \to u_e(s)\,, \qquad h \to h_e(s) \quad for \quad z \to \infty.
\end{gathered} \tag{7}$$

In the transformed equations of the system (7), the nondimensional function Q and Prandtl number Pr are determined with the expressions:

$$Q = \frac{\rho\,\mu}{\rho_w \mu_w}\,; \qquad Pr = \frac{\mu\,c_p}{\lambda}\,; \tag{8}$$

where λ is the coefficient of thermal conductivity and c_p is the specific ionized gas heat at constant pressure.

From the first two equations of the system (4), by the usual procedure just alike to the incompressible fluid flow [2] - by integration transversally to the boundary layer and by applying the transformations of the variables (5), it is relatively easy to derive the momentum equation. For this problem of the ionized gas flow, this equation can be easily written in any of the following three forms:

$$\frac{dZ^{**}}{ds} = \frac{F_{mp}}{u_e}, \quad \frac{df}{ds} = \frac{u_e'}{u_e} F_{mp} + \frac{u_e''}{u_e'} f, \quad \frac{dg}{ds} = \frac{N_\sigma}{u_e} F_{mp} + \frac{N_\sigma'}{N_\sigma} g\,; \qquad (9)$$

where the prime ($'$) denotes a derivative per the newly introduced variable s.

In the expressions (9) F_{mp} denotes the characteristic boundary layer function for the case of a porous contour of the body within the fluid. With the studied ionized gas flow problem this function is:

$$F_m = 2\left[\zeta - (2+H)\ f\right] - 2H_1 g - 2\Lambda\,. \qquad (10)$$

During the course of the derivation of the momentum equation we introduce the usual values: the parameter of the form $f(s)$, the magnetic parameter $g(s)$, the POROSITY PARAMETER $\Lambda(s)$, the nondimensional friction function $\zeta(s)$, the conditional displacement thickness $\Delta^*(s)$, the conditional momentum thickness $\Delta^{**}(s)$, the conditional thickness $\Delta_1^*(s)$, as well as other values and characteristics of the boundary layer. These values are defined as:

$$Z^{**} = \frac{\Delta^{**2}}{\nu_0}, \qquad f(s) = u_e'\, Z^{**} = f_1(s), \qquad g\,(s) = N_\sigma Z^{**} = g_1\,(s)\,,$$

$$\Delta^*(s) = \int_0^\infty \left(\frac{\rho_e}{\rho} - \frac{u}{u_e} \right) dz\,, \qquad \Delta^{**}(s) = \int_0^\infty \frac{u}{u_e} \left(1 - \frac{u}{u_e} \right) dz\,,$$

$$\Delta_1^*(s) = \int_0^\infty \frac{\rho_e}{\rho} \left(1 - \frac{u}{u_e}\right) dz \,,$$

$$H = \frac{\Delta^*}{\Delta^{**}} \,, \quad H_1 = \frac{\Delta_1^*}{\Delta^{**}} \,, \quad N_\sigma(s) = \frac{\rho_0 \, \mu_0}{\rho_w \, \mu_w} \bar{N} \,; \tag{11}$$

$$\bar{N}(s) = \frac{\sigma \, B_m^2}{\rho_e} \,, \qquad \zeta(s) = \left[\frac{\partial \,(u/u_e)}{\partial \,(z/\Delta^{**})}\right]_{z=0} \,,$$

$$\Lambda(s) = -\frac{v_w \Delta^{**}}{\nu_0} \frac{\mu_0}{\mu_w} = -\frac{V_w \Delta^{**}}{\nu_0} = \Lambda_1(s) \,; \qquad V_w(s) = \frac{\mu_0}{\mu_w} v_w = \tilde{v}_w \,.$$

The porosity parameter $\Lambda(s)$ is defined with the value $V_w(s)$. This value is in this paper called CONDITIONAL TRANSVERSAL VELOCITY on the inner edge of the boundary layer on the porous wall of the body within the fluid.

The boundary condition for the transversal velocity on the inner edge of the boundary layer (underlined in (7)) is a characteristic of the porous wall. With this problem of the ionized gas flow the boundary conditions can be kept the same as with the non-porous wall. Therefore, analogously to incompressible fluid [2], we introduce a new stream function $\psi^*(s,\, z)$, which is related to $\psi(s,\, z)$ as:

$$\psi(s,\, z) = \psi_w(s) + \psi^*(s,\, z) \,; \qquad \psi^*(s,\, 0) = 0 \,. \tag{12}$$

In the relation (12), the value $\psi_w(s) = \psi(s,\, 0)$ stands for the stream function along the body wall ($z = 0$).

After the stream function $\psi^*(s,\, z)$has been introduced, the equation system (7) transforms into the following system:

$$\frac{\partial \psi^*}{\partial z} \frac{\partial^2 \psi^*}{\partial s \partial z} - \frac{\partial \psi^*}{\partial s} \frac{\partial^2 \psi^*}{\partial z^2} - \underline{\frac{d\psi_w}{ds} \frac{\partial^2 \psi^*}{\partial z^2}} = \frac{\rho_e}{\rho} u_e u_e' +$$

$$\nu_0 \, \frac{\partial}{\partial z} \, \left(Q \, \frac{\partial^2 \psi^*}{\partial z^2} \right) + \frac{\rho_0 \, \mu_0}{\rho_w \, \mu_w} \, \frac{\sigma B_m^2}{\rho} \, \left(u_e - \frac{\partial \psi^*}{\partial z} \right) ,$$

