THEORETICAL AND APPLIED MECHANICS
vol. 28-29, pp. 77-92, Belgrade 2002
Issues dedicated to memory of the Professor Rastko Stojanović

Spin driven motion in intense spacetime wave geometries

D. A. Burton* **R.W. Tucker†** **C. Wang‡**

Abstract

The motion of a massive particle with intrinsic spin in a gravitational Einstein vacuum plane wave background is explored using the pole-dipole approximation to the Dixon multipole expansion for matter with compact support. Motivated by application to astrophysical processes the dynamical behaviour of the spin and particle motion is described by numerically solving a system of non-linear first order ordinary differential equations. Some results are displayed in a reference frame adapted to the transverse nature of a monochromatic polarised gravitational wave of arbitrary intensity.

1 Introduction

In strong gravitational fields the motion of massive test particles with intrinsic spin is not expected to follow timelike geodesic worldlines [6]. Furthermore the behaviour of continuously distributed matter with "angular momentum" in such fields is relevant to many astrophysical phenomena that have been observed. It is widely believed that the origin of γ-ray bursts, astrophysical jets, X-ray emitters and other exotic processes may be due to the dynamics of electrically charged relativistic matter with "angular momentum" in strong electromagnetic and gravitational fields. Models of

*Department of Physics, Lancaster University, UK(email: d.burton@lancaster.ac.uk)
†Department of Physics, Lancaster University, UK(email: r.tucker@lancaster.ac.uk)
‡Department of Physics, Lancaster University, UK(email: c.wang@lancaster.ac.uk)

such systems can be attempted in terms of the relativistic Einstein-Maxwell-Boltzmann equations [13] provided some basic mechanism for the coupling of charge and "angular momentum" distributions to such fields is adopted. In principle such tidal interactions follow from the stress tensor and current vector used as sources for extended matter in the Einstein-Maxwell equations. Solving such equations for even simple matter models is often impossible and recourse to approximation schemes is necessary in order to gain insight. Some years ago Dixon suggested [1, 2, 3, 4] that for continuous matter with compact support on spacetime a collection of mass and charge multipoles along a suitable worldline might offer such a scheme. He suggested that the worldline could be determined in terms of a finite set of such multipoles in an arbitrary background spacetime.

Although the resulting equations of motion offer a consistent dynamical scheme [9] for the classical behaviour of "spinning matter" [7, 8] they are difficult to solve analytically in all but the simplest gravitational fields and in the lowest pole-dipole approximation [10], [11] to the full multipole series. Truncation of this series leads to further concerns since for sufficiently strong spacetime curvature the initial motion of massive particles with spin is not guaranteed to remain timelike. If such approximation schemes are to find application in more complex astrophysical scenarios it becomes necessary to examine their limits of validity in such gravitational backgrounds. In this article we report on preliminary results in which the motion of a massive electrically neutral test particle with spin is determined numerically in a class of gravitational plane wave geometries in the simplest pole-dipole approximation. The aim is to gain some insight into the resulting motion and explore how it depends on the specific spin to mass ratio of the particle, the strength of the gravitational wave and the nature of its profile. Such information is relevant to astrophysical situations where spinning matter (e.g. particles in plasmas or accretion discs) is irradiated by strong gravitational waves.

2 Equations of Motion in the Pole-Dipole Approximation

The equations of motion under consideration were derived by Dixon from the divergence of the matter stress-energy tensor. A canonical derivation

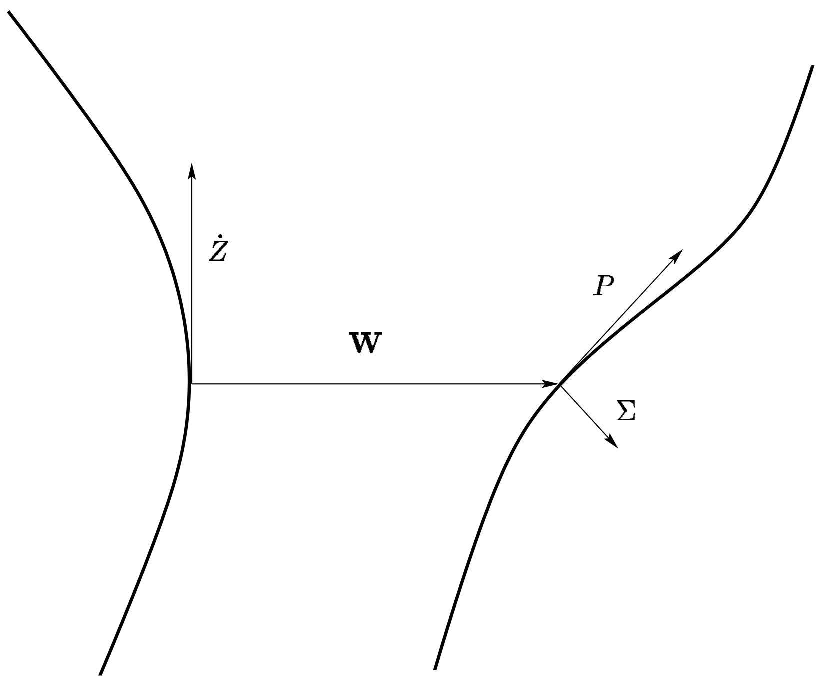

Figure 1: Spacetime figure illustrating the vectors used to define total angular momentum of the particle with history on the right as determined by the observer with history on the left in flat spacetime

