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On an evaluation of the drag force of a growing vapor bubble at rectilinear accelerated ascension

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Abstract

By applying the inviscid approximation, a simple relation for predicting the drag coefficient of a growing vapor bubble at rectilinear accelerated ascension in uniformly-superheated pure liquids was developed. The relation is valid in both regions: inertia controlled and heat diffusion controlled bubble growth, respectively. The drag coefficient decreases with time for all accelerations, as well as with augmentation of the bubble acceleration at each instant of time, independently of the internal vapor parameters.

Notations

A	a parameter, defined by Eq. (12);
a_1	thermal diffusivity of fluid;
B	a parameter, defined by Eq. (13);
h_{fg}	latent heat of evaporation;
J_a	Jakob number;
R	bubble radius;
$\dot{R}$	bubble growing velocity;
R^+	dimensionless bubble radius (10);
T	temperature;

ΔT	liquid superheat, $T_\infty - T_{sat}$;
T_{sat}	saturation temperature (corresponding to p_∞);
$\dot{z}_0$	translation velocity of a bubble;
$\ddot{z}_0$	bubble acceleration;
r	radial coordinate;
t	time;
t^+	dimensionless time (11).

Greekletters

ρ	density;
θ	spherical polar angle;
φ	velocity potential.

Subscripts

l	liquid;
v	vapor;
∞	infinity.

1 Introduction

Bubble growth rates were extensively investigated in the last few decades. Generally, the work has been initially divided into the following two main regions: growth rates controlled by inertia forces, applicable in the range of a relatively low pressure and high Jakob numbers, e.g. Rayleigh [1] and growth rates for heat diffusion controlled growth, e.g. Plesset and Zwick [2], Birkhoff et al. [3]. Lien and Griffith [4] experimentally investigated bubble growth in uniformly superheated water covering the low pressure range. They concluded that the bubble growth at very low pressures is controlled solely by inertia forces and that, as the pressure increases heat diffusion becomes a predominant factor, which at the upper part of their pressure range completely controlled the bubble growth. They also found that the interface resistance at the vapor-liquid interface is never a significant factor in bubble growth. Motivated by these experimental results, Mikic et al.[5] developed one single analytical relation applicable in the entire range of bubble growth in a uniformly superheated liquid. This relation is later slightly improved and extended by Miyatake et al. [6] for predicting the growth rate of a vapor bubble also in binary solutions with a non-volatile solute.

A knowledge of the heat and mass transfer associated with a moving bubble (or droplet) is of importance to a variety of industrial processes. Boussinesque [7] has been the first to obtain a solution for the heat transfer rate from a fluid sphere of uniform and constant surface temperature, moving at a constant speed in another fluid of infinite extent. Ruckenstein [8] studied the heat transfer between a vapor bubble in motion and the liquid from which the bubble was generated. Amongst the relatively small number of papers on deforming bubbles in movement, the most often is used an impulsively started motion in a quiescent liquid initially at rest. So, in [11] for instance, the simultaneous solutions of the unsteady boundary-layer equations for the both outside and inside flows of the bubble in an impulsive ascension are obtained by using the method of successive approximations. Generally speaking, the viscous effect is small when the Reynolds number exceeds two or three hundred. It may be of interest to note that if the hydrodynamic boundary layers are developing simultaneously with the thermal boundary layer, the inviscid approximation is even better [9]. That is why, we are going to use hereafter the inviscid approximation for the external fluid flow.

2 On an accelerated rectilinear movement of a spherical undeformable bubble in an inviscid fluid

Let a fluid non-deforming sphere be put into an accelerated movement along a rectilinear trajectory in an inviscid liquid initially at rest. The movement of the bubble will be described in an inertial system of coordinates shown in Fig. 1. So the bubble surface at each instant of time is given by:

$$x^2 + y^2 + [z - z_0(t)]^2 - R^2 = r^2 - R^2 = 0$$

where R designates the radius of the bubble.

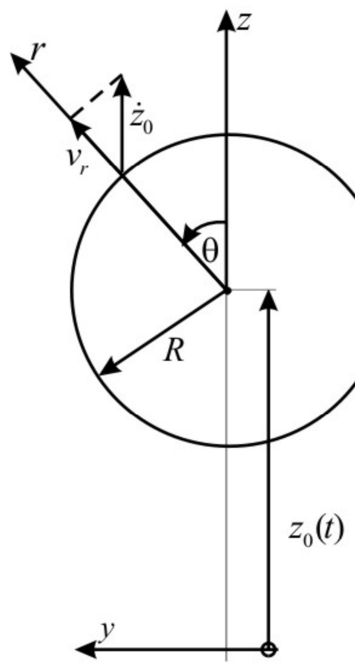

Figure 1: Inertial system of coordinates

In the case of an inviscid external fluid flow, the only condition is that the normal velocity components of the interface and of the fluid flow are identical:

$$v_r = \dot{z}_0(t)\cos\theta, \tag{1}$$

where r and θ are the spherical coordinates linked with the bubble (Fig. 1).

