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Ionized gas flow in the boundary layer for different forms of the electroconductivity variation law

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Abstract

Ionized gas flow for different forms of the law of its electroconductivity variation is studied in this paper. At first, it was considered that the gas electroconductivity was determined by the law $\sigma = \sigma(x)$. Then, under the influence of MHD, it is accepted that the electroconductivity variation law is determined by the expression $\sigma = \sigma_0(1 - u/u_e)$. At the end, ionized gas flow is studied in the case when the electroconductivity of the gas is a function of the longitudinal velocity gradient. For all these cases boundary layer equations are, by means of suitable transformations, brought to a generalized form. The obtained universal equations are numerically solved in the so-called three parametric approximation. Based on the obtained solutions, behaviour of physical values and boundary layer characteristics are graphically presented. Conclusions are drawn about the influence of certain parameters on distribution of physical values in the boundary layer.

1 Introductory studies, starting equations

In MHD boundary layer theory, different problems of a body within conductive, incompressible fluid are studied in details (by application of the general similarity method). In the studies considering this field, [1, 2, 7], flow problems in the cases when the external magnetic field is perpendicular to the contour of the body are solved. Here, the fluid electroconductivity is either constant or different forms of the electroconductivity variation law [7] are used. In these mentioned (and other) studies, the importance of electroconductivity variation for practical usage, as well as for theory and methodology, is stressed.

In that sense, this paper studies ionized gas flow in the boundary layer at the body of any shape for several different possible forms of the law of its electroconductivity variation. Since we do not know the exact form of the electroconductivity variation law of the gas, by analogy with the electroconductive liquid, different electroconductivity variation laws of the ionized gas are suggested in this paper. They have the form of the following expressions:

$$\begin{aligned} &(a) \qquad \sigma = \sigma(x), \\ &(b) \qquad \sigma = \sigma_0(1 - u/u_e), \\ &(c) \qquad \sigma = \sigma_0 \frac{v_0}{u_e^2}\frac{\partial u}{\partial y}, \qquad (\sigma_0, v_0 = const)\,. \end{aligned} \tag{1}$$

As it is seen in the given presumed laws, the ionized gas electroconductivity is: a function only of the longitudinal coordinate x (the expression 1a), a function of the velocities ratio (the case 1b), or a function of the longitudinal velocity gradient (1c). Based on the laws (1b) and (1c), it is concluded that in these cases the ionized gas electroconductivity disappears at the outer edge of the boundary layer (for which $u = u_e(x)$ and $\partial u/\partial y = 0$), that is, $\sigma = \sigma_e = 0$ at this boundary.

Due to ionizations, under the influence of the outer magnetic field $B_m = B_m(x)$, an electric stream appears in the gas. Because of this, Lorentz force and Joule's heat appear. Due to these two effects new terms [4] appear in the corresponding boundary layer equations, which is not the case with the homogenous gas.

If, by the usual procedure we exclude the pressure from the starting boundary layer equations [4], the equations of the steady plane laminar boundary layer, for small values of magnetic Reynolds number and under the conditions of the so-called balanced ionization, have the following form:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0.$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho_e u_e \frac{d u_e}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \underline{-\sigma B_m^2 u},$$

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = -u \rho_e u_e \frac{d u_e}{dx} + \mu \left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\left(\frac{\mu}{\mathrm{Pr}} \frac{\partial h}{\partial y}\right) \underline{+\sigma B_m^2 u^2}; \tag{2}$$

$$y = 0: \qquad u = v = 0, \qquad h = h_w,$$

$$y \to \infty: \qquad u \to u_e(x), \qquad h \to h_e(x).$$

In these equations as well as in the corresponding boundary conditions, symbols common in the boundary layer theory have been used for certain physical values. So, we have the following symbols: $u(x, y)$ - longitudinal projection of velocity in the boundary layer, $v(x, y)$ - transversal projection, ρ - density, p - pressure, $h(x, y)$ - ionized gas enthalpy, μ - dynamic viscosity and Pr - Prandtl number. The indices represent: e - conditions at the outer edge of the boundary layer, w - values at the wall of the body within fluid and o - constant physical values.