was later given by Kunzle [5]. In this paper we are concerned with these equations in background spacetimes with a class of metric tensors g that satisfy the vacuum Einstein equation and describe gravitational plane waves. The history of a particle of mass $m \neq 0$ will be described by a future pointing initially timelike parametrised curve with parameter τ and tangent vector $V(\tau)$. In addition to this vector the dynamics of the particle is also determined by a second timelike vector $P(\tau)$ and a spacelike vector $\Sigma(\tau)$. In the following it proves expedient to relate elements in the tangent space at each point on the worldline of the particle to particular elements in the dual space. Thus for any such vector Z in the tangent space we define $z = \widetilde{Z}$ where $\widetilde{Z} = g(Z, -)$. Similarly for any covector z in the associated cotangent space we define $Z = \widetilde{z} = G(z, -)$ where G is the inverse of g. Furthermore the use of exterior methods streamlines many computations, especially those involving the Hodge map, $\star$, associated with g.

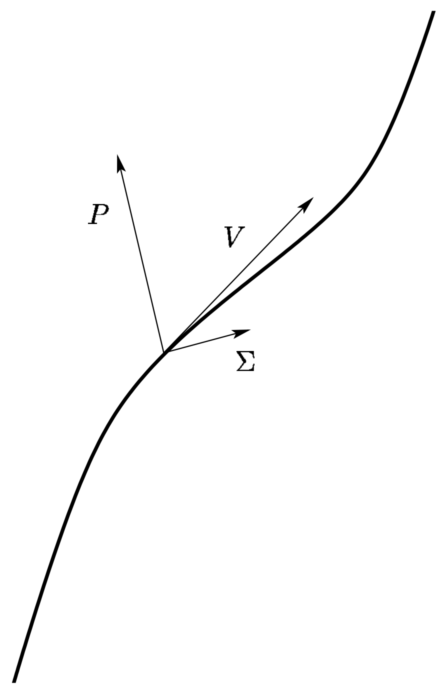

Figure 2: Spacetime figure illustrating the vectors used to define intrinsic angular momentum of a particle with history in a general spacetime

A comment on the notion of "angular momentum" of a massive particle in an arbitrary gravitational field is in order, since such a concept is properly associated with rotational isometries. In a Minkowski spacetime background (that possesses such isometries) such a particle worldline has an associated linear momentum vector $P \equiv mc^2 V$ collinear with V where $g(V,V) = -1$. A Minkowski observer with 4-velocity $\dot{Z}(\tau)$ and $g(\dot{Z},\dot{Z}) = -1$ can use his local rest space orthogonal to $\dot{Z}$ to uniquely specify a spacelike vector $\mathbf{W}$ connecting any event on his worldline to the particle worldline (since Minkowski spacetime is isomorphic to an affine vector space). The *orbital* angular momentum covector of the particle at that instant observed by $\dot{Z}$ is then defined to be

$$l^{orb} = \star(\widetilde{P} \wedge \widetilde{\dot{Z}} \wedge \widetilde{\mathbf{W}}). \tag{1}$$

In terms of the spacelike momentum $\mathbf{P}$ and energy $\mathcal{E}$ of the particle with respect to $\dot{Z}$,

$$P = c\mathbf{P} + \mathcal{E}\dot{Z} \qquad (2)$$

where $g(\dot{Z}, \mathbf{W}) = 0$, one has:

$$l^{orb} = -c\#(\widetilde{\mathbf{W}} \wedge \widetilde{\mathbf{P}}) \qquad (3)$$

in terms of the induced Hodge map given by $\star 1 = \widetilde{\dot{Z}} \wedge \#1$ and the speed of light c. The "intrinsic" spin covector l^{spin} of the particle is given in terms of a 2-form $s(\tau)$:

$$l^{spin} = -\frac{1}{2} \star (u \wedge s) \qquad (4)$$

where $P = mc^2 U$ and $g(U, U) = -1$. This ensures that $i_U l^{spin} = 0$. In Minkowski spacetime one may define the *total* angular momentum covector with respect to $\dot{Z}$ as:

$$j \equiv l^{orb} + l^{spin}. \qquad (5)$$

For a particle in a general spacetime, $\mathbf{W}$ and therefore l^{orb} cannot be so naturally defined but one may continue to call l^{spin} the instantaneous angular momentum or classical intrinsic spin of the particle, independent of any $\dot{Z}$. Such a mathematical formulation however says nothing about the origin of l^{spin}. Indeed in the Dixon scheme l^{spin} may be regarded as a distributional approximation to the collective history of matter about the worldline defined by P and $\Sigma \equiv \frac{\widetilde{l}}{m} \equiv \widetilde{\sigma}$ with $l \equiv -\frac{1}{2} \star (u \wedge s)$ in any background.