$$\frac{\partial \psi^*}{\partial z} \, \frac{\partial h}{\partial s} \; - \; \frac{\partial \psi^*}{\partial s} \, \frac{\partial h}{\partial z} - \underline{\frac{d\psi_w}{ds} \, \frac{\partial h}{\partial z}} = - \, \frac{\rho_e}{\rho} u_e u_e' \, \frac{\partial \psi^*}{\partial z} +$$

$$\nu_0 \, Q \, \left(\frac{\partial^2 \psi^*}{\partial z^2} \right)^2 + \nu_0 \, \frac{\partial}{\partial z} \, \left(\frac{Q}{Pr} \, \frac{\partial h}{\partial z} \right) +$$

$$+ \frac{\rho_0 \, \mu_0}{\rho_w \, \mu_w} \, \frac{\sigma B_m^2}{\rho} \, \left[\frac{\partial \psi^*}{\partial z} \left(\frac{\partial \psi^*}{\partial z} - u_e \right) \right] ; \qquad (13)$$

$$\psi^* = 0 \, , \qquad \frac{\partial \psi^*}{\partial z} = 0 \, , \qquad h = h_w \qquad for \quad z = 0 \, ,$$

$$\frac{\partial \psi^*}{\partial z} \rightarrow u_e(s) \, , \qquad h \rightarrow h_e(s) \qquad for \quad z \rightarrow \infty \, .$$

Therefore, the corresponding boundary conditions for the stream function on the porous wall are the same as with the non-porous wall. This is very important for the application of the General Similarity Method. However, each of the equations of the system (13) contains on the left hand side a single term in which $d\psi_w/ds$ appears. Again, note that

$$\frac{d\psi_w}{ds} = \left(\frac{\partial \psi}{\partial s} \right)_{z=0} = - \, \tilde{v}_w = - \, \frac{\mu_0}{\mu_w} \, v_w = - \, V_w(s) \, .$$

3 Generalized boundary layer equations

The obtained equation system (13) is analyzed in detail. Analogously to the papers [3 - 6], and in order to apply the General Similarity Method, instead of the function $\psi^*(s,\, z)$, a new stream function $\psi(s,\, \eta)$ has been introduced. After a huge and relatively complicated investigation it has been determined that if we use the new variable η and the functions $\Phi(s,\, \eta)$ and $\bar{h}(s,\, \eta)$ it is not possible to obtain the so called generalized boundary layer equations for the study of ionized gas flow problem. In order to derive these generalized boundary layer equations it is necessary to introduce transformations:

$$s = s\,, \qquad \eta\,(s,\, z) \;=\; \frac{u_e^{b/2}}{K(s)}\; z\,,$$

$$\psi^*\,(s,\, z) = u_e^{1-b/2} K(s)\;\Phi\,[\;\eta\,,\;\kappa\,,\;(f_k),\;(g_k),\;(\Lambda_k)\,]\;\;, \tag{14}$$

$$h(s,\, z) = h_1\;\cdot\;\bar{h}\;[\;\eta\,,\;\kappa\,,\;(f_k),\;(g_k)\,,\;(\Lambda_k)\,]\;\;, \qquad h_e + \frac{u_e^2}{2} = h_1 = const.,$$

$$K(s) = \left(a\,\nu_0 \int\limits_0^s u_e^{b-1}\,ds\right)^{1/2}\;, \qquad a,\; b = const.$$

In here defined so-called similarity transformations, $\eta(s,\, z)$ stands for the newly introduced transversal variable, Φ- for the nondimensional stream function and $\bar{h}$- for the nondimensional enthalpy.

Based on the expressions for the function $K(s)$ and the expressions for the transversal variable $\eta(s,\, z)$, and according to the previously defined relations and characteristics (11), it is easy to obtain the following important relations:

$$\Delta^{**}(s) = \frac{K(s)}{u_e^{b/2}}\,B(s)\,, \qquad B(s) = \int\limits_0^\infty \frac{\partial\Phi}{\partial\eta}\;\left(1\;-\;\frac{\partial\Phi}{\partial\eta}\right)d\eta\,,$$

$$\frac{\Delta^*(s)}{\Delta^{**}(s)} = H = \frac{A(s)}{B(s)}\,, \qquad A(s) = \int\limits_0^\infty \left(\frac{\rho_e}{\rho}\;-\;\frac{\partial\Phi}{\partial\eta}\right)d\eta\,,$$

$$\frac{\Delta_1^*(s)}{\Delta^{**}(s)} = H_1 = \frac{A_1(s)}{B(s)}\,; \qquad A_1(s) = \int\limits_0^\infty \frac{\rho_e}{\rho}\left(1 - \frac{\partial\Phi}{\partial\eta}\right) d\eta\,, \quad (15)$$

$$\frac{f}{B^2} = \frac{a\,u_e'}{u_e^b}\int\limits_0^s u_e^{b-1}\,ds\,, \qquad \zeta(s) = B\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)_{\eta=0}.$$

At that, it is presumed that the values A, A_1 and B are continual functions of the longitudinal variable s.