The problem of a rectilinear and accelerated movement of a sphere in an inviscid stationary fluid concerns the determination of the flow potential φ around a sphere put in movement with an unsteady velocity $\dot{z}(t)$. Or, the instantaneous boundary condition at the sphere interface, suitable to (1), should be written as follows:

$$(\frac{\partial\varphi}{\partial r})_{r=R} = \dot{z}(t)\cos\theta, \tag{2}$$

where r and θ are the instantaneous spherical coordinates and $\dot{z}(t)$ is the instantaneous velocity of the bubble (Fig. 1). The function φ has to satisfy also the condition:

$$\varphi \to 0 \quad \text{for} \quad r \to \infty. \tag{3}$$

As the boundary condition (2) is presented in the form of separated variables, the solution of the Laplace equation, written in spherical coordinates, can be obtained by the well known method of separation of variables. Hence, this solution is:

$$\varphi = \sum_{n=0}^{\infty} A_n r^{-n-1} P_n(\cos\theta), \tag{4}$$

where $P_n(\cos\theta)$ are the Legendre ordinary polynomials.. By replacing (4) into (2):

$$\left(\frac{\partial\varphi}{\partial r}\right)_{r=R} = \sum_{n=0}^{\infty} A_n(n+1)R^{-n-2}P_n(\cos\theta) = \dot{z}_0(t)\cos\theta \tag{5}$$

it results that:

$$A_0 = 0, \quad A_1 = -\frac{1}{2}\dot{z}_0 R^3, \quad A_2 = A_3 = ... = 0$$

In this way, the velocity potential φ becomes:

$$\varphi = -\frac{1}{2}\dot{z}_0 R^3 r^{-2}\cos\theta \tag{6}$$

verifying directly the condition (3).

3 Radial expansion of a spherical bubble in an inviscid stationary fluid

In the case of a spherical bubble of the radius $R(t)$ in expansion in an inviscid fluid initially at rest, the boundary condition is:

$$\left(\frac{\partial\varphi}{\partial r}\right)_{r=R} = \dot{R}. \tag{7}$$

Then, the only coefficient in the development (5) is obtained as:

$$A_0 = -R^2 \dot{R}.$$

Consequently, the velocity potential φ for a sphere in radial expansion, after (4), becomes:

$$\varphi = -R^2 \dot{R} r^{-1}, \tag{8}$$

verifying also, evidently, the condition (3).

4 Drag force of a growing vapor bubble at rectilinear accelerated ascension

Consider a growing vapor bubble at a rectilinear accelerated ascension in liquid (Fig. 2) at a pressure p_∞ and initially uniform temperature T_∞. Let p_v and T_v represent the vapor pressure and the vapor temperature inside a growing bubble, respectively. Let us further assume that the vapor is in equilibrium with the liquid, hence p_v and T_v would represent a saturation state for the considered fluid. We are going to use herein the solution of Mikic et al [5]. Neglecting gravitational effects, work done by viscous forces, surface-tension regime, bubble growth acceleration effects and irreversible conversion to internal energy, they proposed [5] for a bubble growth in a uniformly superheated liquid, the following expression (taking account of inertia controlled and diffusion controlled growth, respectively):

$$R^+ = \frac{2}{3}\left[(t^+ + 1)^{3/2} - (t^+)^{3/2} - 1\right], \tag{9}$$

where:

$$R^+ = \frac{R}{B^2/A}, \tag{10}$$

$$t^+ = \frac{t}{B^2/A^2}, \tag{11}$$

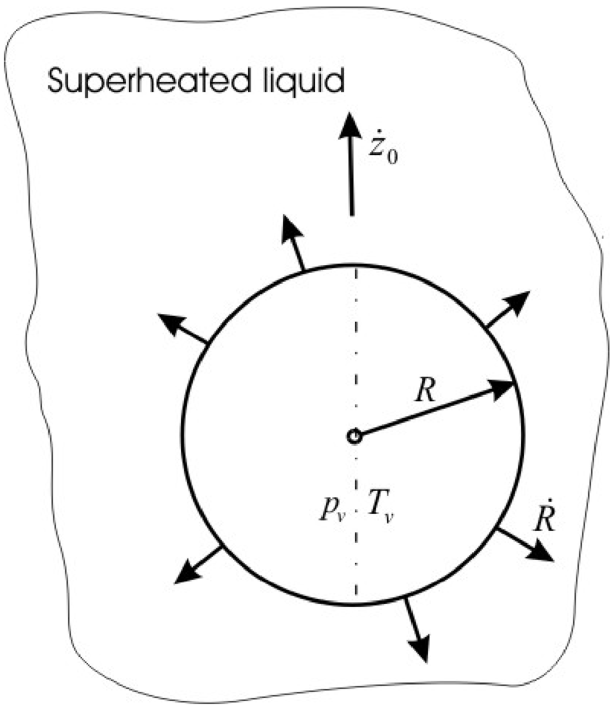

Figure 2: Growth of a spherical vapor bubble

$$A = \left[\frac{2}{3}\frac{\rho_v h_{fg}\Delta T}{\rho_l T_{sat}}\right]^{1/2}, \tag{12}$$