In terms of these definitions, P, s and V must satisfy the equations

$$\dot{P} = \widetilde{i_V f}, \qquad (6)$$

$$\dot{s} = 2\widetilde{P} \wedge \widetilde{V}, \qquad (7)$$

$$i_P s = 0 \qquad (8)$$

where for any tensor $Q(\tau)$ along the worldline, $\dot{Q} \equiv \nabla_V Q$ in terms of the Levi-Civita connection restricted to this curve and i_V denotes interior contraction with V. The tidal bivector $\widetilde{f}$ is given in any local cobasis $\{e^a\}$ along the worldline as:

$$f = -\frac{1}{4} \star (R_{ab} \wedge \star s)\; e^a \wedge e^b \qquad (9)$$

where $\{R_{ab}\}$ denote the curvature 2-forms of ∇ in this cobasis.

To reduce these equations to a system of evolution equations for $P(\tau), \sigma(\tau)$ and the particle worldline it is convenient to adopt a parameterisation such that $g(U, V) = -1$ where $P(\tau) = m(\tau)\, c^2 U(\tau)$ for $g(U, U) = -1$. Then after some calculation the spin transport law for σ becomes:

$$(1 + u \wedge i_U)\, \dot{\sigma} = 0. \tag{10}$$

Furthermore one may verify from 6, 7, 8 that m and $s \wedge \star s$ are constant along the world line and that v can be expressed in terms of $u \equiv \widetilde{P}/(mc^2)$ as

$$v = \frac{u - i_\Sigma \Lambda}{1 + i_U i_\Sigma \Lambda} \tag{11}$$

where $c^4 \Lambda \equiv \star(R_{ab} \wedge \sigma \wedge u) \star (e^a \wedge e^b)$. The system 6,8,10,11 constitutes the differential algebraic system that we propose to analyse numerically in a gravitational wave background. A judicious choice of spacetime coordinates $\{t, x, y, z\}$ for this calculation will assist in the interpretation of the results. An exact Einstein vacuum metric for a gravitational wave can be written in these coordinates as:

$$g = -c^2\, d\,t \otimes d\,t + d\,x \otimes d\,x + d\,y \otimes d\,y + d\,z \otimes d\,z + 2\mathcal{F} d\,(ct - z) \otimes d\,(ct - z) \tag{12}$$

where $\mathcal{F}(t, x, y, z) = \frac{1}{2} f_1(ct - z)\,(x^2 - y^2) + f_2(ct - z)\, xy$ in terms of arbitrary smooth g-wave profiles f_1 and f_2 describing the polarisation states of the wave. This form of the metric is particularly useful for interpreting the behaviour of a spinning particle in regions of spacetime where g departs negligibly from Minkowski spacetime. A convenient g-ortho-normal cobasis to use below in calculating the forms f and Λ is

$$e^0 = (\rho_3 + \rho_4)/\sqrt{2} \tag{13}$$

$$e^1 = d\,x \tag{14}$$

$$e^2 = d\,y \tag{15}$$

$$e^3 = (\rho_3 - \rho_4)/\sqrt{2} \tag{16}$$

where $\rho_3 = \frac{1}{2}(c d\,t + d\,z) - \mathcal{F}\, \rho_4$ with $\rho_4 = d\,(ct - z)$

3 Reduction

The first step is to accommodate the algebraic condition 8. Since $g(\Sigma, \Sigma)$ is a constant of the motion with Σ spacelike and $g(U, U) = -1$ by definition, we can write:

$$u = \cosh(\beta)\, e^0 + \sinh(\beta) \left(\cos(\theta)\, e^1 + \sin(\theta) \left(\cos(\phi)\, e^2 + \sin(\phi)\, e^3\right)\right) \tag{17}$$

$$\sigma = S_0 \Big(\sinh(\alpha)\, e^0 + \cosh(\alpha) \left(\cos(\xi)\, e^3 + \sin(\xi) \left(\cos(\eta)\, e^2 + \sin(\eta)\, e^1\right)\right)\Big). \tag{18}$$

[1] The condition $g(U, \Sigma) = 0$ then gives

$$\frac{\tanh(\alpha)}{\tanh(\beta)} = \cos(\theta)(\sin(\eta)\sin(\xi)) + \sin(\theta)\{\sin(\phi)\cos(\xi) + \cos(\phi)\sin(\xi)\cos(\eta)\}. \tag{19}$$