With the nondimensional functions Φ and $\bar{h}$ (14), from the very beginning, besides the so-called local compressibility parameter $\kappa = f_0$, we introduce three sets of parameters. They are: a set of parameters of the form f_k of Loitsianskii type [2], a set of magnetic parameters g_k [4] and a set of POROSITY PARAMETERS Λ_k [7]. In the considered flow case the mentioned similarity parameters are determined with the expressions:

$$\kappa = f_0(s) = \frac{u_e^2}{2h_1}\,, \qquad f_k(s) = u_e^{k-1}\,u_e^{(k)}\,Z^{**k}, g_k(s) = u_e^{k-1}\,N_\sigma^{(k-1)}\,Z^{**k},$$

$$(16)$$

$$\Lambda_k(s) = -u_e^{k-1}\left(\frac{V_w}{\sqrt{\nu_0}}\right)^{(k-1)} Z^{**k-1/2}, \qquad (\,k = 1,\ 2,\ 3,\ ...\,)$$

Each of the sets of parameters (16) satisfies the recurrent simple differential equation of the form:

$$\frac{u_e}{u_e'}\; f_1\,\frac{d\kappa}{ds} = 2\,\kappa\, f_1 \equiv \theta_0\,,$$

$$\frac{u_e}{u_e'}\; f_1\,\frac{df_k}{ds} = [(k-1)\;f_1 + k\,F_{mp}]\;f_k + f_{k+1} \equiv \theta_k,$$

$$\frac{u_e}{u_e'}\; f_1\,\frac{dg_k}{ds} = [\,(k-1)\;f_1 + k\,F_{mp}\,]\;g_k + g_{k+1} \equiv \gamma_k,$$

$$\frac{u_e}{u_e'}\; f_1\,\frac{d\Lambda_k}{ds} = \{\,(k-1)\;f_1 + [\,(2k-1)\,/2\,]\,F_{mp}\}\;\Lambda_k + \Lambda_{k+1} \equiv \chi_k.$$

$$(\,k = 1,\;2,\;3\,,\;...\,) \tag{17}$$

By application of similarity transformations (14) to the equation system (13), we obtain the so-called generalized equation system of the considered problem of the compressible fluid flow. This equation system is:

$$\frac{\partial}{\partial\eta}\left(Q\,\frac{\partial^2\Phi}{\partial\eta^2}\right) + \frac{aB^2+(2-b)\,f_1}{2\,B^2}\;\Phi\,\frac{\partial^2\Phi}{\partial\eta^2} + \frac{f_1}{B^2}\left[\frac{\rho_e}{\rho} - \left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]$$

$$+\frac{g_1}{B^2}\,\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right) + \underline{\frac{\Lambda_1}{B}\,\frac{\partial^2\Phi}{\partial\eta^2}} =$$

$$= \frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial^2\Phi}{\partial\eta\,\partial f_k} - \frac{\partial\Phi}{\partial f_k}\,\frac{\partial^2\Phi}{\partial\eta^2}\right) + \right.$$

$$\sum_{k=1}^{\infty}\gamma_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial^2\Phi}{\partial\eta\,\partial g_k} - \frac{\partial\Phi}{\partial g_k}\,\frac{\partial^2\Phi}{\partial\eta^2}\right) +$$

$$\left. + \underline{\sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial^2\Phi}{\partial\eta\,\partial\Lambda_k} - \frac{\partial\Phi}{\partial\Lambda_k}\,\frac{\partial^2\Phi}{\partial\eta^2}\right)}\,\right]\;,$$

$$\frac{\partial}{\partial\eta}\left(\frac{Q}{Pr}\frac{\partial\bar{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)\,f_1}{2\,B^2}\,\Phi\,\frac{\partial\bar{h}}{\partial\eta}-\frac{2\kappa f_1}{B^2}\frac{\rho_e}{\rho}\frac{\partial\Phi}{\partial\eta}+$$

$$2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2-\frac{2\kappa g_1}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}+\underline{\frac{\Lambda_1}{B}\frac{\partial\bar{h}}{\partial\eta}}=$$

$$\frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial f_k}-\frac{\partial\Phi}{\partial f_k}\frac{\partial\bar{h}}{\partial\eta}\right)+\sum_{k=1}^{\infty}\gamma_k\right. \tag{18}$$

$$\left.\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial g_k}-\frac{\partial\Phi}{\partial g_k}\frac{\partial\bar{h}}{\partial\eta}\right)+\underline{\sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial\Lambda_k}-\frac{\partial\Phi}{\partial\Lambda_k}\frac{\partial\bar{h}}{\partial\eta}\right)}\right];$$

$$\Phi=0\,,\qquad\frac{\partial\Phi}{\partial\eta}=0\,,\qquad\bar{h}=\bar{h}_w=const.\qquad for\qquad\eta=0\,;$$

$$\frac{\partial\Phi}{\partial\eta}\to 1\,,\qquad\bar{h}\to\bar{h}_e=1-\kappa\qquad for\qquad\eta\to\infty\,.$$

As seen, the velocity distribution on the outer edge of the boundary layer $u_e(s)$appears neither in the obtained equation system (18) nor in the corresponding boundary conditions. The equation system (18), is, in that sense, universal [2], i.e. generalized. This system, therefore, represents the general mathematical model for the considered problem of the ionized gas flow in the boundary layer - the general mathematical model for the flow along the porous wall of the body within the fluid.

Both equations of the system (18) contain a single term on the left hand side which depends on the porosity parameter Λ_1 (11). Each of the equations also contains a sum of terms on the right hand side which are multiplied with the function χ_k (17). For the case of a non-porous wall of the body within the fluid ($v_w = 0$) all the porosity parameters are equal to zero. That is why the mentioned terms are also equal to zero. In that case the equation system (18), obtained in this paper, reduces to the corresponding equation system [3] concerned with the case of the flow along a non-porous wall of the body within the fluid.