Here, the term $\sigma B_m^2 u$ represents already mentioned Lorentz force, and the term $\sigma B_m^2 u^2$ represents Joule's heat. Since the ionized gas electroconductivity σ appears only in these terms, they are different for different forms of the electroconductivity variation law (1). Therefore, these terms are underlined in the equations (2). The boundary layer equations (2) correspond to the electroconductivity variation laws (1 b) and (1 c). In the case when $\sigma = \sigma(x)$, the underlined terms have these forms in the dynamic and energy equation:

$$(a) \qquad \underline{+\sigma B_m^2 (u_e - u)}, \qquad \underline{+\sigma B_m^2 \left(u^2 - u u_e\right)}. \tag{2'}$$

2 Transformations of the variables

For solving the differential equation systems (2) and (2'), the general similarity method [6] has been used.

Unlike other methods [8], parametric methods of solution of boundary layer equations, of compressible as well as of incompressible fluid, are based on the introduction of the corresponding sets of parameters. These parameters, as it is known, represent so-called, similarity parameters and play a role of new independent variables. Depending on the flow problem, these parameters may be dynamic, thermodynamic or diffusional or magnetic. However, the introduction of the similarity parameters is based on the application of the momentum equation. In order for the momentum equation of the considered ionized gas flow problem, to have the simplest form, new purposeful variables s, z and the stream function $\psi(s,z)$ are introduced in the form of these relations:

$$s(x) = \frac{1}{\rho_0\mu_0}\int_0^x \rho_w\mu_w dx, \qquad z(x,y) = \frac{1}{\rho_0}\int_0^y \rho dy,$$

$$u = \frac{\partial\psi}{\partial z}, \qquad \tilde{v} = \frac{\rho_0\mu_0}{\rho_w\mu_w}\left(u\frac{\partial z}{\partial x} + v\frac{\rho}{\rho_0}\right) = -\frac{\partial\psi}{\partial s}. \tag{3}$$

They are also introduced with similar flow problems [5].

By means of newly introduced transformations (3), the boundary layer equation system (2) comes down to:

$$\frac{\partial\psi}{\partial z}\frac{\partial^2\psi}{\partial s\partial z} - \frac{\partial\psi}{\partial s}\frac{\partial^2\psi}{\partial z^2} = \frac{\rho_e}{\rho}u_e\frac{du_e}{ds} + v_0\frac{\partial}{\partial z}\left(Q\frac{\partial^2\psi}{\partial z^2}\right) \underline{-\frac{\rho_0\mu_0}{\rho_w\mu_w}\frac{\sigma B_m^2}{\rho}\frac{\partial\psi}{\partial z}},$$

$$\frac{\partial\psi}{\partial z}\frac{\partial h}{\partial s} - \frac{\partial\psi}{\partial s}\frac{\partial h}{\partial z} = -\frac{\rho_e}{\rho}u_e\frac{du_e}{ds}\frac{\partial\psi}{\partial z} + v_0 Q\left(\frac{\partial^2\psi}{\partial z^2}\right)^2 +$$

$$+v_0\frac{\partial}{\partial z}\left(\frac{Q}{\Pr}\frac{\partial h}{\partial z}\right) \underline{+\frac{\rho_0\mu_0}{\rho_w\mu_w}\frac{\sigma B_m^2}{\rho}\left(\frac{\partial\psi}{\partial z}\right)^2}; \tag{4}$$

$$z = 0: \qquad \psi = \frac{\partial \psi}{\partial z} = 0, \qquad h = h_w,$$

$$z \to \infty: \qquad \frac{\partial \psi}{\partial z} \to u_e(s), \qquad h \to h_e(s).$$

In the obtained equations (4) the non-dimensional function Q and Prandtl number Pr are determined by the expressions:

$$Q = \frac{\rho\mu}{\rho_w \mu_w}; \qquad \mathrm{Pr} = \frac{\mu c_p}{\lambda} \tag{5}$$

The obtained equations (4) are applied in the cases of ionized gas electroconductivity variation (1 b) and (1 c). If the electroconductivity variation law is determined by the relatively more generalized expression (1 a), then, after the introduction of the transformations (3), we obtain equations which differ from the obtained only for the underlined terms. Then these terms are:

$$(a) \qquad \begin{array}{l} \underline{+\frac{\rho_0\mu_0}{\rho_w\mu_w}\frac{\sigma B_m^2}{\rho}\left(u_e - \frac{\partial \psi}{\partial z}\right);} \\ \\ \underline{+\frac{\rho_0\mu_0}{\rho_w\mu_w}\frac{\sigma B_m^2}{\rho}\left[\left(\frac{\partial \psi}{\partial z}\right)^2 - u_e\frac{\partial \psi}{\partial z}\right].} \end{array} \tag{4'}$$