$$B = \left[\frac{12}{\pi} a_l J_a^2\right]^{1/2}. \tag{13}$$

In order to study the problem resulting of two movements of a vapor bubble, discussed in §2 an §3, we will use the principle of superposition [10]. Hence, the potential φ of the external flow around a growing vapor bubble at rectilinear accelerated motion in an inviscid liquid, can be written by adding (6) and (8):

$$\varphi = -\frac{1}{2}\dot{z}_0(t)R^3 r^{-2}\cos\theta - R^2\dot{R}r^{-1}, \tag{14}$$

which verifies directly the condition: $\varphi \to 0 \quad \text{for} \quad r \to \infty$.

Let us calculate now the pressure distribution on the bubble surface by using the Bernouilli equation for the external unsteady flow:

$$\left[p+\frac{1}{2}\rho_l(v_r^2+v_\theta^2)+\rho_l\frac{\partial\varphi}{\partial t}\right]_{r=R}=\left[p+\frac{1}{2}\rho_l(v_r^2+v_\theta^2)+\rho_l\frac{\partial\varphi}{\partial t}\right]_{r\to\infty},$$

whence:

$$(p)_{r=R}=p_\infty-\frac{1}{2}\rho_l(v_r^2+v_\theta^2)_{r=R}-\rho_l\left[\frac{\partial\varphi}{\partial t}\right]_{r=R}. \tag{15}$$

From (14), we have:

$$v_r=\frac{\partial\varphi}{\partial r}=R^2\dot{R}r^{-2}+\dot{z}_0R^3r^{-3}\cos\theta\rightarrow(v_r)_{r=R}=\dot{R}+\dot{z}_0\cos\theta,$$

$$v_\theta=\frac{1}{r}\frac{\partial\varphi}{\partial\theta}=\frac{1}{2}\dot{z}_0R^3r^{-3}\sin\theta\rightarrow(v_\theta)_{r=R}=\frac{1}{2}\dot{z}_0\sin\theta,$$

$$\left(\frac{\partial\varphi}{\partial t}\right)_{r=R}=-2\dot{R}^2-R\ddot{R}-\frac{5}{2}\dot{R}\dot{z}_0\cos\theta-\frac{1}{2}R\ddot{z}_0\cos\theta-\dot{z}_0^2\cos^2\theta,$$

then replacing to (15), it results that:

$$\begin{aligned}(p)_{r=R}=p_\infty+\frac{1}{2}\rho_l\dot{z}_0^2(1-\frac{9}{4}\sin^2\theta)+\\+\rho_l(R\ddot{R}+\frac{3}{2}\dot{R}^2)+\frac{1}{2}\rho_l(R\ddot{z}_0+3\dot{R}\dot{z}_0)\cos\theta.\end{aligned} \tag{16}$$

The total drag force of a growing vapor bubble at an accelerated ascension is different from zero even in an inviscid fluid (due to the instationarity), and may be evaluated as:

$$D=-2\pi R^2\int_0^\pi(p)_{r=R}\cos\theta\sin\theta d\theta+\rho_lg\frac{4}{3}\pi R^3 \tag{17}$$

where the last term represents the buoyancy force experienced by the bubble. By replacing (16) into (17), after integration, we obtain:

$$D=-\frac{2}{3}\pi\rho_lR^3\ddot{z}_0-2\pi\rho_lR^2\dot{R}\dot{z}_0+\frac{4}{3}\pi\rho_lgR^3. \tag{18}$$

In this kind of problems, sometimes the so-called "added mass" is utilized , as follows:

$$m_a(t) = \frac{2}{3}\pi R^3 \rho_l,$$

which represents the (variable) mass of the liquid sphere, simultaneously accelerated with the vapor bubble during its movement. It is still to be noted that the relation (18) could be also established as follows:

$$D = \frac{4}{3}\pi R^3 \rho_l g - \frac{d}{dt}(m_a \dot{z}_0) = \frac{4}{3}\pi R^3 \rho_l g - \frac{2}{3}\pi \rho_l R^3 \ddot{z}_0 - 2\pi \rho_l R^2 \dot{R} \dot{z}_0.$$