Since on the right hand sides of the system (18), three sets of parameters appear the numerical solution of this system is practically possible only when there is relatively few similarity parameters. That is why the solution is sought in the so-called n-parametric approximation. If we presume that all the similarity parameters are equal zero starting from the second one, i.e. if

$$\kappa = f_0 \neq 0\,,\ f_1 = f \neq 0\,,\ g_1 = g \neq 0\,,\ \Lambda_1 = \Lambda \neq 0\,;$$

$$f_2 = f_3 = ... = 0\,,\ g_2 = g_3 = ... = 0\,,\ \Lambda_2 = \Lambda_3 = ... = 0\,,$$

then, according to (17), the system (18) simplifies remarkably and takes the following form:

$$\begin{gathered}
\frac{\partial}{\partial\eta}\left(Q\,\frac{\partial^2\Phi}{\partial\eta^2}\right) + \frac{aB^2+(2-b)\,f_1}{2\,B^2}\;\Phi\,\frac{\partial^2\Phi}{\partial\eta^2} + \frac{f_1}{B^2}\left[\frac{\rho_e}{\rho} - \left(\frac{\partial\Phi}{\partial\eta}\right)^2\right] + \\
\frac{g_1}{B^2}\,\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right) + \underline{\frac{\Lambda_1}{B}\,\frac{\partial^2\Phi}{\partial\eta^2}} = \\
\frac{1}{B^2}\left[2\kappa f_1\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial^2\Phi}{\partial\eta\,\partial\kappa} - \frac{\partial\Phi}{\partial\kappa}\,\frac{\partial^2\Phi}{\partial\eta^2}\right) + \right. \\
F_{mp}f_1\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial^2\Phi}{\partial\eta\,\partial f_1} - \frac{\partial\Phi}{\partial f_1}\,\frac{\partial^2\Phi}{\partial\eta^2}\right) + \\
\left. F_{mp}g_1\left(\frac{\partial\Phi}{\partial\eta}\,\frac{\partial^2\Phi}{\partial\eta\,\partial g_1} - \frac{\partial\Phi}{\partial g_1}\,\frac{\partial^2\Phi}{\partial\eta^2}\right)\right] +
\end{gathered} \tag{19}$$

$$
\underline{+\frac{1}{2}F_{mp}\Lambda_1\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\,\partial\Lambda_1}-\frac{\partial\Phi}{\partial\Lambda_1}\frac{\partial^2\Phi}{\partial\eta^2}\right)}\Bigg],
$$

$$
\frac{\partial}{\partial\eta}\left(\frac{Q}{Pr}\frac{\partial\bar{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)\,f_1}{2\,B^2}\;\Phi\;\frac{\partial\bar{h}}{\partial\eta}-
$$

$$
\frac{2\kappa f_1}{B^2}\frac{\rho_e}{\rho}\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2-
$$

$$
-\frac{2\kappa g_1}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}+\underline{\frac{\Lambda_1}{B}\frac{\partial\bar{h}}{\partial\eta}}=
$$

$$
\frac{1}{B^2}\Bigg[2\kappa f_1\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial\kappa}-\frac{\partial\Phi}{\partial\kappa}\frac{\partial\bar{h}}{\partial\eta}\right)+F_{mp}f_1
$$

$$
\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial f_1}-\frac{\partial\Phi}{\partial f_1}\frac{\partial\bar{h}}{\partial\eta}\right)+F_{mp}g_1\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial g_1}-\frac{\partial\Phi}{\partial g_1}\frac{\partial\bar{h}}{\partial\eta}\right)\Bigg]
$$

$$
\underline{+\frac{1}{2}F_{mp}\Lambda_1\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial\Lambda_1}-\frac{\partial\Phi}{\partial\Lambda_1}\frac{\partial\bar{h}}{\partial\eta}\right)}\Bigg]
$$

If, as it is usually done in the papers on the boundary layer theory, we neglect the derivatives per compressibility, magnetic and porosity parameters ($\partial/\partial\kappa=0$, $\partial/\partial g_1=0$, $\partial/\partial\Lambda_1=0$), the equation system for numerical integration in the four-parametric "three times localized" approximation becomes:

$$\frac{\partial}{\partial \eta}\left(Q\,\frac{\partial^2 \Phi}{\partial \eta^2}\right)+\frac{aB^2+(2-b)\,f}{2\,B^2}\;\Phi\,\frac{\partial^2 \Phi}{\partial \eta^2}+\frac{f}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial \Phi}{\partial \eta}\right)^2\right]+$$

$$\frac{g}{B^2}\,\frac{\rho_e}{\rho}\left(1-\frac{\partial \Phi}{\partial \eta}\right)+\underline{\frac{\Lambda}{B}\,\frac{\partial^2 \Phi}{\partial \eta^2}}=\frac{F_{mp}f}{B^2}\left(\frac{\partial \Phi}{\partial \eta}\,\frac{\partial^2 \Phi}{\partial \eta\,\partial f}-\frac{\partial \Phi}{\partial f}\,\frac{\partial^2 \Phi}{\partial \eta^2}\right)\,,$$

$$\frac{\partial}{\partial \eta}\left(\frac{Q}{Pr}\,\frac{\partial \bar{h}}{\partial \eta}\right)+\frac{aB^2+(2-b)\,f}{2\,B^2}\;\Phi\,\frac{\partial \bar{h}}{\partial \eta}-\frac{2\kappa f}{B^2}\,\frac{\rho_e}{\rho}\,\frac{\partial \Phi}{\partial \eta}+$$