Using the variables (3), by the usual procedure of integration of the dynamic equation with respect to the transversal coordinate within the boundary layer, the corresponding momentum equation can be easily derived. This equation has the same form for all three forms of electroconductivity variation law (1) of the ionized gas, and it is:

$$\frac{dZ^{**}}{ds} = \frac{F_m}{u_e} \tag{6}$$

However, in each of these cases the characteristic boundary layer universal function F_m has a different form:

$$\begin{array}{ll} (a) & F_m = 2[\zeta - (2 + H)f] - 2H_1 g \\ (b) & F_m = 2[\zeta - (2 + H)f] + 2H_1 g \\ (c) & F_m = 2[\zeta - (2 + H)f] + g \end{array} \tag{7}$$

In obtaining the momentum equation, in further studies we introduce: the parameter of the form f, the magnetic parameter g, the non-dimensional friction function ζ , the conditional displacement thickness Δ^*, the conditional momentum loss thickness Δ^{**}, the conditional thicknesses Δ_1^* and Δ_1^{**}, as well as other physical and characteristic values of the boundary layer. They are denoted in the usual way and they are:

$$Z^{**}=\frac{\Delta^{**2}}{v_0},\quad f(s)=u_e'Z^{**},\quad \zeta(s)=\left[\frac{\partial(u/u_e)}{\partial(z/\Delta^{**})}\right]_{z=0},\quad H=\frac{\Delta^*}{\Delta^{**}},$$

$$\Delta^*(s)=\int_0^\infty\left(\frac{\rho_e}{\rho}-\frac{u}{u_e}\right)dz,\quad \Delta^{**}(s)=\int_0^\infty\frac{u}{u_e}\left(1-\frac{u}{u_e}\right)dz,$$

$$\Delta_1^*(s)=\int_0^\infty\frac{\rho_e}{\rho}\left(1-\frac{u}{u_e}\right)dz,\quad \Delta_1^{**}(s)=\int_0^\infty\frac{u}{u_e}\left(1-\frac{u}{u_e}\right)\frac{\rho_e}{\rho}dz, \tag{8}$$

$$(a)\quad H_1=\frac{\Delta_1^*}{\Delta^{**}},\quad N_\sigma=\frac{\rho_0\mu_0}{\rho_w\mu_w}\frac{\sigma B_m^2}{\rho_e},\quad g(s)=N_\sigma Z^{**},$$

$$(b)\quad H_1=\frac{\Delta_1^{**}}{\Delta^{**}},\quad N_\sigma=\frac{\rho_0\mu_0}{\rho_w\mu_w}\frac{\sigma_0 B_m^2}{\rho_e},\quad g(s)=N_\sigma Z^{**},$$

$$(c)\quad N_\sigma=\frac{\rho_0\mu_0}{\rho_w\mu_w}\frac{\sigma_0 B_m^2}{\rho_0},\quad g(s)=N_\sigma\frac{\Delta^{**}}{u_e}=u_e^{-1}N_\sigma v_0^{1/2}Z^{**1/2},$$

For further studies of the ionized gas problem, another transformation of variables is used. It is also used with electroconductive liquid. As a matter of fact, new variables, like Saljnikov's, are introduced in the form of:

$$s=s,\quad \eta(s,z)=\frac{u_e^{b/2}}{K(s)}z,$$

$$\phi(s,\eta)=\frac{\psi(s,z)}{u_e^{1-b/2}K(s)},\quad \bar{h}(s,\eta)=\frac{h(s,z)}{h_1},\tag{9}$$

$$K(s)=\left(av_0\int_0^s u_e^{b-1}ds\right)^{1/2},\quad h_1=\frac{u_e^2}{2}+h_e=const.;$$

in which $\phi(s,\eta)$ represents the newly introduced stream function, $\bar{h}$ stands for the non-dimensional enthalpy, while a and b are arbitrary constants. Using the newly introduced transformations (9), some characteristic values (8) of the boundary layer can be written in the form:

$$\zeta=B\left(\frac{\partial^2\phi}{\partial\eta^2}\right)_{\eta=0},\quad H=\frac{A}{B},$$

$$A=\int_0^\infty\left(\frac{\rho_e}{\rho}-\frac{\partial\phi}{\partial\eta}\right)d\eta,\quad B=\int_0^\infty\frac{\partial\phi}{\partial\eta}\left(1-\frac{\partial\phi}{\partial\eta}\right)d\eta,\tag{10}$$

$$(a)\quad H_1=\frac{A_1}{B},\quad A_1=\int_0^\infty\frac{\rho_e}{\rho}\left(1-\frac{\partial\phi}{\partial\eta}\right)d\eta,$$

$$(b)\quad H_1=\frac{A_1}{B},\quad A_1=\int_0^\infty\frac{\partial\phi}{\partial\eta}\left(1-\frac{\partial\phi}{\partial\eta}\right)\frac{\rho_e}{\rho}d\eta.$$

