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MHD Couette flow with heat transfer between two horizontal plates in the presence of a uniform transverse magnetic field

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Abstract

The problem of an unsteady two-dimensional flow of a viscous incompressible and electrically conducting fluid between two parallel plates in the presence of a uniform transverse magnetic field has been analyzed, when in case-I the plates are at different temperatures and in case-II the upper plate is considered to move with constant velocity where as the lower plate is adiabatic. Fluid velocities and temperatures are obtained and plotted graphically.

1 Introduction

Borkakati and Bharali [1] have discussed the flow and heat transfer between two horizontal parallel plates, where the lower plate is a stretching sheet and the upper one is a porous solid plate in the presence of a transverse magnetic field. The heat transfer in an axisymmetric flow between two parallel porous disks under the effect of a transverse magnetic field is studied by Bharali and Borkakati [2]. Also, they discussed the hydrodynamic flow and heat transfer between two horizontal parallel plates, where the lower one is a stretching sheet and the upper one is a porous solid plate in the presence of a transverse magnetic field [3]. Shih-I-Pai

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[4] studied an unsteady motion of an infinite flat insulated plate sets impulsively into the uniform motion with velocity in its own plane in the presence of a transverse uniform magnetic field.

The objective of the present paper, is to investigate an unsteady flow of an incompressible and electrically conducting fluid between two horizontal parallel plates, one of which is at rest, other moving in its own plane with a velocity u_0 in the presence of a uniform transverse magnetic field is analyzed.

2 Formulation of the problem

The unsteady laminar flow of an incompressible viscous electrically conducting fluid between two horizontal parallel non-conducting plates at a distance $2h$ apart is considered under the action of transverse magnetic field. The flow is assumed to be in the X'-axis along the horizontal direction through the central line of the channel and Y'-axis is normal to it. The plates of the channel are at $y' = \pm h$ and that the relative velocity between the two plates is $2u_0$ and also, there is no pressure gradient in the flow field. The uniform magnetic field B_0 makes an angle θ with X'-axis induced a magnetic field $B(y)$ or the imposed magnetic field makes an angle θ to the free stream velocity [1, 2]. The plate at $y' = -h$ is maintained at temperature T_0, while the other plate $y' = +h$ is kept at temperature $T_1 (T_1 > T_0)$ and the plates are electrically non-conducting. The components of the velocities and the magnetic field are given [4] as follows:

$$u' = \{u, v, w\} = \{u(y,t), 0, 0\},$$

$$B' = \{B_x, B_y, B_z\} = \{\lambda B(y,t), (1-\lambda^2)^{1/2} B_0, 0\}$$

and $p =$ constant, where $\lambda = \cos\theta$ is imposed and t is the time.

In order to derive the governing equations of the problem, we are to assume that the fluid is finitely conducting and the viscous dissipation and the Joule heat are neglected, and the Hall effect and polarization effect are negligible.

Under the above conditions the governing equations are as follows[5]:

$$\rho \frac{D\vec{u}'}{Dt'} = -\nabla p + \mu \nabla^2 \vec{u}' + \vec{J} \times \vec{B} + \vec{X} \tag{1}$$

and

$$\rho c_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} \ . \tag{2}$$

Here the third term in the right hand side of equation (1) is the magnetic body force and $\vec{J}$ is the current density due to the magnetic field and $\vec{X}$ is the force due to the buoyancy, $\vec{X} = \rho g \beta (T' - T_0)$. Where ρ is the density of the fluid, σ is the electrically conductivity, k is the thermal conductivity, $\upsilon = \frac{\mu}{\rho}$ is the kinematics viscosity, μ is the coefficient of viscosity, c_p is the specific heat at constant pressure and β is the coefficient of thermal expansion.

Using velocity and magnetic field distributions as stated above, the equations (1) and (2) are as followed;

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}(1 - \lambda^2) u' + g\beta (T' - T_0) \tag{3}$$

and

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} \ . \tag{4}$$

We consider here two cases [3]:

(i) *when the plates are maintained at different temperatures;*

(ii) *when the lower plate is adiabatic and the upper plate is maintained at a constant temperature.*

Case(i): *when the plates are at different temperatures*, the boundary conditions are

$$t' > 0 : \quad \begin{cases} u' = u_0, & T' = T_1, \qquad at\ y' = +h, \\ u' = -u_0, & T' = T_0, \qquad at\ y' = -h. \end{cases} \tag{5}$$

Consider the non-dimensional parameters as

$$u = \frac{u'}{u_0}, \quad y = \frac{y'}{h}, \quad t = \frac{t' u_0}{h}, \quad T = \frac{T' - T_0}{T_1 - T_0} \ . \tag{6}$$