It is customary to introduce the drag coefficient C_z, defined by:

$$C_z = \frac{D}{\frac{1}{2}\rho_l \dot{z}_0^2 \pi R^2}.$$

From Eq. (18), one finds:

$$C_z = \frac{8}{3}\frac{g}{\ddot{z}_0^2}\frac{R}{t^2} - \frac{4}{3\ddot{z}_0}\frac{R}{t^2} - \frac{4}{\ddot{z}_0}\frac{\dot{R}}{t}, \tag{19}$$

i.e., taking into account (9):

$$\frac{B^2 \ddot{z}_0}{A^3} C_z = \frac{8}{9}(2\frac{g}{\ddot{z}_0} - 1)\frac{\left[(t^+ + 1)^{3/2} - (t^+)^{3/2} - 1\right]}{(t^+)^2} - \\ -4\frac{\left[(t^+ + 1)^{1/2} - (t^+)^{1/2}\right]}{t^+}. \tag{20}$$

Fig. 3 shows the variation of the drag coefficient with non dimensional time ($i.e. t^+$) for some arbitrarily assigned values of the acceleration parameter: $(g/\ddot{z}_0) \in 4, 5, 6$. Another values may be used as well but the main features of the finding as described in the next section would not be affected.

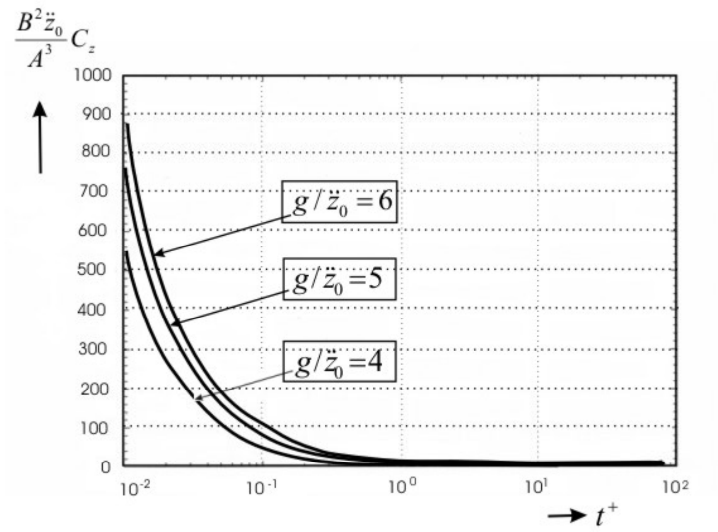

Figure 3: Evolution of the drag coefficient of a vapor bubble for different accelerations $\ddot{z}_0$

5 Concluding remarks

Besides the hypothesis of the non viscous external flow, all the results presented in Fig. 3 seem to be acceptable. Due to the buoyancy effect, the drag coefficient decreases with time for all values of the acceleration non-dimensional parameter $g/\ddot{z}_0$ as well as with the augmentation of acceleration at each instant of time t^+. Of course, thermal characteristics of the growing vapor bubble are implicitly present through dimensionless parameters t^+ and $B^2\ddot{z}_0/A^3$ (or $A^3/B^2\ddot{z}_0$).

It is to be noticed that the analysis cannot give the correct prediction at very early stages of a bubble growth, where the assumptions incorporated in the analysis [5] are not justifiable. In particular, a vapor bubble cannot exist in a thermodynamic equilibrium with the surrounding liquid for radii smaller than certain critical radius determined by the fluid properties and the liquid superheat. Then, the evaluation of the vapor temperature inside the growing bubble is not also correct in the initial stages of the bubble growth. Nevertheless, as Mikić et al. [5] showed, the uncertainty related to the early life history of a vapor bubble did not noticeably affect the prediction for most of its life range. Anyway, in a perspective, we will try to include also into the computation of the drag force of a growing vapor bubble at accelerated ascension some other effects such as this initial surface tension controlled bubble growth regime as well as the influence of the fluid viscosity.

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Približni proračun sile otpora pri translatornom ubrzanom kretanju rastućeg parnog mehura

UDK 532.526; 532.15

Primenom neviskozne aproksimacije, u ovom radu je dobijena jedna relativno prosta relacija za približno odredjivanje koeficijenta otpora pri translatornom ubrzanom usponu rastućeg parnog mehura u uniformno pregrejanoj tecnosti. Dobijena relacija važi u oba slučaja rasta mehura: kako u slučaju rasta kontrolisanog inercijom, tako i u slučaju rasta kontrolisanog toplotnom difuzijom. Koeficijent otpora opada sa vremenom pri svakom ubrzanju, kao i pri povećanju ubrzanja uspona - nezavisno od unutrašnjih parametara parnog mehura - a u svakom posebnom vremenskom trenutku.