3 Generalized equations of ionized gas boundary layer

Using the newly introduced transformations (9), the equation system (4) i.e. (4') can be, after complicated transformation, brought to a new form in which there will still be the outer velocity $u_e(s)$ and the magnetic field power $B_m(s)$. In order for the equation system to be explicitly independent from these values, which characterize each concrete problem, it is necessary, just like with MHD, to introduce two sets of parameters.

As with ionized gas flow problems, it has been proven that the sets of dynamic and magnetic parameters should have the following form:

$$
\begin{aligned}
(a,b) \quad & \kappa = f_0 = u_e^2/2h_1, \quad f_k = u_e^{k-1} u_e^{(k)} Z^{**k}, \\
& g_k = u_e^{k-1} N_\sigma^{(k-1)} Z^{**k}, \\
(c) \quad & \kappa = f_0 = u_e^2/2h_1, \quad f_k = u_e^{k-1} u_e^{(k)} Z^{**k}, \\
& g_k = u_e^{k-2} N_\sigma^{(k-1)} v_0^{1/2} Z^{**k-1/2},
\end{aligned} \tag{11}
$$

$$(k = 1, 2, 3, ...).$$

The first parameters (11) represent the already (8) introduced parameter of the form f and the magnetic parameter g, i.e. $f_1 = f(s)$ and $g_1 = g(s)$; while κ represents the so-called local compressibility parameter. This parameter satisfies the simple differential equation

$$\frac{u_e}{u_e'} f_1 \frac{d\kappa}{ds} = 2\kappa f_1 = \theta_0, \tag{12}$$

and it represents a function of the coordinate s, which is set in advance.

Using the momentum equation of the considered problem, it is relatively easily found, by differentiation of the parameters, that the introduced parameters satisfy recurrent simple differential equations. For all three cases of electroconductivity variation law, these equations are:

$$
\begin{aligned}
(a) \quad & \frac{u_e}{u_e'} f_1 f_k' = [(k-1)f_1 + kF_m] f_k + f_{k+1} = \theta_k, \\
& \frac{u_e}{u_e'} f_1 g_k' = [(k-1)f_1 + kF_m] g_k + g_{k+1} = \gamma_k, \\
(b) \quad & \frac{u_e}{u_e'} f_1 f_k' = [(k-1)f_1 + kF_m] f_k + f_{k+1} = \theta_k, \\
& \frac{u_e}{u_e'} f_1 g_k' = [(k-1)f_1 + kF_m] g_k + g_{k+1} = \gamma_k,
\end{aligned} \tag{13}
$$

$$(c)\quad \begin{aligned} &\frac{u_e}{u_e'} f_1 f_k' = [(k-1) f_1 + k F_m] f_k + f_{k+1} = \theta_k, \\ &\frac{u_e}{u_e'} f_1 g_k' = [(k-2) f_1 + (k-1/2)\, F_m] g_k + g_{k+1} = \gamma_k, \end{aligned}$$

$$(k = 1, 2, 3, ...)$$

where, as before, the prim stands for a derivation with respect to s.

If, from the very beginning, we introduce two sets of parameters (together with the compressibility parameter), equations of ionized gas boundary layer in so-called general similarity variables will be obtained. Because of that, instead of the variables (9) we use the similarity transformations in the following form:

$$s = s, \quad \eta(s,z) = \frac{B(s)}{\Delta^{**}(s)} z,$$

$$\psi(s,z) = \frac{u_e(s)\Delta^{**}(s)}{B(s)} \phi[\eta, \kappa, (f_k), (g_k)], \tag{14}$$

$$h(s,\eta) = h_1 \cdot \bar{h}[\eta; \kappa, (f_k), (g_k)], \quad (k = 1, 2, 3, ...).$$

With the new transformations, as it is seen, the function ϕ and the non-dimensional enthalpy $\bar{h}$ are not directly dependent on the longitudinal variable s but indirectly through the parameters κ, f_k and g_k.

