

Drag of a growing bubble at rectilinear accelerated ascension in pure liquids and binary solutions

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Abstract

The problem of predicting the drag coefficient of a growing bubble at rectilinear accelerated ascension in uniformly superheated pure liquids and in binary solutions with a non-volatile solute at large Reynolds and Peclet numbers is discussed. In the case of pure liquids, the general solution for the drag coefficient of an accelerated growing bubble from its inception at the critical radius and through the surface-tension-, inertia-, and heat-diffusion-controlled regimes is established, as well as some necessary adaptations in the case of binary solutions with a non-volatile solute. Two particular limiting regimes in the case of pure liquids, inertia-controlled and heat-diffusion-controlled regimes, respectively, are analyzed in details, with satisfactory results.

Keywords: *Growing bubble, accelerated ascension, pure liquid, drag*

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Nomenclature

R	bubble radius [m]
R_c	critical bubble radius expressed by equation (6) [m]
R^*	dimensionless bubble radius defined by equation (3)
t	time [s]
t^*	dimensionless time defined by equation (2)
t_d	bubble growth delay period [s]
t_u	upper limit of time during the period concerned [s]
t_r^*	dimensionless time for estimating T_r
A^*	parameter defined by equation (4) [ms^{-1}]
B^*	parameter defined by equation (5) [$ms^{-1/2}$]
c_1	specific heat of pure liquid or of solvent [$Jkg^{-1}K^{-1}$]
h_{fg}	latent heat of vaporization [Jkg^{-1}]
p	vapor pressure of pure liquid or of solution [Pa]
p_∞	pressure in pure liquid or solution far from the bubble [Pa]
T	temperature [$^{\circ}C$]
T_e	equilibrium temperature of solution corresponding to p_∞ [$^{\circ}C$]
T_i	bubble wall temperature [$^{\circ}C$]
T_r	reference temperature at which ρ_v is evaluated [$^{\circ}C$]
T_s	saturation temperature [$^{\circ}C$]
T_∞	temperature of pure liquid or of solution far from the bubble

Greek symbols

ρ_1	density of pure liquid or of solvent [kgm^{-3}]
ρ_v	density of pure liquid vapor or of solvent vapor [kgm^{-3}]
α_1	thermal diffusivity of pure liquid or of solvent [m^2s^{-1}]
σ	surface tension [Nm^{-1}]
ω	mass fraction of solute in solution
ω_∞	mass fraction of solute in solution far from the bubble, dimensionless

1 Introduction

The role of gas bubbles is an important one in many physical operations and chemical processes involving interaction between liquid and gaseous systems. In some operations the transfer of mass between gas bubbles and the continuous liquid phase is the very essence of the operation. In others the kinematical behavior of gas bubbles is bound up with secondary aspects of the operation. Examples of the former include rectification, absorption and stripping in bubble-cap and perforated-plate contacting devices and chemical reactions between liquid and gaseous reactants. Boiling is an operation in which the formation and movement of gas bubbles appear as a secondary aspect of the heat-transfer operation.

A knowledge of the heat and mass transfer associated with a moving bubble (or droplet) is of importance to a variety of industrial processes. Boussinesque [1] has been the first to obtain a solution for the heat transfer rate from a fluid sphere of uniform and constant surface temperature, moving at a constant speed in another fluid of infinite extent. Ruckenstein [2] studied the heat transfer between a vapor bubble in motion and the liquid from which the bubble was generated. Amongst the relatively small number of papers on deforming bubbles in movement, the most often is used an impulsively started motion in a quiescent liquid initially at rest. So in [13] for instance, the simultaneous solutions of the unsteady boundary-layer equations for the both outside and inside flows of a radially deforming bubble in an impulsive ascension were obtained by using the method of perturbations. Generally speaking, the viscous effect is small when the Reynolds number exceeds two or three hundred. It may be of interest to note that if the hydrodynamic boundary layers are developing simultaneously with the thermal boundary layer, the inviscid approximation is even better [3]. On the other hand, under the condition of large Peclet numbers, the thermal boundary layers are very thin except the region close to the rear stagnation. As the present paper concerns the unsteady flow around a deforming bubble with particular emphasis on the drag coefficient, we are going to use hereafter the inviscid approximation [15] for the external fluid flow.