$$2\kappa Q\left(\frac{\partial^2 \Phi}{\partial \eta^2}\right)^2-\frac{2\kappa g}{B^2}\,\frac{\rho_e}{\rho}\left(1-\frac{\partial \Phi}{\partial \eta}\right)\frac{\partial \Phi}{\partial \eta}+ \tag{20}$$

$$\underline{\frac{\Lambda}{B}\,\frac{\partial \bar{h}}{\partial \eta}}=\frac{F_{mp}f}{B^2}\left(\frac{\partial \Phi}{\partial \eta}\,\frac{\partial \bar{h}}{\partial f}-\frac{\partial \Phi}{\partial f}\,\frac{\partial \bar{h}}{\partial \eta}\right);$$

$$\Phi=0\,,\qquad \frac{\partial \Phi}{\partial \eta}=0\,,\qquad \bar{h}=\bar{h}_w=const.\qquad for\qquad \eta=0\,,$$

$$\frac{\partial \Phi}{\partial \eta}\to 1\,,\qquad \bar{h}\to\bar{h}_e=1-\kappa\qquad for\qquad \eta\to\infty\,.$$

4 Numerical solution of the equation system and the obtained results

The obtained system of differential partial equations of the third order (20) has been numerically solved using the finite difference method. Applying the usual transformation

$$\frac{u}{u_e} = \frac{\partial \Phi}{\partial \eta} = \varphi = \varphi\left(\eta\,,\ \kappa\,,\ f,\ g,\ \Lambda\right) \tag{21}$$

the order of the momentum equation has been decreased.

According to the known scheme of the finite differences [6], some derivatives of the variables are replaced with the corresponding finite difference relations. This way the solution of the obtained differential equations (20) has been brought down to the solution of the corresponding system of algebraic equations. This system, as with similar problems of the boundary layer flow [3, 8], is to be solved using an iterative procedure.

For the numerical solution of the obtained equation system it is necessary, from the very beginning, to determine analytic forms of distributions of certain physical values which are a part of these equations. It is also necessary to determine certain numerical values. Since the primary goal of this study is to solve the obtained generalized equation (20), approximate relations are adopted for the function Q and the density ratio ρ_e/ρ [3]:

$$Q = Q\left(\bar{h}\right) \approx \left(\frac{\bar{h}_w}{\bar{h}}\right)^{1/3}; \quad \frac{\rho_e}{\rho} \approx \frac{\bar{h}}{1-\kappa}\,. \tag{22}$$

Obtaining the exact laws on the distribution of these values demands a detailed investigation of the considered problem using the corresponding thermodynamic tables. Prandtl number is constant here, and the value of this number for air is held at $Pr = 0.712$. The constants a and b are considered to have their usual values [2, 8]: $a = 0.4408$; $b = 5.7140$.

The concrete numerical solution of the system (20), i.e. of the corresponding algebraic equation system, has been obtained by means of the written program in FORTRAN program language, the basis of which is the program used in the paper [6].

By localization per the compressibility, porosity and magnetic parameters, these parameters have got the role of constant parameters. Therefore, the equation system (20) has been solved by the usual procedure [2, 6], starting from the value $f = 0.00$ (flat plate) for the in advance given values of the parameters κ, g and Λ. Numerical solutions of the equation system (20) are obtained and presented in the form of adequate tables. For example, Table 1 represents the solution of the system (20) at the cross-section of the boundary layer which is defined with $f = 0.10$ and for the values of the magnetic and porosity parameters $g = 0.02$, $\Lambda = 0.04$. This paper gives only some of the results and they are in the form of suitable diagrams. The diagrams presented are for: the nondimensional velocity u/u_e (Fig. 1), the nondimensional enthalpy $\bar{h}$ (Fig. 2), the value B (Fig. 3), the nondimensional friction function ζ (Fig. 4), the characteristic function F_{mp} (Fig. 5) and the nondimensional function ζ for different values of the porosity parameter Λ (Fig. 6). Finally, Fig. 7 shows the distribution of the nondimensional velocity u/u_e for different values of the porosity parameter Λ.

5 Conclusions

- Based on the shown diagrams we can draw a general conclusion that the profiles of the obtained solutions of the boundary layer equations (20), according to their behavior, according to their behavior, are very much alike to the other similar problems of compressible fluid flow [4].
- According to the shown diagram (Fig.1) and others that are not shown, we can conclude that the nondimensional velocity u/u_e of the considered flow problem at different cross-sections very quickly converges towards unity.
- The magnetic parameter g has a great influence on the boundary layer values B, ζ and F_{mp} (Fig.3 - Fig.5).
- The diagram (Fig.6) shows that the porosity parameter Λ has a great influence on the friction function ζ, and hence the boundary layer separation point.

Finally, we can point out that we would benefit much if we numerically solved the boundary layer equations of the considered problem without localization. This means that the equation system (19) should be solved without localization per parameters κ, g and Λ. This way we would get more accurate quantitative results. Furthermore, we can also realize the influence of the porosity parameter on the nondimensional enthalpy, which has an important role in the dissociated gas flow in the boundary layer. Of course, it is of an interest to find out more accurate laws on distribution of the physical values (22), which are a part of the boundary layer equations of the considered problem.

We can also conclude that the application of the General Similarity Method to the compressible fluid flow is associated with certain difficulties. They are usually of mathematical nature.