By means of the variables (14) the basic equation system (4) is brought to this form:

$$\frac{\partial}{\partial \eta}\left(Q \frac{\partial^2 \phi}{\partial \eta^2}\right) + \frac{aB^2 + (2-b) f_1}{2B^2} \phi \frac{\partial^2 \phi}{\partial \eta^2} + \frac{f_1}{B^2}\left[\frac{\rho_e}{\rho} - \left(\frac{\partial \phi}{\partial \eta}\right)^2\right] \underline{- \frac{g_1}{B} \frac{\partial \phi}{\partial \eta} \frac{\partial^2 \phi}{\partial \eta^2}} =$$

$$= \frac{1}{B^2}\left[\sum_{k=0}^{\infty} \theta_k \left(\frac{\partial \phi}{\partial \eta} \frac{\partial^2 \phi}{\partial \eta \partial f_k} - \frac{\partial \phi}{\partial f_k} \frac{\partial^2 \phi}{\partial \eta^2}\right) + \sum_{k=1}^{\infty} \gamma_k \left(\frac{\partial \phi}{\partial \eta} \frac{\partial^2 \phi}{\partial \eta \partial g_k} - \frac{\partial \phi}{\partial g_k} \frac{\partial^2 \phi}{\partial \eta^2}\right)\right],$$

$$\frac{\partial}{\partial\eta}\left(\frac{Q}{\Pr}\frac{\partial\bar{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\phi\frac{\partial\bar{h}}{\partial\eta}-\frac{2\kappa f_1}{B^2}\frac{\rho_e}{\rho}\frac{\partial\phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\phi}{\partial\eta^2}\right)^2+$$

$$\underline{+\frac{2\kappa g_1}{B}\left(\frac{\partial\phi}{\partial\eta}\right)^2\frac{\partial^2\phi}{\partial\eta^2}}= \tag{15}$$

$$=\frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\phi}{\partial\eta}\frac{\partial\bar{h}}{\partial f_k}-\frac{\partial\phi}{\partial f_k}\frac{\partial\bar{h}}{\partial\eta}\right)+\sum_{k=1}^{\infty}\gamma_k\left(\frac{\partial\phi}{\partial\eta}\frac{\partial\bar{h}}{\partial g_k}-\frac{\partial\phi}{\partial g_k}\frac{\partial\bar{h}}{\partial\eta}\right)\right];$$

$$\eta=0:\qquad \phi=\frac{\partial\phi}{\partial\eta}=0,\qquad \bar{h}=\bar{h}_w=const.,$$

$$\eta\to\infty:\qquad \frac{\partial\phi}{\partial\eta}\to 1,\qquad \bar{h}\to\bar{h}_e=1-\kappa.$$

Since the distribution of the outer velocity $u_e(s)$ appears neither in the equation system (15) nor in the corresponding boundary conditions, the equation system (15) is in that sense universal, i.e. generalized. Also, the magnetic field power $B_m(s)$ does not appear explicitly in the obtained system, but indirectly through magnetic parameters.

It is clear that the generalized equation system (15) differs in certain (underlined) terms for different given forms of the ionized gas electroconductivity variation law (1). The equation system (15), written here, is suitable in the case when the electroconductivity σ changes according to the law (1 c). For the law forms (1 a) and (1 b), the underlined terms in the dynamic and energy equations are:

$$(a)\qquad \underline{+\frac{g_1}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\phi}{\partial\eta}\right)}\qquad \underline{-\frac{2\kappa g_1}{B^2}\frac{\rho_e}{\rho}\frac{\partial\phi}{\partial\eta}\left(1-\frac{\partial\phi}{\partial\eta}\right)} \tag{15'}$$

$$(b)\qquad \underline{-\frac{g_1}{B^2}\frac{\rho_e}{\rho}\frac{\partial\phi}{\partial\eta}\left(1-\frac{\partial\phi}{\partial\eta}\right)}\qquad \underline{+\frac{2\kappa g_1}{B^2}\frac{\rho_e}{\rho}\left(\frac{\partial\phi}{\partial\eta}\right)^2\left(1-\frac{\partial\phi}{\partial\eta}\right)}, \tag{15''}$$

respectively.