Using the conditions (6) in the equations (3) and (4), we get

$$\frac{\partial u}{\partial t} = \frac{1}{\mathrm{Re}} \frac{\partial^2 u}{\partial y^2} - Ha \ \mathrm{Re}(1 - \lambda^2) u + \frac{Gr}{\mathrm{Re}^2} T \tag{7}$$

and

$$\frac{\partial T}{\partial t} = \frac{1}{Pe} \frac{\partial^2 T}{\partial y^2}, \tag{8}$$

where

$Ha = \sigma B_0^2 \upsilon / \rho u_0^2$	magnetic Hartmann number,
$\mathrm{Re} = h u_0 / \upsilon$	Reynolds number,
$Gr = g \beta h^3 (T_1 - T_0) / \upsilon^2$	Grashoff number,
$\alpha = k / \rho c_p$	thermal diffusivity,
$\mathrm{Pr} = \upsilon / \alpha$	Prandtl number and
$Pe = \mathrm{Pr}\mathrm{Re}$	Peclet number.

For the relation (6), the boundary conditions (5) become

$$t > 0 : \quad \begin{cases} u = 1, & T = 1, \qquad at \ y = +1, \\ u = -1, & T = 0, \qquad at \ y = -1. \end{cases} \tag{9}$$

In order to solve equations (7) and (8), we consider

$$u = f(y) e^{-nt} \qquad \text{and} \qquad T = g(y) e^{-nt}, \tag{10}$$

where n is the decay constant.

Substituting (10) in equations (7) and (8), they become

$$f''(y) - \mathrm{Re}\{Ha \ \mathrm{Re}(1 - \lambda^2) - n\} f(y) = -\frac{Gr}{\mathrm{Re}} g(y) \tag{11}$$

and

$$g''(y) + nPe \ g(y) = 0 \ . \tag{12}$$

The corresponding boundary conditions are

$$t > 0 : \quad \begin{cases} f = e^{nt}, & g = e^{nt}, \quad at \;\; y = +1, \\ f = -e^{nt}, & g = 0, \quad\;\; at \;\; y = -1. \end{cases} \tag{13}$$

Solving the equations (11) and (12) with the help of conditions (13), and substituting in the relations (10), we get

$$\begin{aligned} u = & \frac{[2\text{Re}(a_1^2 + a_2^2) - Gr] \sinh a_2 y}{2\text{Re}(a_1^2 + a_2^2) \sinh a_2} - \\ & \frac{Gr \cosh a_2 y}{2\text{Re}(a_1^2 + a_2^2) \cosh a_2} + \frac{Gr \;\; \sin(1+y)a_1}{\text{Re} \; (a_1^2 + a_2^2) \;\; \sin 2a_1} \end{aligned} \tag{14}$$

and

$$T = \frac{\sin(1+y)a_1}{\sin 2a_1}, \tag{15}$$

where $a_1 = (nPe)^{1/2}$ and $a_2 = (\text{Re}\{Ha \; \text{Re}(1 - \lambda^2) - n\})^{1/2}$.

Case(ii): *when the lower plate is adiabatic*, then the boundary conditions are

$$t' > 0 : \quad \begin{cases} u' = u_0, & T' = T_1, \quad\quad at \; y' = +h, \\ u' = -u_0, & \dfrac{\partial T'}{\partial y'} = 0, \quad at \; y' = -h. \end{cases} \tag{16}$$

For the relation (6), the boundary conditions (16) become

$$t > 0 : \quad \begin{cases} u = 1, & T = 1, \quad\quad at \;\; y = +1, \\ u = -1, & \dfrac{\partial T}{\partial y} = 0, \quad at \;\; y = -1, \end{cases} \tag{17}$$

whereas for (10), the corresponding boundary conditions are given by

$$t > 0 : \quad \begin{cases} f = e^{nt}, & g = e^{nt}, \quad\quad at \;\; y = +1, \\ f = -e^{nt}, & \dfrac{\partial g}{\partial y} = 0, \quad at \;\; y = -1. \end{cases} \tag{18}$$

Solving the equations (11) and (12) with the help of (18), and substituting in the relations (10), we get

$$u = \left[\frac{Gr(1-\cos 2a_1)}{2\mathrm{Re}(a_1^2+a_2^2)\cos 2a_1 \sinh a_2} + \frac{1}{\sinh a_2}\right]\sinh a_2 y - \frac{Gr\cosh a_2 y}{2\mathrm{Re}(a_1^2+a_2^2)\cos 2a_1 \cosh a_2} + \frac{Gr\cos(1+y)a_1}{\mathrm{Re}(a_1^2+a_2^2)\cos 2a_1} \tag{19}$$

and

$$T = \frac{\cos(1+y)a_1}{\cos 2a_1}\ . \tag{20}$$

3 Results and discussion

Numerical solutions for case I. by equations (14) and (15) are analyzed for different values of λ, where $\lambda = \cos\theta$ which varies as $\theta = 45^0$, 60^0, 75^0.