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Table 1. Solution of the ionized gas boundary layer equations

$f = 0.1000000\,D+\,00$	$B = 0.6221930$	$\Delta\,f = 0.1000000\,D\,-\,02$
$A = -0.5514339$	$H = -0.8862746$	$B^2 = 0.3871241$
$f\ /\ B^2 = 0.2583151$	$H_1 = 0.7852932$	$\partial^2\Phi\ /\ \partial\eta^2 = 0.6512277$
$F_{mp} = 0.4762197$		$\zeta = 0.4051883$
$\kappa = 0.0200000$	$\Lambda = 0.0400000$	$g = 0.0200000$

M	η	u/u_e	Φ	$\bar{h}$	Q
1	0.0000000	0.0000000	0.0000000	0.0152000	1.0000000
9	0.4000000	0.1231830	0.0257073	0.0847194	0.5640102
17	0.8000000	0.2644884	0.1026390	0.1806311	0.4382114
25	1.2000000	0.4132161	0.2382488	0.2908294	0.3738825
33	1.6000000	0.5532208	0.4320185	0.4066144	0.3343646
41	2.0000000	0.6751754	0.6783965	0.5203691	0.3079716
49	2.4000000	0.7748022	0.9691662	0.6255981	0.2896339
57	2.8000000	0.8514006	1.2951567	0.7174514	0.2767050
65	3.2000000	0.9067865	1.6474511	0.7931364	0.2676077
73	3.6000001	0.9443918	2.0182121	0.8519883	0.2612983
81	4.0000001	0.9683563	2.4011465	0.8951648	0.2570279
89	4.4000001	0.9827064	2.7916183	0.9250570	0.2542290
97	4.8000001	0.9908021	3.1864817	0.9446004	0.2524634
105	5.2000001	0.9951205	3.5837601	0.9566797	0.2513964
113	5.6000001	0.9973077	3.9822969	0.9637467	0.2507804
121	6.0000001	0.9983638	4.3814574	0.9676651	0.2504414
129	6.4000001	0.9988518	4.7809133	0.9697263	0.2502638

M	η	u/u_e	Φ	$\bar{h}$	Q
137	6.8000001	0.9990683	5.1805033	0.9707560	0.2501753
145	7.2000001	0.9991606	5.5801517	0.9712449	0.2501333
153	7.6000001	0.9991986	5.9798247	0.9714656	0.2501144
161	8.0000001	0.9992137	6.3795077	0.9715604	0.2501063
169	8.4000001	0.9992194	6.7791945	0.9715992	0.2501029
177	8.8000001	0.9992215	7.1788827	0.9716144	0.2501016
185	9.2000001	0.9992223	7.5785715	0.9716200	0.2501012
193	9.6000001	0.9992225	7.9782605	0.9716220	0.2501010
201	10.0000001	0.9992226	8.3779495	0.9716227	0.2501009
209	10.4000002	0.9992227	8.7776386	0.9716230	0.2501009
217	10.8000002	0.9992227	9.1773277	0.9716232	0.2501009
225	11.2000002	0.9992227	9.5770167	0.9716234	0.2501009
233	11.6000002	0.9992228	9.9767058	0.9716236	0.2501008
241	12.0000002	0.9992228	10.3763950	0.9716240	0.2501008
249	12.4000002	0.9992229	10.7760841	0.9716244	0.2501008
257	12.8000002	0.9992230	11.1757733	0.9716250	0.2501007
265	13.2000002	0.9992231	11.5754625	0.9716258	0.2501007
273	13.6000002	0.9992233	11.9751518	0.9716269	0.2501006
281	14.0000002	0.9992236	12.3748412	0.9716284	0.2501004
289	14.4000002	0.9992239	12.7745307	0.9716304	0.2501003
297	14.8000002	0.9992243	13.1742203	0.9716330	0.2501000
305	15.2000002	0.9992249	13.5739101	0.9716364	0.2500998
313	15.6000002	0.9992256	13.9736002	0.9716408	0.2500994
321	16.0000002	0.9992266	14.3732907	0.9716466	0.2500989
329	16.4000002	0.9992278	14.7729816	0.9716541	0.2500982
337	16.8000003	0.9992294	15.1726730	0.9716638	0.2500974
345	17.2000003	0.9992315	15.5723652	0.9716763	0.2500963
353	17.6000003	0.9992342	15.9720583	0.9716924	0.2500949
361	18.0000003	0.9992376	16.3717526	0.9717132	0.2500932
369	18.4000003	0.9992420	16.7714485	0.9717402	0.2500908
377	18.8000003	0.9992477	17.1711464	0.9717753	0.2500878
385	19.2000003	0.9992551	17.5708469	0.9718217	0.2500839
393	19.6000003	0.9992652	17.9705508	0.9719055	0.2500767
401	20.0000003	1.0000000	18.3702924	0.9800000	0.2493862

