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MHD flow and heat transfer of a dusty viscoelastic stratified fluid down an inclined channel in porous medium under variable viscosity

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Abstract

The laminar convection flow of an incompressible electrically conducting second order visco-elastic stratified fluid in porous medium down a parallel plate channel inclined at an angle θ to the horizontal surface in presence of uniform transverse magnetic field is studied. The plates are maintained at temperatures which decay with time. The fluid properties density and viscosity are assumed to be variable among different layers within the channel. Solutions are obtained using perturbation method for fluid velocity, particle velocity, and the fluid temperature distribution within the channel. Their variations with the channel are shown graphically for different values of stratification factor at constant values of magnetic field parameter and porosity parameter. Expressions for viscous drag, the rate of heat transfer at the plates, flow flux for fluid and particle are also obtained.

1 Introduction

In recent years many authors have studied the flow of immiscible viscous electrically conducting fluids and their different transport phenomena. These fluids also known as non-Newtonian fluids are molten plastics, pulps, emulsion etc. and large variety of industrial products having visco-elastic behavior in their motion. Such fluids are often embedded with spherical non-conducting dust particles in the form of impurities. This king of fluid is then called dusty Rivlin- Ericksen second order fluid. The influence of dust particles on visco-elastic fluid flow has its importance in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of Crude oil, pulp, paper industry, textile industry and in different Geophysical cases etc. In these cases stratification effect is often observed which are under the action of geomagnetic field.

The study of dusty visco-elastic fluids under different physical conditions have been carried out by several authors like Kapur et al. [1], Bagchi [2], Gupta et al. [3], Purkait [4], Lahiri et al. [5], Sengupta et al. [8] and some others. Sisodia and Gupta [6] have studied the unsteady flow of a dusty Rivlin- Ericksen fluid through circular and coaxial ducts. Lal et al. [7] have studied MHD transient flow of second order Rivlin-Ericksen fluid through porous medium down an inclined channel. Recently Singh et al. [9] have studied MHD flow and heat transfer of a dusty visco-elastic liquid down an inclined channel in porous medium.

In the present paper we are trying to investigate the stratification effect of an unsteady flow of dusty visco-elastic electrically conducting fluid down an inclined parallel plate channel in porous medium in presence of uniform magnetic field applied externally transverse to the direction of flow. Assuming the plates are maintained are at temperatures which decay exponentially with time, the expressions for fluid velocity, dust particle velocity and temperature distribution, fluid and dust particle flux, heat transfer, viscous drag at the boundary layers are obtained. The velocity distribution of fluid and dust particle, the temperature distribution are shown graphically for different values of stratification factor at constant magnetic Hartmann and porosity parameter.

2 Formulation of the problem

We consider fully developed flow of an incompressible, dusty Rivlin-Ericksen fluid of electrically conducting material through a parallel plate channel separated by $2h$, inclined horizontally by an angle θ. The plates are maintained at two different temperatures which decay exponentially with time. Let the central line of the channel as the x-axis while y-axis is perpendicular to it. The uniform magnetic field B_0 is applied normal to the plates. So that the velocity and magnetic field distributions are $V = [u(y,t), 0, 0]$ and $B = [0, B_0, 0]$. The inertial force experienced by the dust particles is equal and opposite to that experienced by the dust particles due to the fluid motion.

3 Assumptions

To write down the governing equations following assumptions are made.

(i) The plates are infinitely long, so that the fluid velocity (u) and dust particles velocity (v) are functions of y and t only.

(ii) There are neither chemical reactions, mass transfer nor heat radiation among the dust particles.

(iii) The number density of dust particles is constant and has small value throughout the fluid motion.

(iv) Dust particles are solids, elastic spheres, identical and symmetrical in size, electrically non-conducting, and are distributed uniformly within the fluid motion.

(v) Hall effect, Polarization effect, and the effect due to Buoyancy are negligible.

(vi) Initially (i.e. at time $t = 0$) there is no flow and plates are at two different temperatures (i.e. at $t = 0 \;\; T = T_0$, at $y = -h$ and $t > 0 \;\; T = T_1$, at $y = +h$).

(vii) The fluid under consideration is finitely conducting so that Joule heat due to the presence of external magnetic field is negligible.

(viii) The value of magnetic Reynolds number (Rm) is small enough so that the induced field is negligible.

(ix) The fluid properties density and viscosity under consideration are varying along y-axis throughout the channel and are given as

$$\rho = \rho_0 \mathbf{e}^{-n(y/h+1)} \tag{1}$$

$$\mu = \mu_0 \mathbf{e}^{-n(y/h+1)} \tag{2}$$

where n is the stratification factor of the fluid and μ_0 the coefficient of density and

viscosity respectively on the line of the channel at $y = -h$.