2 An improved bubble growth equation for pure liquids

Bubble growth in superheated fluids is of key interest in boiling phenomena in general and in flash evaporation in particular. Most of the large amount of research on such bubble growth has been conducted for pure liquids (see reviews in refs [4], [5], [6] for instance). The past research shows that bubble growth in superheated liquids can be characterized as progressing in three consecutive regimes: at first, just when the bubble has nucleated, surface tension is dominant, impeding significant growth for a certain “delay period”. After the nucleus grew somewhat, say doubled its diameter, inertia forces become dominant and the bubble grows primarily due to the difference between the vapor pressure inside the bubble (p_v) and the exterior pressure (p_∞). As the bubble grows further and its wall temperature consequently drops, causing an increased temperature difference between the surrounding liquid and the bubble wall, its growth rate becomes dominated by heat transfer from the surrounding liquid which causes addition of vapor to the bubble by evaporation at the interface.

In an experimentally-validated numerical study, Miyatake and Tanaka [7], [8] have developed an improved equation for bubble growth in pure liquids, which reflects reality more closely, by including the following effects:

(a) The initial, surface-tension-controlled bubble growth regime, which occurs immediately after the nucleation of a bubble, and which causes an initial lag in bubble growth (the “delay period”, t_d , was added to the inertia- and heat-transfer-controlled regimes taken into account in the previous solution by Mikic et al. [9]. Consequently, the new equation now covers the entire bubble life span.

(b) Consistently with improvement (a) above, growth was considered to start when the bubble radius was just larger than the critical radius R_c (at which the bubble nucleus is sustained as a result of equilibrium between surface tension and the pressure difference across the bubble wall), specifically here at $R(0) = 1,0001R_c$.

(c) The correct, non-linear relationship between the vapor pressure and temperature, obtained from the steam tables, was used, eliminating the linear relationship assumption used in [9].

(d) In addition to these assumptions, we are going to use also the following one taking into account of the bubble movement: since only large Reynolds numbers are of interest, the inviscid approximation can be used. This is particularly true if the internal circulation is vigorous. Accordingly, the external flow can be considered as irrotational. As it was said above, the thermal boundary layers are thin for large Peclet numbers.

The new general equation for bubble growth in pure liquids [7], between a dimensionless bubble radius (R^*) and dimensionless time (t^*), which was shown in ref. [10] to represent experimental data very well, is

$$R^* = \frac{2}{3} \left\{ 1 + \frac{1}{3} t^* \exp [-(t^* + 1)^{1/2}] \right\} [(t^* + 1)^{3/2} - (t^*)^{3/2} - 1] \quad (1)$$

where

$$t^* = (A^*/B^*)^2 \left\{ t - t_d \left[1 - e^{-(t/t_d)^2} \right] \right\} \quad (2)$$

and

$$R^* = \frac{A^*}{(B^*)^2} (R - R_c) \quad (3)$$

$$A^* = \left[\frac{2 \Delta p_0}{3 (\rho_l)_{T_\infty}} \right]^{1/2} \quad (4)$$

$$B^* = \left(\frac{12}{\pi} \right)^{1/2} \left[\frac{\Delta T_s}{(\rho_v)_{T_r}} \right] \left(\frac{\alpha_l^{1/2} c_l \rho_l}{h_{fg}} \right)_{T_s} \quad (5)$$

where

$$R_c = 2(\sigma)_{T_\infty} \Delta p_0 \quad (6)$$

$$t_d = 6R_c A^*. \quad (7)$$

In the above equations the properties are based on T_∞ , T_s and T_r of the liquid. T_r is a reference temperature at which the temperature-sensitive saturation density (ρ_v) of the vapor is evaluated, and is defined as

$$T_r = T_s + (T_\infty - T_s) \left\{ 1 - 2(t_r^*)^{1/2} [(t_r^* + 1)^{1/2} - (t_r^*)^{1/2}] \right\} \quad (8)$$

where

$$t_r^* = \frac{1}{2} (A^*/B^*)^2 (t_u - t_d) \quad (9)$$

and where t_u is the upper limit of the time period during which the bubble growth is investigated by these equations.