4 Solutions of the obtained equations

Solution of the obtained equation system (15), i.e., (15') or (15") is practically possible only when there is only a small number of similarity parameters (n-parametric approximation). If it is presumed that all the parameters, starting from the second one equal zero, and if the derivations with respect to compressibility and magnetic parameters are neglected, the system is remarkably simplified. In the so-called three-parametric twice-localized approximation ($\partial/\partial\kappa = 0$, $\partial/\partial g_1 = 0$) the equation system (15) reduces to:

$$\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\phi}{\partial\eta^2}\right) + \frac{aB^2 + (2-b)f_1}{2B^2}\phi\frac{\partial^2\phi}{\partial\eta^2} + \frac{f_1}{B^2}\left[\frac{\rho_e}{\rho} - \left(\frac{\partial\phi}{\partial\eta}\right)^2\right] \underline{- \frac{g_1}{B}\frac{\partial\phi}{\partial\eta}\frac{\partial^2\phi}{\partial\eta^2}} =$$

$$= \frac{F_m f_1}{B^2}\left(\frac{\partial\phi}{\partial\eta}\frac{\partial^2\phi}{\partial\eta\partial f_1} - \frac{\partial\phi}{\partial f_1}\frac{\partial^2\phi}{\partial\eta^2}\right),$$

$$\frac{\partial}{\partial\eta}\left(\frac{Q}{\Pr}\frac{\partial\bar{h}}{\partial\eta}\right) + \frac{aB^2 + (2-b)f_1}{2B^2}\phi\frac{\partial\bar{h}}{\partial\eta} - \frac{2\kappa f_1}{B^2}\frac{\rho_e}{\rho}\frac{\partial\phi}{\partial\eta} + 2\kappa Q\left(\frac{\partial^2\phi}{\partial\eta^2}\right)^2 +$$

$$\underline{+\frac{2\kappa g_1}{B}\left(\frac{\partial\phi}{\partial\eta}\right)^2\frac{\partial^2\phi}{\partial\eta^2}} = \frac{F_m f_1}{B^2}\left(\frac{\partial\phi}{\partial\eta}\frac{\partial\bar{h}}{\partial f_1} - \frac{\partial\phi}{\partial f_1}\frac{\partial\bar{h}}{\partial\eta}\right); \qquad (16)$$

$$\eta = 0: \qquad \phi = \frac{\partial\phi}{\partial\eta} = 0, \qquad \bar{h} = \bar{h}_w = const,$$

$$\eta \to \infty: \qquad \frac{\partial\phi}{\partial\eta} \to 1, \qquad \bar{h} \to \bar{h}_e = 1 - \kappa.$$

For electroconductivity variation laws (1 a) and (1 b), the underlined terms change and have a form as with (15') i.e. (15"). Under these conditions, the expression for the characteristic function F_m is also remarkably simplified. In our studies, while solving the equation system (16), the exact expression (7) has been used; of course according to the corresponding form of the electroconductivity variation law. By analogy with dissociated gas [3], the following approximate formulae are used for the non-dimensional function Q and for the densities ratio ρ_e/ρ :

$$Q = Q(\bar{h}) \approx \left(\frac{\bar{h}_w}{h}\right)^{1/3}, \quad \frac{\rho_e}{\rho} \approx \frac{\bar{h}}{1-\kappa}; \tag{17}$$

while the usual values are accepted for Prandtl number Pr $= 0.712$ (air), and the constants a and b are: $a = 0.4408;\ b = 5.7140$.

The parametric equation system (16) has been numerically solved using the finite differences method.

The system is, according to the written program in FORTRAN language [6], solved for several different values of the parameters κ and g_1, given in advance, as well as for the enthalpy $\bar{h}_w$. Out of many obtain results in a form of tables, only some, in a form of the corresponding diagrams (Fig. 1. - Fig. 6.) are given here, and those are for the presumed ionized gas electroconductivity laws.