This equation for α (and its derivative along the worldline) can be inserted in 6, 10, 11 and the system reduced to a first order set of non-linear ordinary evolution equations of the form:

$$\frac{d\,A(\tau)}{d\,\tau} = K(A(\tau)) \tag{20}$$

where $A(\tau) = (T(\tau), X(\tau), Y(\tau), Z(\tau), \beta(\tau), \theta(\tau), \phi(\tau), \chi(\tau), \eta(\tau))$ and

$$\dot{V} = \dot{T}(\tau)\partial_{ct} + \dot{X}(\tau)\partial_x + \dot{Y}(\tau)\partial_y + \dot{Z}(\tau)\partial_z.$$

Given an initial condition $A(0)$ these equations can be integrated for a choice of constant S_0 and profiles f_1, f_2.

4 Interpretation

One may interpret the results of integration of the system above in different reference frames associated with the geometry of the problem [12]. Provided the motion of the particle remains timelike one may construct a local basis of three spacelike vectors along the particle history orthogonal to either U or V since in general they will not be collinear. A more natural basis

[1] For a metric with physical dimensions of $length^2$ it follows from the definitions above that the constant S_0/c has physical dimensions [angular momentum]/[mass]. Thus in units with $c = 1$ one finds for an electron that $S_0 \simeq \frac{1}{2}$. On the other hand for a homogeneous massive sphere rotating about an axis through its centre with Newtonian angular speed ω, $S_0 = \frac{2}{5}a^2\, c\,\omega$.

perhaps is that offered by the spacetime propagation direction of the g-wave itself. The null vector $Y \equiv \partial_{ct} + \partial_z$ is parallel with respect to ∇ and hence Killing. It determines the gravitational wave fronts which at every event yield hypersurfaces containing non-zero vectors orthogonal to Y. Let $\dot{Z}$ be some unit timelike vector along the particle's timelike worldline and W_3 be the unique spacelike unit vector obtained by normalising the projection of Y (restricted to the worldline for each τ) onto the local rest space of $\dot{Z}$. Then any observer with 4-velocity coinciding with $\dot{Z}$ at that instant detects a g-wave propagating in the direction W_3 with a 2-dimensional wave front orthogonal to that direction. In the local rest space of $\dot{Z}$ an ortho-normal basis can be constructed:

$$W_3 = -\frac{Y}{\mathcal{E}} + \dot{Z} \tag{21}$$

$$W_2 = \partial_y - \frac{1}{\mathcal{E}} g(\dot{Z}, \partial_y)\, Y \tag{22}$$

$$W_1 = \partial_x - \frac{1}{\mathcal{E}} g(\dot{Z}, \partial_x)\, Y \tag{23}$$

where here $\mathcal{E} = -g(\dot{Z}, Y)$. In this g-wave adapted basis an observer congruent with $\dot{Z}$ at any instant measures a transverse g-wave in which all relative 3-accelerations are orthogonal to W_3. For a choice of $\dot{Z}$ we shall use this frame to describe the motion of the projected spin vector $\Pi_{\dot{Z}}\Sigma$ where $\Pi_{\dot{Z}}$ projects onto the subspace orthogonal to $\dot{Z}$. In general $\dot{Z}$ need not be parallel along the world line. If it is, then since Y is Killing it follows that $\mathcal{E}$ is a constant of the motion proportional to the energy of the gravitational wave observed by $\dot{Z}$.

5 Implementation

The initial value problem above has been analysed numerically for a class of g-wave profiles and specific spins S_0. The simplest wave to consider is a linearly polarised monochromatic wave with angular frequency $\omega = \Omega\, c$ Hz. e.g.

$$f_1 = 0, \;\; f_2 = \lambda_2 \, \sin \Omega t \tag{24}$$

in units with $c = 1$ for some constants $\{\lambda_2\, , S_0\}$. The choice of such constants is made so that, on output, $g(V, V) \leq 0$ and is future pointing over at least a period of the gravitational wave. Since $g(V, V)$ is periodic in τ for monochromatic waves this ensures that the history of the particle remains within the forward light cone of such a spacetime. For such motion V can be normalised if necessary and the motion given in terms of proper time.

The magnitude of the constant λ_2 determines the intensity of the gravitational wave. For a metric with the physical dimensions of $length^2$ this constant has the same dimensions so we require a dimensionless measure of relative intensity.