The initial pressure difference between the bubble interior and exterior is expressed by

$$\Delta p_0 = (\Delta p)_{T_\infty} - p_\infty \quad (10)$$

in which p is the vapor pressure of the liquid, and the subscript is indicating the temperature T_∞ at which the vapor pressure is evaluated.

The agreement between this general bubble growth equation (equations (1)-(5)) and the experimentally-validated numerical solution of Miyatake and Tanaka [7], [8] is excellent, and the capability of the equation to predict bubble growth from its inception at the critical radius and through the surface-tension-, inertia-, and heat-transfer-controlled regimes is clearly demonstrated.

3 Drag coefficient of a growing bubble at rectilinear accelerated ascension

Consider a spherical bubble of the growing radius R after the relation (1) at a rectilinear ascension with the constant acceleration \ddot{z}_0 in an incompressible fluid initially at rest (figure 1). Let p_v and T_v represent the vapor pressure and the vapor temperature inside a growing bubble, respectively.

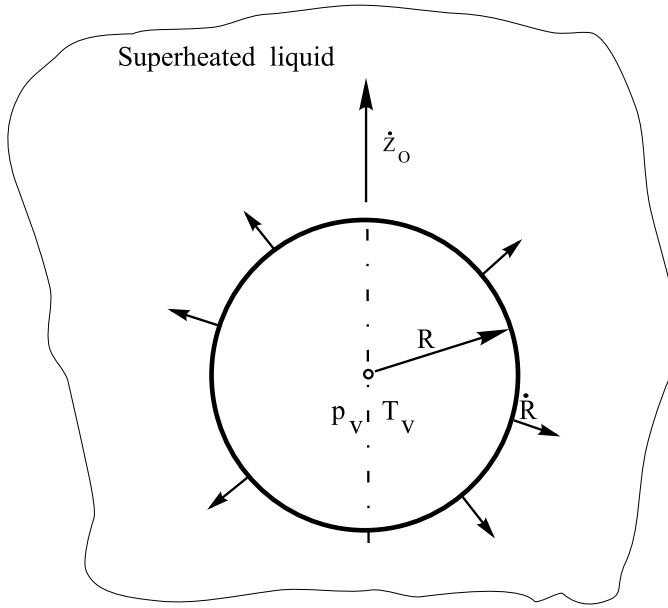


Figure 1: Growth of a spherical bubble

Due to the instationarity, the total drag force of a growing bubble at an accelerated ascension is different from zero even in an inviscid fluid and may be evaluated [15] as

$$D = \frac{4}{3}\pi\rho_l g R^3 - \frac{2}{3}\pi\rho_l R^3 \ddot{z}_0 - 2\pi\rho_l R^2 \dot{R}\dot{z}_0. \quad (11)$$

It is customary to introduce the drag coefficient C_z defined by

$$C_z = \frac{D}{\frac{1}{2}\rho_l \pi R^2 \dot{z}_0^2},$$

i.e. from Eq.(11), one finds:

$$C_z = \frac{8}{3} \frac{g}{\dot{z}_0^2} \frac{R}{t^2} - \frac{4}{3} \frac{R}{\dot{z}_0} \frac{1}{t^2} - \frac{4}{\dot{z}_0} \frac{\dot{R}}{t}. \quad (12)$$