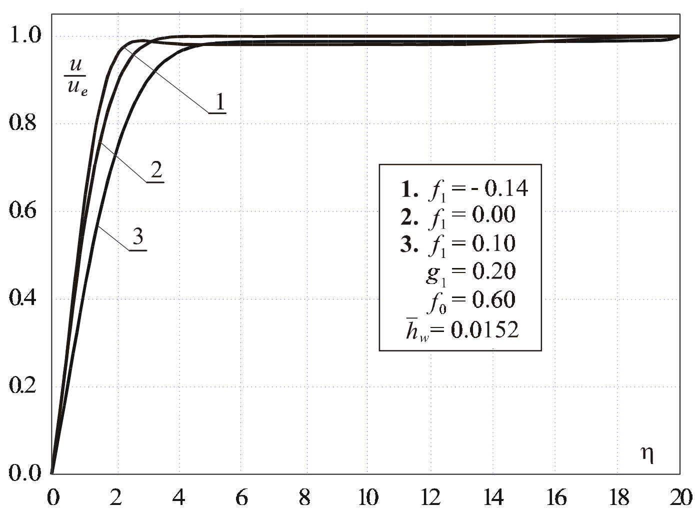

Fig. 1. (a) Distribution of the non-dimensional velocity

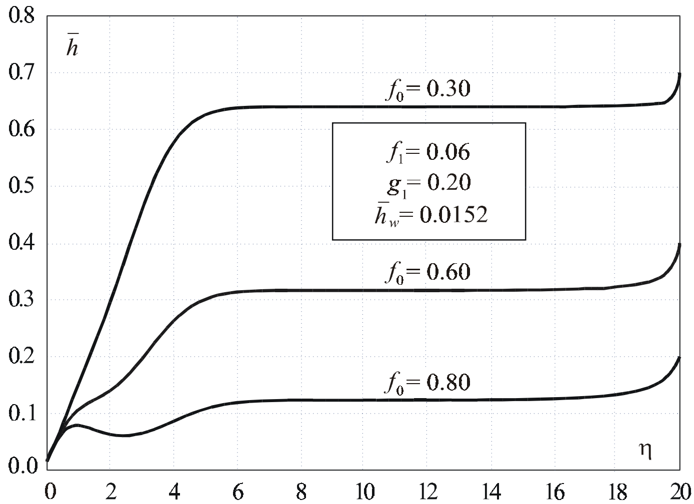

Fig. 2. (a) Distribution of the non-dimensional enthalpy for different values of the parameter f_0