The figure 1. shows the nature of the fluid velocity for the variation of θ. The values of the velocity distribution increase with the increase of λ. The velocity distribution increases near the plates and then decreases very slowly at the central portion between the two plates.

The figure 2. is obtained by plotting the temperature distribution against the variable y for different values of Prandtl number Pr, while Pr= 0.71, 1 and 2. The temperature distribution between the plates increases gradually with the increase of Pr. However the values of temperature increase towards the plate $y > 0$ and decreases towards the plate $y < 0$. Also the values of the temperature due to the increase of Pr are very closed - that is why the plotted graphs are touching among the three curves which are drawn by taking the values of Pr= 0.71, 1 and 2.

Numerical solutions for case II by means of the equations (19) and (20) are obtained for different values of λ, where $\lambda = \cos\theta$ which varies again as $\theta = 45^0$, 60^0, 75^0.

The figure 3. shows the fluid velocity and the values of the velocity distribution increase also with the increase of λ. The figure 4. is made by plotting the temperature distribution against the variable y for different values of Prandtl number Pr, while the values of Pr varies as the same

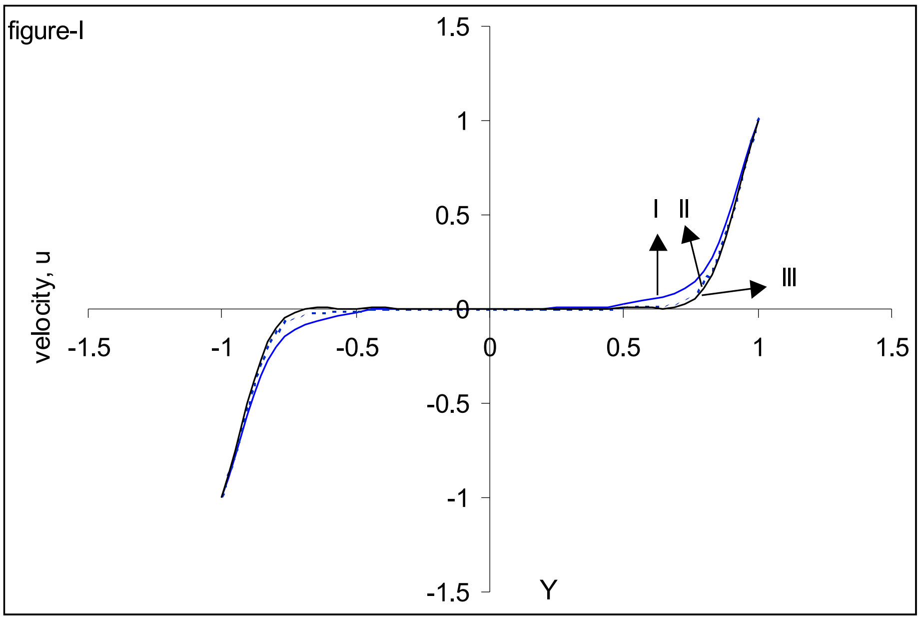

Figure 1: Velocity, u for I: $\theta = 45^0$, II: $\theta = 60^0$and III: $\theta = 75^0$

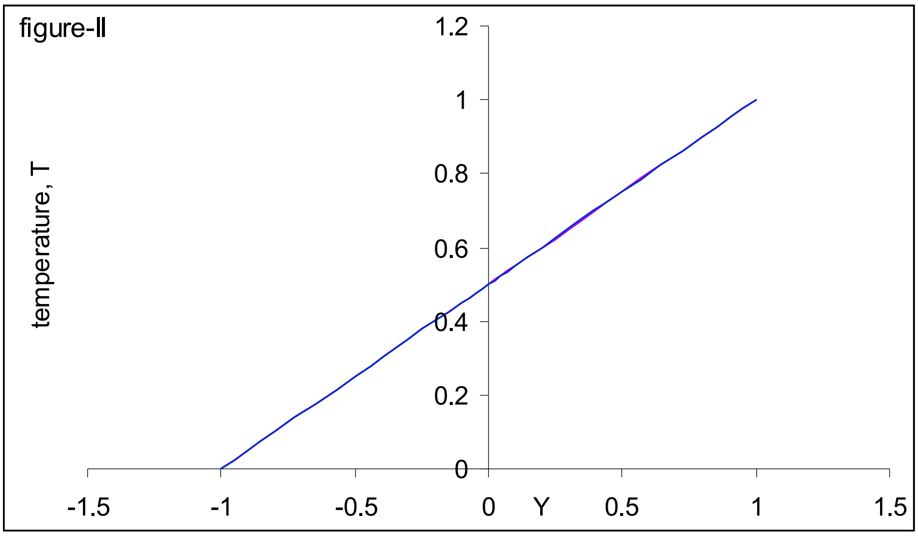

Figure 2: Temperature, T for $Pr = 0.71$, 1,2

as in Figures 1. and 2. The temperature distribution between the plates increases gradually with the increase of Pr.