At any event a family of neighbours to any observer will experience tidal accelerations due to the curvature of spacetime. For a neighbour modelled as a spacelike vector field $\mathbf{W}$ along the observer worldline with unit tangent $\dot{Z}$, a measure of the neighbour's relative acceleration $\mathcal{A}$ is $(\nabla^F_{\dot{Z}}(\nabla^F_{\dot{Z}}\mathbf{W}))$ in terms of the Fermi-Walker covariant derivative ∇^F along the curve. This can be expressed in terms of the ambient curvature operator as $R_{\dot{Z}\mathbf{W}}\dot{Z}$. If $\mathbf{W}$ is a dimensionless vector then ortho-normal components of $c^2\,\mathcal{A}$ have the dimensions of Newtonian acceleration. A gravitational wave will be said to be intense at some point if the ratio of this acceleration field at the point to, say, the Newtonian acceleration due to gravity $\mathcal{G}$ at the earth's surface is large. Note that this definition is observer dependent. In the following we shall use the same choice of $\dot{Z}$ as in the construction of the g-wave adapted basis above. An averaged estimate of the intensity can be obtained by averaging over a a spatial sphere in the rest space of $\dot{Z}$ at each τ. Thus we define the dimensionless specific intensity $\mathcal{I}$ by the relation:

$$\mathcal{I}^2 = \frac{c^4}{\mathcal{G}^2}\, g(\mathcal{A},\mathcal{A}) \tag{25}$$

and, with $\dot{Z} = U$ (timelike), average over directions:

$$\bar{\mathcal{I}^2} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\mathcal{I}^2\sin(\theta)d\,\theta\,d\,\phi. \tag{26}$$

With

$$\mathbf{W} = r_1W_1 + r_2W_2 + r_3W_3 \tag{27}$$

one finds for the plane polarised wave above:

$$\mathcal{I}_{RMS} = \frac{c^2}{\mathcal{G}}\,\frac{1}{\sqrt{5}}\,\mathcal{R}\lambda_2\,\sin\left(\Omega(cT(\tau) - Z(\tau))\right)\sqrt{\cosh^2\beta(\tau) - \sinh^4\beta(\tau)} \tag{28}$$

where $\mathcal{R} = \sqrt{r_1^2 + r_2^2}$. A plot of

$$\kappa(\tau) \equiv \frac{\mathcal{G}}{c^2\,\mathcal{R}}\mathcal{I}_{RMS}(\tau)$$

versus τ, for chosen f_1, f_2 and $\dot{Z}$, enables one to make an estimate of the tidal intensity in the vicinity $\mathcal{R}$ of the particle relative to $\mathcal{G}$.

6 Conclusions

Numerical solutions of the system (20) can be generated for a range of initial conditions, gravitational wave profiles f_1, f_2 and values of S_0. These permit one to explore the nature of the orbit of the particle and behaviour of the spin vector in various frames attached to the particle worldline. An illustration of such results is displayed in Figures 4-118. These are generated with

$$\begin{gathered} A(0) = \big(T(0) = 0, X(0) = 0.1, Y(0) = 0.1, \\ Z(0) = 0.1, \beta(0) = 0.1, \theta(0) = 0.1, \phi(0) = 0.1, \\ \chi(0) = 0.1, \eta(0) = 0.2\big), \end{gathered}$$

a monochromatic wave (24) with $\lambda_2 = 3, \Omega = 5$ and a particle with $S_0 = 1$. An estimate of the associated relative intensity $\mathcal{I}_{RMS}$ over a range of the parameter τ can been seen from Figure 3. Figure 18 indicates that the motion is timelike throughout this interval. The results indicate that the particle executes an overall mean spiral motion in space (Figure 12) with a superimposed oscillatory motion about the mean spiral. The drift coordinate is given by $Z(\tau)$ (Figure 11) and the superimposed oscillations follow from the $X(\tau), Y(\tau)$ solutions (Figures 9-10). The overall motion in space in bounded with aspects reminiscent of the motion of an electrically charged particle in certain electromagnetic fields. With $\dot{Z} = U$, the spin motion is found to be orthogonal to the W_3 direction of the $\{W_1, W_2, W_3\}$ frame along the orbit. Figure 15 illustrates the bounded motion of the spin vector $\Sigma(\tau)$ over the same interval of τ and projected into the $W_1 - W_2$ subspace. Thus in this frame the spin simply oscillates with finite directional amplitude. By contrast the locus of the spin vector in the frame $\{X_1, X_2, X_3\}$ is more complicated (e.g. Figure 16). With $\dot{Z} = V$, the motion of the spin vector in the $\{W_1, W_2, W_3\}$ frame is indicated in Figure 17 and is not restricted to the

$W_1 - W_2$ plane. Although these results refer to plane polarised monochromatic gravitational waves the methods are readily adapted to more complex polarisations and gravitational wave pulses.

Such computations offer a valuable insight into the dynamics of spinning particles in strong gravitational plane waves and display the degree to which the motion of such particles deviates from that determined by timelike geodesics. They enable one to bound the parameters that enter into f_1, f_2 and S_0 in order that the solutions remain physical and are a starting point for more general equations of motion involving higher multipoles and particles with additional electromagnetic properties. We hope to present the results of such generalisations elsewhere.