4 Governing equations

The governing equations under above assumptions are:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \left(n\frac{v_1}{h}\right)\frac{\partial u}{\partial y} + v_1\frac{\partial^2 u}{\partial y^2} + v_2\left\{\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial y^2}\right) - \right.$$

$$-\frac{v_1}{k_1}u + g\sin\theta + K\frac{N}{\rho}(v-u) + \frac{\sigma}{\rho}\left(B_0^2 u\right) \tag{3}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} + g\cos\theta = 0 \tag{4}$$

$$m\frac{\partial v}{\partial t} - K(u-v) = 0 \tag{5}$$

$$\frac{\partial T}{\partial t} = \frac{\alpha}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + v_1\left(\frac{\partial u}{\partial y}\right)^2 + v_2\left\{\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right)^2\right\} \tag{6}$$

where p - fluid pressure; m - mass of the dust particle; v_1 - kinematics coefficient of fluid viscosity; v_2 - kinematics coefficient of viscoelasticity; k_1 - porosity of the medium; N - number density of the dust particles; α - thermal conductivity of the fluid; C_p - specific heat at constant pressure; v_H - magnetic diffusivity; K - proportionality constant; and T - fluid temperature.

We define fluid pressure p as

$$p = \rho g\{x \sin\theta - y\cos\theta\} + \rho\ x\ a(t) + A \tag{7}$$

where A is a constant.

Using (7), equation (3) is written as

$$\frac{\partial u}{\partial t} = -a(t) - n\frac{v_1}{h}\frac{\partial u}{\partial y} + v_1\frac{\partial^2 u}{\partial y^2} + v_2\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) -$$

$$-\frac{v_1}{k_1}\left(K\frac{N}{\rho}\right)(v-u) + \frac{\sigma}{\rho}\left(B_0^2 u\right) \tag{8}$$

The boundary conditions of the problem are

$$\begin{aligned} &u = 0, && v = 0, && T = T_0\mathbf{e}^{-2nt}, && \text{at } y = -h \\ &u = u_0\mathbf{e}^{-nt}, && v = v_0\mathbf{e}^{-nt}, && T = T_1\mathbf{e}^{-2nt}, && \text{at } y = +h \end{aligned} \tag{9}$$

where T_0 and T_1 are the temperatures at the plates $y = +h$ and $y = -h$ respectively.

We consider following non-dimensional parameters

$$\begin{aligned} &u^* = u/u_0, && v^* = v/v_0, && y^* = y/h, && t^* = t\ u_0/h \\ &a^* = ah/u_0^2, && T^* = T/T_0, && k^* = h/\sqrt{k_1}, && \lambda = u_0/v_0 \end{aligned} \tag{10}$$

Substituting (10) in equations (3-7), and then removing asterisks, we get

$$\frac{\partial u}{\partial t} = -a(t) - \frac{n}{R}\frac{\partial u}{\partial y} + \frac{1}{R}\frac{\partial^2 u}{\partial y^2} - \eta\left\{\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right)\right\} -$$

$$-\frac{k^2}{R}\,u+\left\{\frac{C}{R_t}\right)\left\{\frac{v}{\lambda}-u)\right\}+\frac{M^2}{R}u \tag{11}$$

$$R_t\,\frac{\partial v}{\partial t}-(\lambda u-v)=0 \tag{12}$$

$$\frac{\partial^2 T}{\partial y^2}=(RP_r)\frac{\partial T}{\partial t}-\left\{E.P_r\left(\frac{\partial u}{\partial y}\right)^2\right\}+\eta ERP_r\left\{\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right)^2\right\} \tag{13}$$

where

$R=(u_0h/v_1)$ - Reynolds number;
$\eta=(-v_2/h^2)$ - visco-elastic parameter;
$C=(m.N/\rho)$ - dust particle concentration;
$R_t=\{m.u_0/(K.h)\}$- relaxation time parameter of dust particles;
$M=\sqrt{\{(B_0^2h^2\sigma)/\rho v_1\}}$ -magnetic Hartmann number;
$P_r=v_1/\alpha$ - Prandtl number;
$E=\{u_0^2/(C_pT_0)\}$ - Eckert number.

The non-dimensional boundary conditions are

$$u=v=0,\qquad T=\mathbf{e}^{-2nt},\qquad \text{at}\ \ y=-1$$

$$u=\mathbf{e}^{-nt},\qquad v=\mathbf{e}^{-nt},\qquad T=\chi\mathbf{e}^{-2nt},\qquad \text{at}\ \ y=+1 \tag{14}$$

where $\chi=(T_1/T_0)$, is a constant temperature.