Hence, inserting R obtained from (3):

$$R = R_c + \frac{(B^*)^2}{A^*} R^* \quad , \quad (13)$$

with R^* given by (1), we obtain:

$$\begin{aligned} C_z &= \frac{4}{3g} \frac{g}{\ddot{z}_0} \left(2 \frac{g}{\ddot{z}_0} - 1 \right) \frac{1}{t^2} \left\{ R_c + \frac{2(B^*)^2}{3A^*} \left[1 + \frac{1}{3} t^* e^{-(1+t^*)^{1/2}} \right] \right\} \times \\ &\times \left[(1+t^*)^{3/2} - (t^*)^{3/2} - 1 \right] \left\{ - \frac{4}{g} \frac{g}{\ddot{z}_0} \left\{ \frac{2}{9} \left[1 - \frac{1}{2} \frac{t^*}{(1+t^*)^{1/2}} \right] \right\} \times \right. \\ &\times \left. \left[(1+t^*)^{3/2} - (t^*)^{3/2} - 1 \right] e^{-(1+t^*)^{1/2}} + \left[1 + \frac{1}{3} t^* e^{-(1+t^*)^{1/2}} \right] \right\} \times \\ &\times \left. \left[(1+t^*)^{1/2} - (t^*)^{1/2} \right] \right\} \otimes \frac{A^*}{t} \left[1 - 2 \frac{t}{t_d} e^{-(t/t_d)^2} \right] , \quad (14) \end{aligned}$$

where $t = t(t^*)$ is given implicitly by (2) and has to be determined numerically in every particular case.

4 Drag coefficient of a growing bubble in the two limiting cases

Examination of the bubble growth relations by Miyatake and Tanaka equations (1)-(5), shows that

$$R_{t \rightarrow 0} = A^* t , \quad (15)$$

where A^* is the dominant coefficient in the inertia-controlled regime depending on Δp_0 , and

$$R_{t \rightarrow \infty} = B^* t^{1/2} , \quad (16)$$

where B^* depends on $\Delta T_s = T_\infty - T_s$ and is the dominant coefficient in the heat-transfer-controlled regime. Let us calculate now the drag coefficient (12) for these two limiting cases.

4.1 Inertia-controlled regime

Bubble growth rates controlled by inertia forces are applicable in the range of a relatively low pressure and high Jakob numbers. The bubble grows due to the evaporation of the liquid at the vapor-liquid interface. The heat required for the evaporation is supplied from the superheated liquid. The driving temperature potential between the liquid and vapor is presented as $(T_\infty - T_v)$. For the cases when the vapor density is very small, relatively small evaporation will cause substantial bubble growth. So here very little temperature difference $(T_\infty - T_v)$ is needed and to this limiting case $T_v \rightarrow T_\infty$ corresponds the well known Rayleigh solution [11] for the bubble growth controlled by the inertia forces, as (15).

Consequently, after replacing (15) into (12) and some simple transformations, it results that:

$$\frac{(B^*)^2 g}{(A^*)^3} C_z = \frac{8}{3} \left(\frac{g}{\ddot{z}_0} - 2 \right) \frac{g}{\ddot{z}_0} \frac{1}{t^*} \quad . \quad (17)$$

4.2 Heat-transfer-controlled regime

Growth rates for heat-diffusion-controlled bubble growth, corresponding to (16), was previously studied by Plesset and Zwick [12]. So the drag coefficient in this case will be calculated by inserting (16) into (12):

$$\frac{(B^*)^2 g}{(A^*)^3} C_z = \frac{2}{3} \left(4 \frac{g}{\ddot{z}_0} - 5 \right) \frac{g}{\ddot{z}_0} \frac{1}{t^* \sqrt{t^*}} \quad . \quad (18)$$

5 Concluding remarks

Figures 2 and 3 show the variation of the drag coefficient with dimensionless time (i.e. t^*) in two limiting cases for some arbitrarily assigned values of the acceleration parameter $(g/\ddot{z}_0) = 4; 5; 6$. Another values