References

[1] Dixon, G. Proc. Roy. Soc **314**,1970,499.

[2] Dixon, G. Proc. Roy. Soc **319**,1970,509.

[3] Dixon, G. Jour. Gen. Rel. Grav. **4**,1973,193.

[4] Dixon, G. Proc. Roy. Soc **319**,1974,509.

[5] Kunzle H. P. J. Math. Phys. **13**. 1972, 739

[6] Papapetrou, A. Phil. Roy. Soc **A209**,1951,248.

[7] Wald, R. Phys. Rev. D **6**,1972,406.

[8] Wald, R. Ann. of Phys. **83**,1974,548.

[9] Ehlers, J, Rudolph. E. Jour. Gen. Rel. Grav. **8**,1977,197.

[10] Tod, K, De Felice, F. Il Nuovo Cimento **34B**,1976,365.

[11] Mohsen, M, Wang, C, Tucker, R. Class. Quant. Grav. **18**,2001,3007.

[12] Sachs, R. K, Wu H. *General Relativity for Mathematicians*, p.247, Springer Verlag 1977.

[13] Ehlers J. *General Relativity and Kinetic Theory* in Relativity and Cosmology, Proc. Int. School of Physics "Enrico Fermi", Course XLV11, Ed. Sachs B. K., Academic Press, 1971.

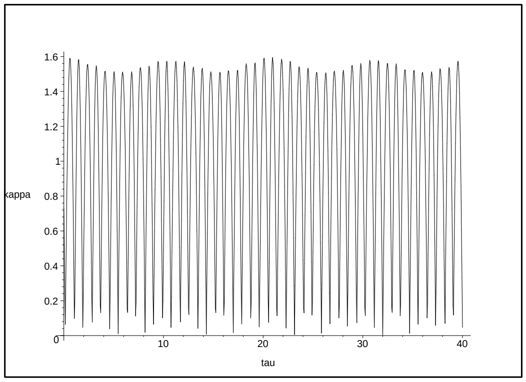

Figure 3: For $\dot{Z} = U$, $\kappa(\tau) \equiv \frac{\mathcal{G}}{c^2\,\mathcal{R}}\mathcal{I}_{RMS}(\tau)$ plotted versus τ where $f_1 = 0$, $f_2 = \lambda_2 \sin \Omega t$ with $\lambda_2 = 3, S_0 = 1$ and $\Omega = 5$. This plot enables one to make an estimate of the tidal acceleration intensity $\mathcal{I}_{RMS}$ in the vicinity $\mathcal{R}$ of the particle relative to any reference acceleration $\mathcal{G}$ (with $c, \mathcal{R}, \mathcal{G}$ in any common units).

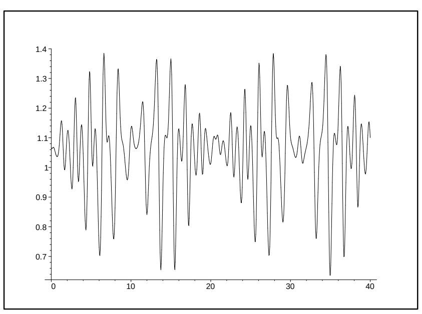

Figure 4: $\dot{T}(\tau) = i_V\, dt$ for $f_1 = 0$, $f_2 = \lambda_2 \sin \Omega t$ with $\lambda_2 = 3, S_0 = 1$ and $\Omega = 5$.

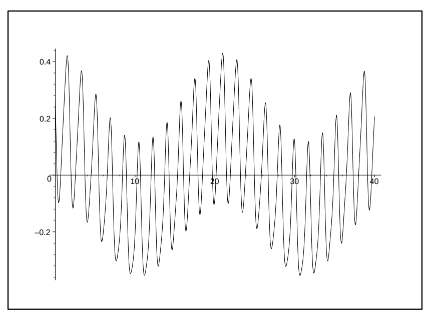

Figure 5: $\dot{X}(\tau) = i_V\, dx$ for $f_1 = 0$, $f_2 = \lambda_2 \sin \Omega t$ with $\lambda_2 = 3, S_0 = 1$ and $\Omega = 5$.

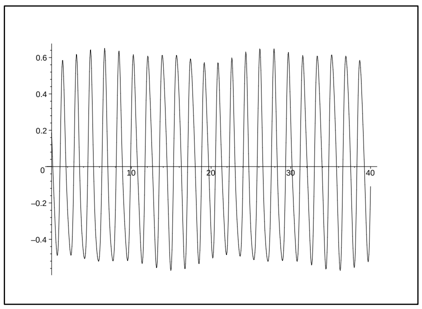

Figure 6: $\dot{Y}(\tau) = i_V\, dy$ for $f_1 = 0$, $f_2 = \lambda_2 \sin \Omega t$ with $\lambda_2 = 3, S_0 = 1$ and $\Omega = 5$.