5 Solutions

In order to solve the equations (10-13) under the boundary conditions (14), we consider

$$u=f(y)\mathbf{e}^{-nt},\quad v=g(y)\mathbf{e}^{-nt},\quad T=F(y)\mathbf{e}^{-2nt},\quad a=a_0\mathbf{e}^{-nt} \tag{15}$$

substituting (14) in equations (10-13), we get

$$\frac{\partial^2 f(y)}{\partial y^2}-A_1\frac{\partial f(y)}{\partial y}+A_2 f(y)-A_3=0 \tag{16}$$

$$g(y) - \lambda \frac{f(y)}{(1 - n.R_t)} = 0 \tag{17}$$

$$\frac{\partial^2 F(y)}{\partial y^2} + A_4 F(y) + A_5 \left(\frac{\partial f(y)}{\partial y} \right)^2 = 0 \tag{18}$$

where

$$A_1 = \frac{n}{1 + n\eta R}; \quad A_3 = \frac{a_0}{1 + n\eta R}; \quad A_4 = (2.n.R.P_r);$$

$$A_2 = \frac{R}{1 + n\eta R} \left(n - \frac{K^2 + M^2}{R} + C.\frac{n}{1 - nR_t} \right);$$

$$A_5 = E.P_r(1 + 2.n.\eta.R); \quad A_6 = \frac{\lambda}{1 - nR_t}.$$

The corresponding boundary conditions are now

$$f(-1) = g(-1) = 0, \quad F(-1) = 1 \text{ and}$$

$$f(+1) = 1, \quad g(+1) = \frac{\lambda}{A_6}, \quad F(+1) = \chi. \tag{19}$$

The solution of equations (16-18) subject to the boundary conditions (19) are

$$f(y) = C_1 \mathbf{e}^{m_1 y} + C_2 \mathbf{e}^{m_2 y} + \frac{A_3}{A_2} \tag{20}$$

$$g(y) = \frac{1}{A_6} \left[C_1 \mathbf{e}^{m_1 y} + C_2 \mathbf{e}^{m_2 y} + \frac{A_3}{A_2} \right] \tag{21}$$

$$F(y) = \left\{ C_3 \cos \sqrt{A_4 y} + C_4 \cos \sqrt{A_4 .y} \right\} -$$

$$- \left[S_1 \mathbf{e}^{2m_1 y} + S_2 \mathbf{e}^{2m_2 y} + S_3 \mathbf{e}^{(m_1 + m_2)y} \right] \tag{22}$$

where

$$m_1 = \frac{A_1 + \sqrt{(A_1^2 - 4A_2)}}{2}; \quad m_2 = \frac{A_1 - \sqrt{(A_1^2 - 4A_2)}}{2};$$

$$A_7 = \frac{\sinh m_1}{\mathbf{e}^{-m_1}\sinh m_2 - \mathbf{e}^{-m_2}\sinh m_1};$$

$$C_2 = A_7\left\{\frac{A_3}{A_2} + \frac{\mathbf{e}^{-m_1}}{\sinh m_2}\right\}; \quad C_1 = \frac{1}{\sinh m_1}\left(\frac{1}{2} - C_2\sinh m_2\right);$$

$$S_1 = \frac{C_1^2 m_1^2}{4m_1^2 + A_4}; \quad S_2 = \frac{C_2^2 m_2^2}{4m_2^2 + A_4};$$

$$S_3 = \frac{2C_1C_2m_1m_2}{(m_1+m_2)^2 + A_4};$$

$$C_3 = \frac{1}{\cos\sqrt{A_4}}\left[\frac{1+\chi}{2} + S_1\cosh 2m_1 + \right.$$

$$\left. S_2\cosh 2m_2 + S_3\cosh(m_1+m_2)\right];$$

$$C_4 = \frac{1}{\sin\sqrt{A_4}}\left[\frac{(\chi-1)}{2} + S_1\sinh 2m_1 + \right.$$

$$\left. S_2\sinh 2m_2 + S_3\sinh(m_1+m_2)\right].$$

6 Skin friction

The viscous drag acting at the plates for fluid (τ_f) and for the particles (τ_p) are

$$\tau_f = \left[\left(\frac{1}{R} - \eta\frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial y}\right)\right]_{y=\pm1} = \tag{23}$$

$$\left[\left(\frac{1}{R}-\eta\frac{\partial}{\partial t}\right)\left(\mathbf{e}^{-nt}\frac{\partial f}{\partial y}\right)\right]_{y=\pm 1}=$$