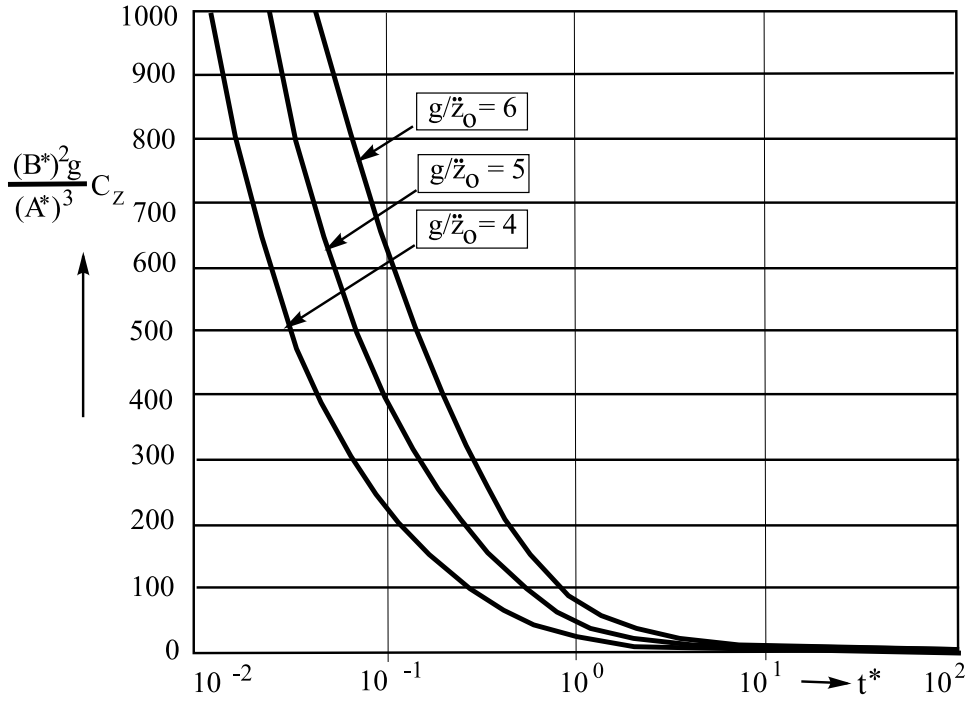


Figure 2: Evolution of the drag coefficient of a bubble for different accelerations in inertia-controlled regime

may be used as well but the main features of the finding here would not be affected. Besides the hypothesis of the irrotational external fluid flow, all the results presented in Figures 2 and 3 seem to be acceptable, the shapes of all curves being compatible to those in refs [11], [12], [14], [15]. Due to the buoyancy effect, the drag coefficient decreases with time for all values of the acceleration dimensionless parameter g/\ddot{z}_0 , as well as with the augmentation of acceleration at each instant of time t^* . Of course, thermal characteristics of the growing vapor bubble are implicitly present through different dimensionless parameters such as t^* and $(B^*)^2 g / (A^*)^3$.