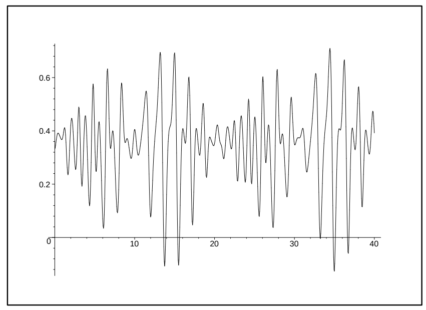

Figure 7: $\dot{Z}(\tau) = i_V\, dz$ for $f_1 = 0$, $f_2 = \lambda_2 \sin \Omega t$ with $\lambda_2 = 3, S_0 = 1$ and $\Omega = 5$.

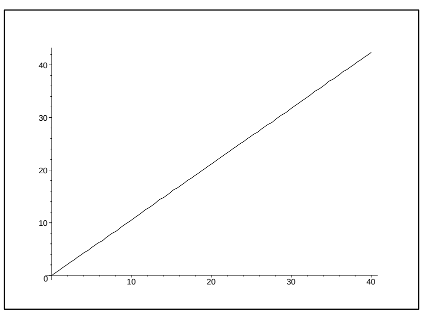

Figure 8: $T(\tau)$ for $f_1 = 0$, $f_2 = \lambda_2 \sin \Omega t$ with $\lambda_2 = 3, S_0 = 1$ and $\Omega = 5$.

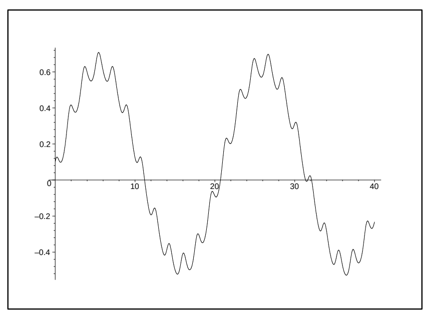

Figure 9: $X(\tau)$ for $f_1 = 0$, $f_2 = \lambda_2 \sin \Omega t$ with $\lambda_2 = 3, S_0 = 1$ and $\Omega = 5$.

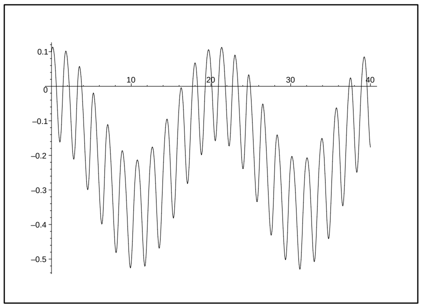

Figure 10: $Y(\tau)$ for $f_1 = 0,\ f_2 = \lambda_2 \sin \Omega t$ with $\lambda_2 = 3, S_0 = 1$ and $\Omega = 5$.

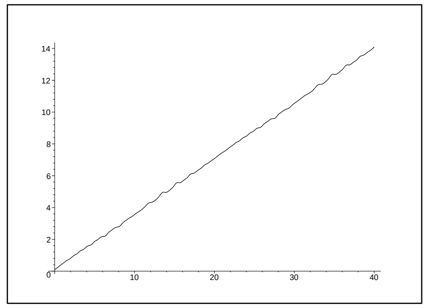

Figure 11: $Z(\tau)$ for $f_1 = 0,\ f_2 = \lambda_2 \sin \Omega t$ with $\lambda_2 = 3, S_0 = 1$ and $\Omega = 5$.

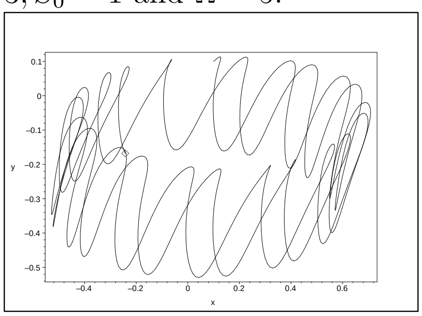

Figure 12: World Line Motion $Y(\tau)$ vs $X(\tau)$ for $f_1 = 0,\ f_2 = \lambda_2 \sin \Omega t$ with $\lambda_2 = 3, S_0 = 1$ and $\Omega = 5$.

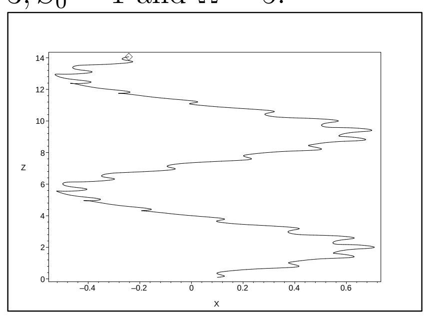

Figure 13: World Line Motion $Z(\tau)$ vs $X(\tau)$ for $f_1 = 0,\ f_2 = \lambda_2 \sin \Omega t$ with $\lambda_2 = 3, S_0 = 1$ and $\Omega = 5$.