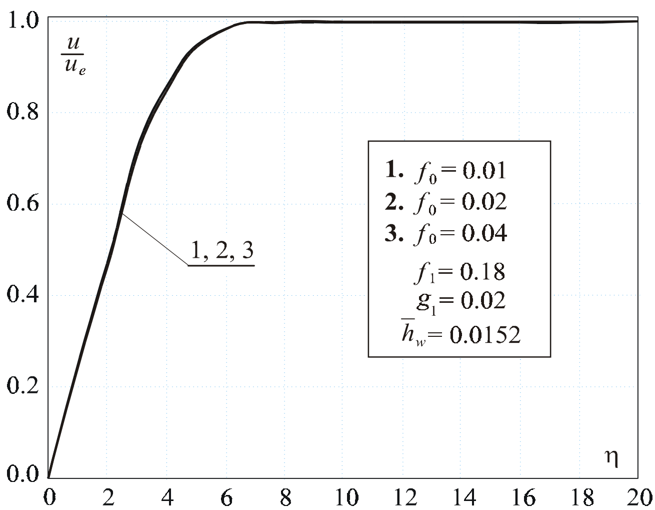

Fig. 3. (b) Distribution of the non-dimensional velocity

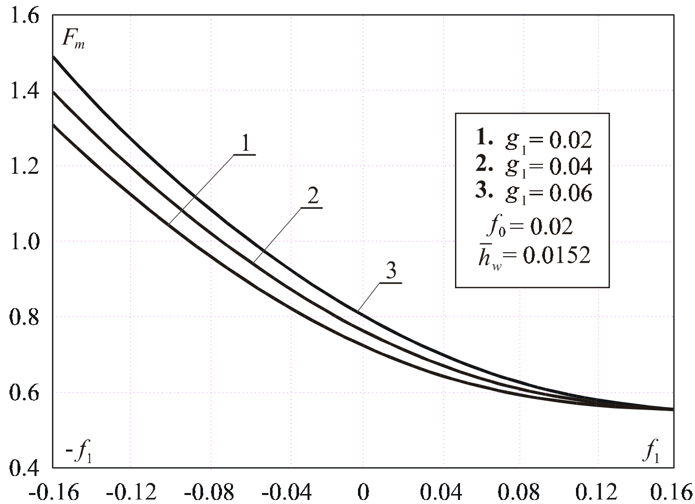

Fig. 4. (b) The characteristic of the boundary layer

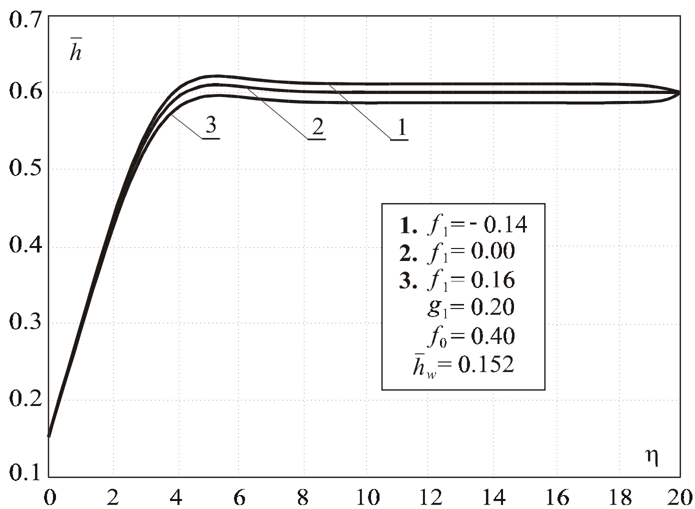

Fig. 5. (c) Distribution of the non-dimensional enthalpy

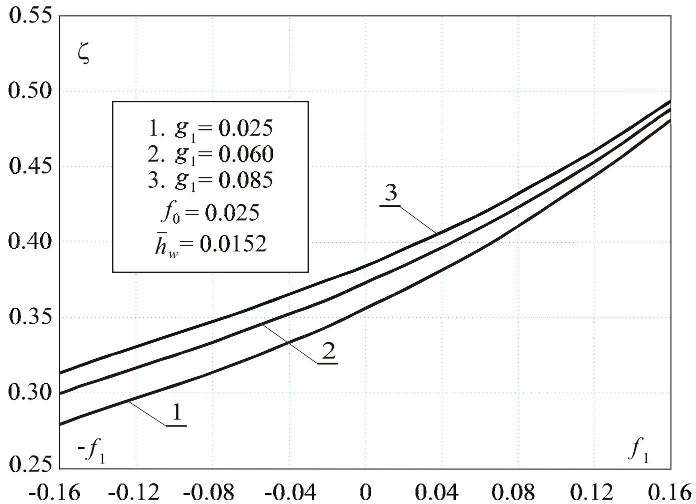

Fig. 6. (c) The non-dimensional function of the friction

5 Conclusions

Based on the given and other diagrams, the following has been concluded:

- The non-dimensional velocity u/u_e converges very fast towards one for all the values of the compressibility parameter and different values of the form parameter and the magnetic parameter.

- Here the compressibility parameter κ has a negligible influence on distribution of the non-dimensional velocity in the boundary layer.

- This parameter has a great influence on distribution of the enthalpy in the boundary layer.

It is pointed out that the given conclusions are in total agreement with the conclusions which are valid for similar compressible fluid flow problems (dissociated gas). These conclusions are valid for all three forms of the ionized gas electroconductivity variation law.

It can be stated, with certainty, that distributions of physical values at the boundary layer for other possible forms of electroconductivity variation law will be identical with the ones obtained in this paper.

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Strujanje jonizovanog gasa u graničnom sloju za različite oblike zakona promene elektroprovodnosti

UDK 533.95; 537.56

U radu se istražuje strujanje jonizovanog gasa za različite oblike zakona promene njegove elektroprovodnosti. Najpre je smatrano da je elektroprovodnost gasa određena zakonom $\sigma = \sigma(x)$. Zatim je, pod uticajem MHD, usvojeno da je zakon promene elektroprovodnosti određen izrazom $\sigma = \sigma_0(1 - u/u_0)$. Na kraju je razmatrano strujanje jonizovanog gasa i za slučaj kada je elektroprovodnost gasa funkcija gradijenta podužne brzine. Za sve navedene slučajeve jednačine graničnog sloja su pogodnim transformacijama dovedene na odgovarajući uopšteni oblik. Dobijene univerzalne jednačine su numerički rešavane u tzv. troparametarskom približenju. Na bazi dobijenih rešenja grafički je prikazano ponašanje fizičkih veličina i karakteristika graničnog sloja. Izvedeni su zaključci o uticaju pojedinih parametara na raspodele fizičkih veličina u graničnom sloju.