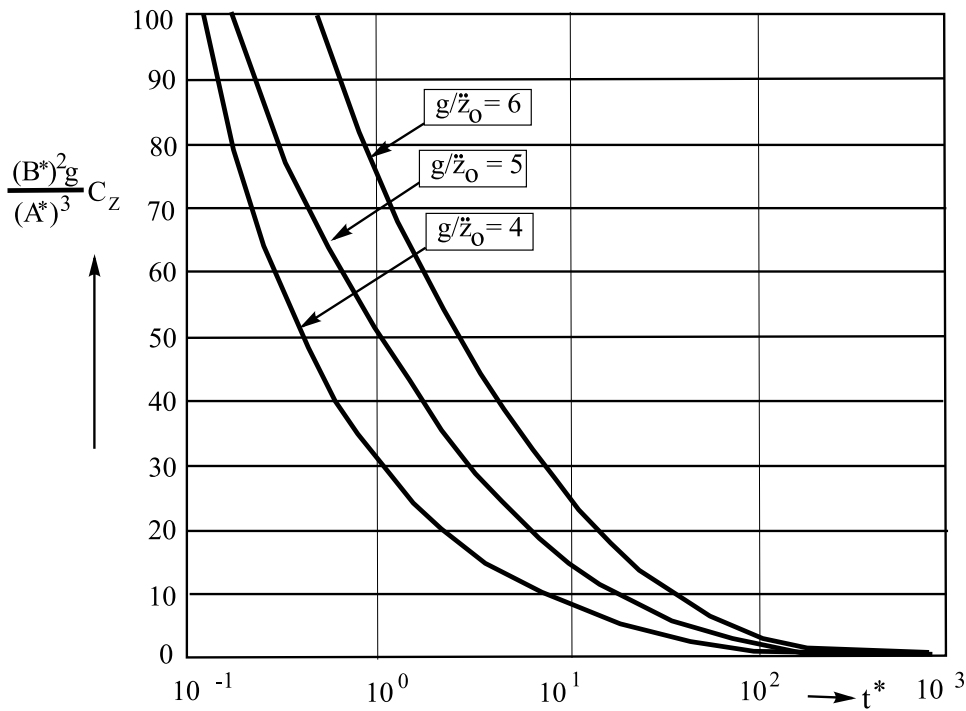


Figure 3: Evolution of the drag coefficient of a bubble for different accelerations in heat-transfer-controlled regime

6 Perspectives: on the drag of a growing bubble in a binary solution with a non-volatile solute

First of all, we will analyze numerically the equation (14) based on an improved bubble growth equation (1)-(5) which covers the entire bubble life span and then we will make comparison with results previously obtained [15] for the analogous problem but using a simplified bubble growth relation given in ref.[9]. We will try also to include into the computation of the drag coefficient of a growing vapor bubble the influence of the fluid viscosity similarly as in [16].

In the contrast to pure liquids where most of the large amount of research on bubble growth has been conducted, very little is known about bubble growth in superheated solutions with a non-volatile so-

lute, a topic of both fundamental and practical importance. It has many interesting applications including a wide variety of separation processes such as water desalination and energy conversion processes such as nuclear reactor safety, geothermal power generation or ocean-thermal energy conversion.

As Miyatake and Tanaka demonstrated [10], the bubble growth equation for a superheated pure liquid, equation (1)-(5), may also be applicable for a superheated binary solution containing a non-volatile solute, after the following adaptations:

- the superheat $\Delta T_s = T_\infty - T_s$ is replaced by that defined by equation $\Delta T_s = T_\infty - T_e$ where T_e is the equilibrium temperature satisfying the relation $p_\infty = (p)_{T_e, \omega_\infty}$ in which p is the vapor pressure of the solution, and the subscripts are indicating the temperature T and mass fraction ω at which the vapor pressure is evaluated;
- the initial pressure difference Δp_0 between the vapor interior and exterior defined by equation (10) is replaced by that defined by equation $\Delta p_0 = (p)_{T_\infty, \omega_\infty} - p_\infty$ where the mass fraction (ω_∞) of the solute has, by definition, no effect on bubble growth in the inertia-controlled regime, and
- the physical properties of the liquid are taken as those of the solvent.

Or, in contrast to the above-discussed bubble growth in pure liquids, bubble growth in uniformly superheated binary solutions with a non-volatile solute is determined not only by the temperature T_∞ and pressure p_∞ of the solution, but also by the mass fraction ω_∞ of the solute. It was found that the concentration ω_∞ has a significant effect on the bubble growth rate when the far-field solution pressure p_∞ is held constant, as it is said above.

Taking into account of all these adaptations, we intend to study numerically the drag coefficient (14) of a growing bubble at ascension in some binary solutions with a non-volatile solute (uniformly superheated aqueous NaCl solutions at different solute mass fractions, for instance).

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U ovom radu se analizira problem odredjivanja koeficijenta otpora pri translatorsnom ubrzanom usponu rastućeg parnog mehura u uniformno-pregrejanom čistoj tecnosti ili u binarnom rastvoru sa neisparljivim primesama pri velikim brojevima Reynolds-a i Peclet-a. U slučaju čiste tecnosti, nađeno je opšte rešenje za koeficijent otpora rastućeg sfernog mehura počev od njegovog nastanka sa kritičnim radijusom preko svih sledećih faza rasta: faze kontrolisane interfacijalnim naponom, faze kontrolisane inercijom i, najzad, faze kontrolisane toplotnom difuzijom. Takodje su definisane neophodne adaptacije predloženog opšteg rešenja za slučaj kretanja rastućeg mehura kroz binarne rastvore sa neisparljivim primesama. Konačno, detaljno su analizirana dva granična režima rasta mehura: kontrolisanog inercijom i kontrolisanog toplotnom difuzijom, sa zadovoljavajućim rezultatima.