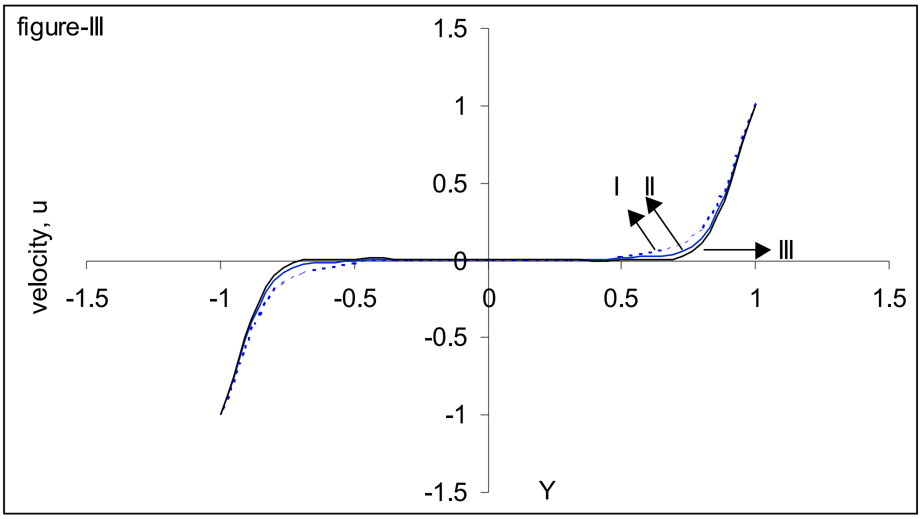

Figure 3: Velocity, u for I: $\theta = 45^0$, II: $\theta = 60^0$and III: $\theta = 75^0$

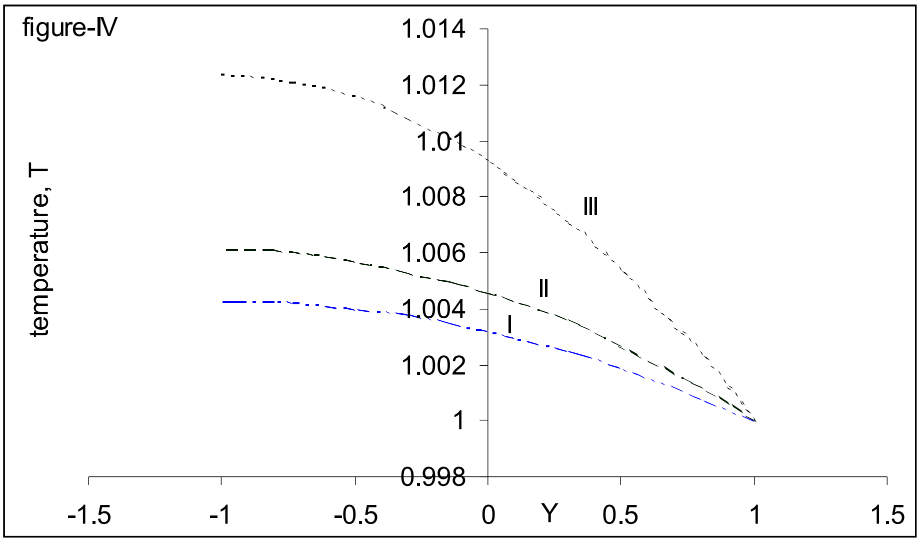

Figure 4: Temperature, T for I: Pr = 0.71, II: Pr =1 and III: Pr = 2.

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MHD Couette-tečenje sa prenosom toplote izmedju dve horizontalne ploče u prisustvu uniformnog poprečnog magnetskog polja

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Analizira se problem nestacionarnog dvodimenzionog tečenja nekog viskoznog nestišljivog elektroprovodnog fluida izmedju dve horizontalne ploče u prisustvu uniformnog poprečnog magnetskog polja. Pritom su u slučaju-I ploče na različitim temperaturama, a u slučaju-II posmatra se kretanje gornje ploče konstantnom brzinom dok je donja ploča adijabatska. Fluidne brzine i temperature su dobijene i prikazane grafički.