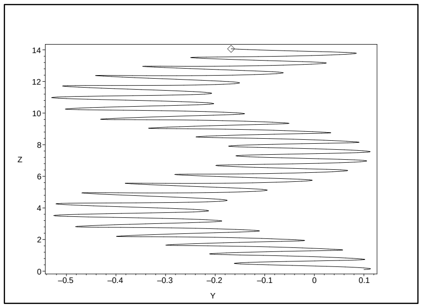

Figure 14: World Line Motion $Z(\tau)$ vs $Y(\tau)$ for $f_1 = 0$, $f_2 = \lambda_2 \sin \Omega t$ with $\lambda_2 = 3, S_0 = 1$ and $\Omega = 5$.

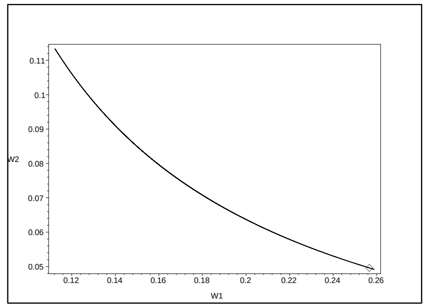

Figure 15: Locus of Spin Motion in $W_1 - W_2$ plane with components of $\Sigma(\tau)$ taken with respect to W_1 and W_2. The $\{W_1, W_2, W_3\}$ frame here has been generated with the choice $\dot{Z} = U$.

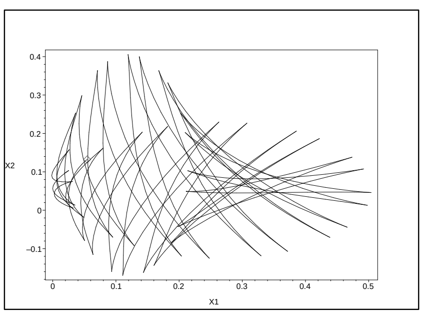

Figure 16: Locus of Projected Spin Motion in $X_1 - X_2$ plane. The spin vector $\Sigma(\tau)$ has here been projected onto the subspace spanned by X_1, X_2, X_3 and its motion displayed in the $X_1 - X_2$ subspace.

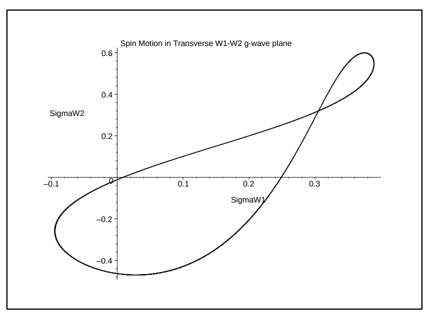

Figure 17: Locus of Spin Motion in $W_1 - W_2$ plane with components of $\Sigma(\tau)$ taken with respect to W_1 and W_2. The $\{W_1, W_2, W_3\}$ frame here has been generated with the choice $\dot{Z} = V$.

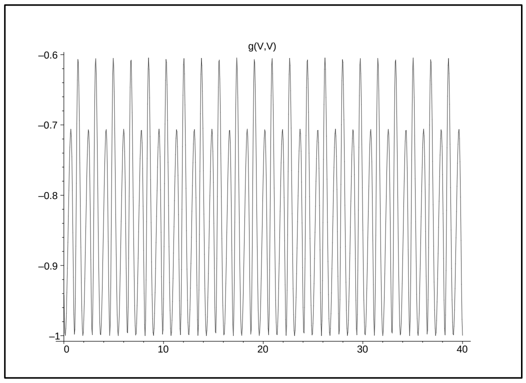

Figure 18: $g(V, V)$ vs τ. Since this invariant remains negative over the range displayed, the particle remains within the local light cone of the gravitational wave spacetime.

Kretanje izazvano spinom u intezivnoj prostor-vremenskoj talasnoj geometriji

UDK 530.12, 514.752, 514.82

Kretanje masivne čestice sa unutrašnjim spinom u nekoj ravansko talasnoj pozadini gravitacionog Einstein-ovog vakuuma se istražuje korišćenjem Dixon-ovog višepolnog razvoja za materiju sa kompaktnim nosačem. Motivisani primenom na astrofizičke procese numerički opisujemo dinamičko ponašanje spina i kretanja čestice rešavanjem jednog sistema nelinearnih običnih diferencijalnih jednačina prvog reda. Neki od rezultata su prikazani u sistemu referencije prilagodjenom poprečnoj prirodi monohromatskog polarizovanog gravitacionog talasa proizvoljne jačine.