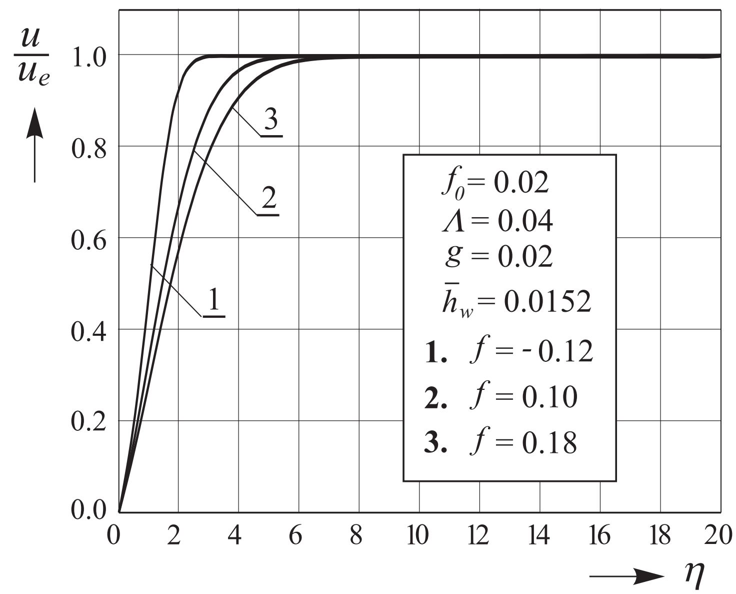

Figure 1: Diagram of the nondimensional velocity u/u_e

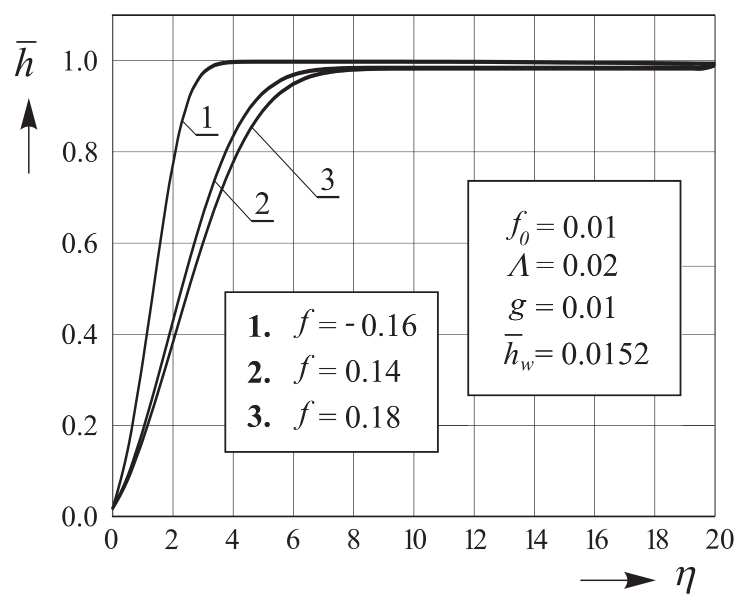

Figure 2: Diagram of the nondimensional enthalpy $\bar{h}$

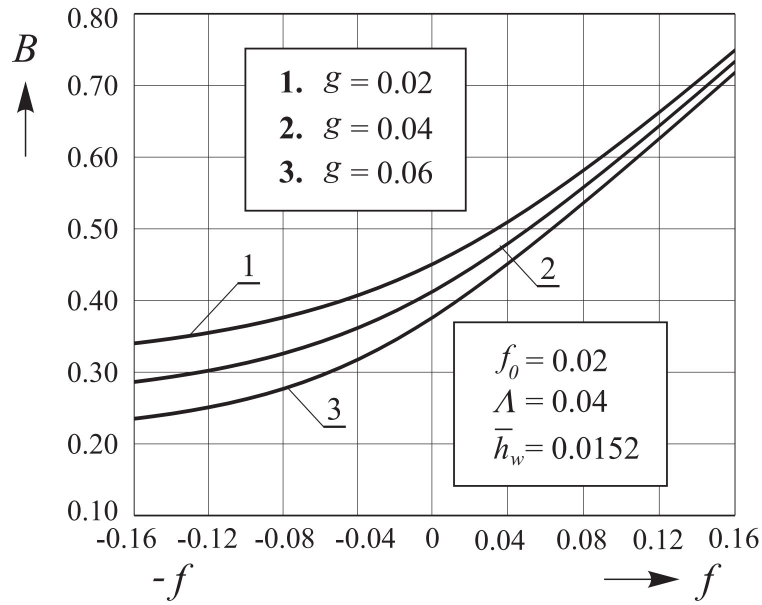

Figure 3: Graphic of the characteristic B of the boundary layer

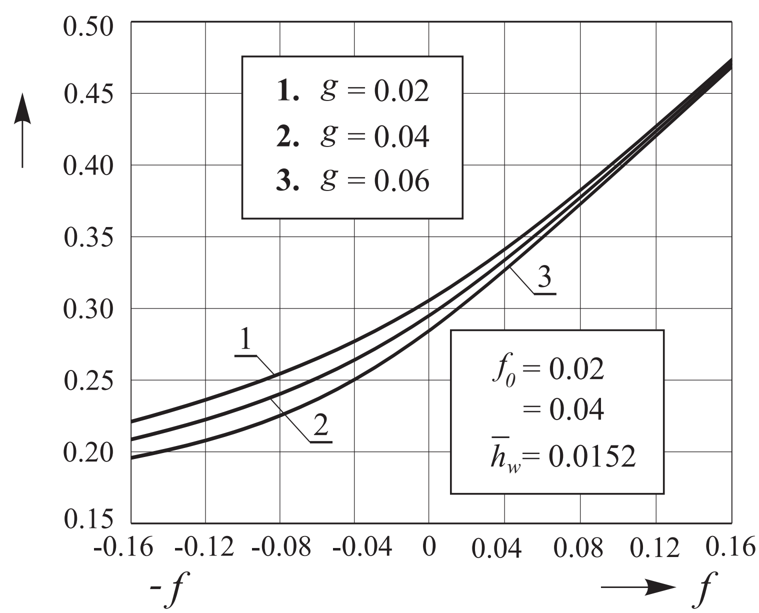

Figure 4: Distribution of the nondimensional friction function ζ

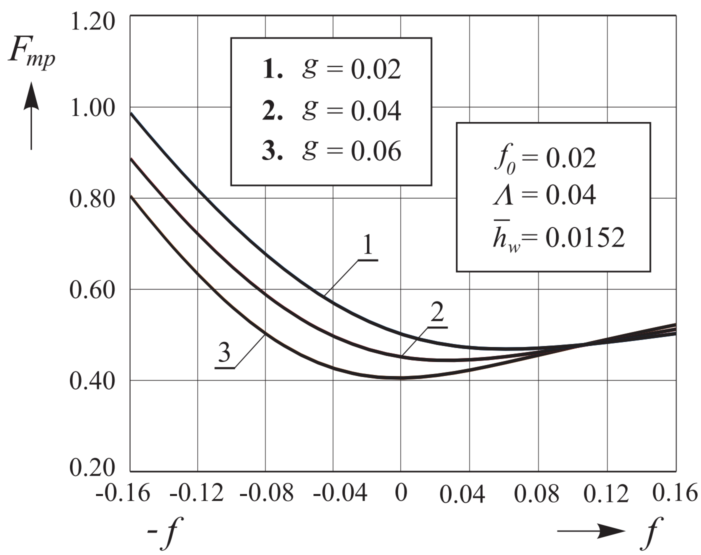

Figure 5: Characteristic boundary layer function F_{mp}

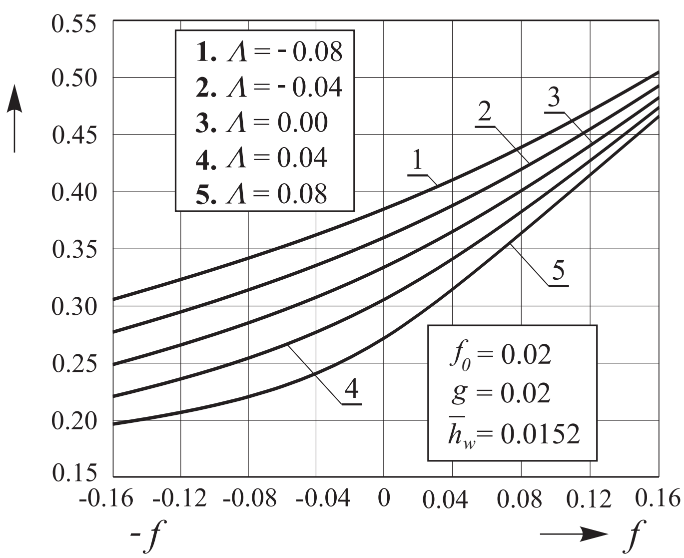

Figure 6: Distribution of the nondimensional friction function ζ for different values of the parameter Λ

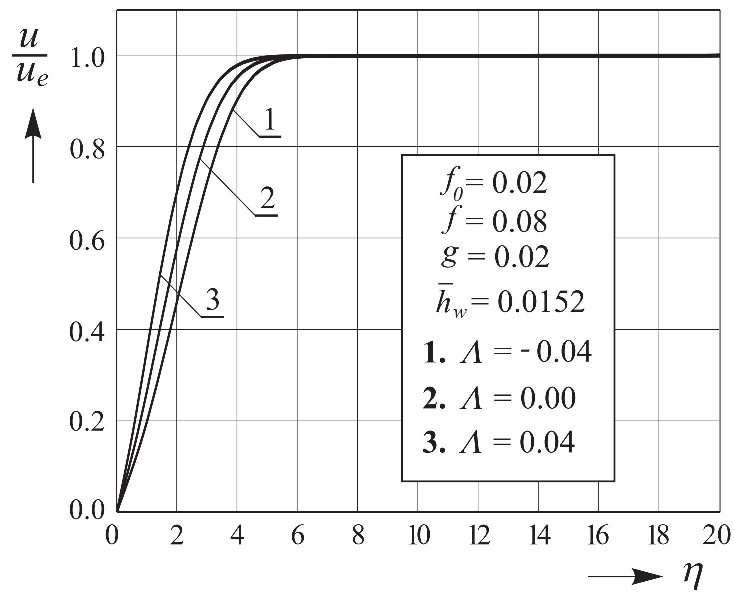

Figure 7: Distribution of the nondimensional velocity u/u_e for different values of the parameter Λ for different values

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Granični sloj jonizovanog gasa na poroznom zidu opstrujavanog tela unutar elektroprovodnog fluida

UDK 532.526; 533.15

U radu se istražuje strujanje jonizovanog gasa u graničnom sloju, pri čemu je kontura opstrujavanog tela porozna. Jonizovani gas je izložen dejstvu spoljašnjeg magnetnog polja jačine $B_m = B_m(x)$ koje je upravno na konturu opstrujavanog tela. Pri tome je pretpostavljeno da je elektroprovodnost jonizovanog gasa funkcija samo podužne koordinate, tj. da je $\sigma = \sigma(x)$. Pomoću pogodnih transformacija, polazne jednačine graničnog sloja su dovedene na uopšteni oblik. Dobijene uopštene jednačine su rešene u četvoro-parametarskom lokalizovanom približenju. Na bazi dobijenih numeričkih rešenja nacrtani su dijagrami važnijih fizičkih veličina i karakteristika graničnog sloja. Izvedeni su i odgovarajući zaključci.