$$\left(\frac{1}{R}+n\eta\right)\left[C_1 m_1\mathbf{e}^{\pm m_1}+C_2 m_2\mathbf{e}^{\pm m_2}\right]\mathbf{e}^{-nt} \tag{24}$$

$$\tau_p=\left[\left(\frac{1}{R}-\eta\frac{\partial}{\partial t}\right)\left(\frac{\partial v}{\partial y}\right)\right]_{y=\pm 1}= \tag{25}$$

$$\left[\left(\frac{1}{R}-\eta\frac{\partial}{\partial t}\right)\left(\mathbf{e}^{-nt}\frac{\partial g}{\partial y}\right)\right]_{y=\pm 1}=$$

$$\left(\frac{1}{R}+n\eta\right)\frac{\lambda}{A_6}\left[C_1 m_1\mathbf{e}^{\pm m_1}+C_2 m_2\mathbf{e}^{\pm m_2}\right]\mathbf{e}^{-nt}. \tag{26}$$

7 Flow flux for fluid and particles

The flux of flow for fluid (ϕ_f) and the particles (ϕ_p) through the channel are represented as

$$\phi_f=\int_{-1}^{+1} u dy = \mathbf{e}^{-nt}\int_{-1}^{+1} f(y)dy \tag{27}$$

$$=2\left[\frac{C_1}{m_1}\sinh m_1+\frac{C_2}{m_2}\sinh m_2+\frac{A_3}{A_2}\right]\mathbf{e}^{-nt}, \tag{28}$$

$$\phi_p=\int_{-1}^{+1} v dy = \mathbf{e}^{-nt}\int_{-1}^{+1} g(y)dy \tag{29}$$

$$=\frac{2\lambda}{A_6}\left(\frac{C_1}{m_1}\sinh m_1+\frac{C_2}{m_2}\sinh m_2+\frac{A_3}{A_2}\right). \tag{30}$$

8 Heat Transfer

The rate of heat transfer i.e., the heat transfer coefficient in terms of Nusselt number (N_u) at the plates is

$$N_u = \left[\frac{\partial T}{\partial y}\right]_{y=\pm 1} = \mathbf{e}^{-2nt}\left[\frac{\partial F}{\partial y}\right]_{y=\pm 1} =$$

$$\left[\sqrt{A_4}\left\{-C_3 \sin\sqrt{A_4} + C_4 \cos\sqrt{A_4}\right\} - \right.$$

$$\left. -2\left\{m_1 S_1 \mathbf{e}^{\pm 2m_1} + m_2 S_2 \mathbf{e}^{\pm 2m_2} + S_3(m_1+m_2)\mathbf{e}^{\pm(m_1+m_2)}\right\}\right]. \quad (31)$$

9 Results and discussion

The aim of our study is to investigate the effect of stratification factor n at different constant values of magnetic field parameters M (magnetic Hartmann number) and the porosity parameter (k) of the medium on both velocity and temperature distribution for fluid and particles. Numerical results of equations (20-22) are obtained for constant values of M (0.5 and 2.5), k (0.1 and 10.0) when $R = 5.0$, $Pr = 0.71$, $\lambda = 1.5$, $\eta = 0.1$, $E = 0.01$, $C = 0.1$, $a_0 = 1.0$ and $\chi = 2.0$, are shown in the Figures 1.-2.

Following results are observed from the figures.

(i) At the lower value of M ($= 0.5$) and k ($= 1.0$), f and g decrease steadily with increase of stratification factor ($n \cong 0.1$ to 5.0);

(ii) With the increase of M ($\cong 0.5$ to 2.5), f and g decrease for all values of n within ($n \cong 0.1$ to 1.0) but gradually increase for all values of n within ($n \cong 0.1$ to 5.0).These values are minimum at $n = 5.0$ when $M = 0.5$ and at $n = 0.1$ when $M = 2.5$.

(iii)With the increase of k (0.1 to 10.0), f and g decrease sharply for all values of n within ($n \cong 0.1$ to 5.0). At $k = 10.0$, theses values are very small. They are minimum at $n = 5.0$ when $k = 0.1$ and at $n = 0.1$ when $k = 10.0$.

10 Conclusions

- The variations of fluid and particle velocity, and the temperature with the variation of stratification factor, significantly depend upon both the applied magnetic field as well as the porosity of the medium.

- With the increase applied field, field velocity, particle velocity and the temperature decrease in the lower range of the stratification factor but increase in the higher range of it.

- The fluid velocity, particle velocity and the temperature fall rapidly with the increase of porosity of the medium for all values of stratification factor. At higher values of porosity, fluid velocity is small enough.

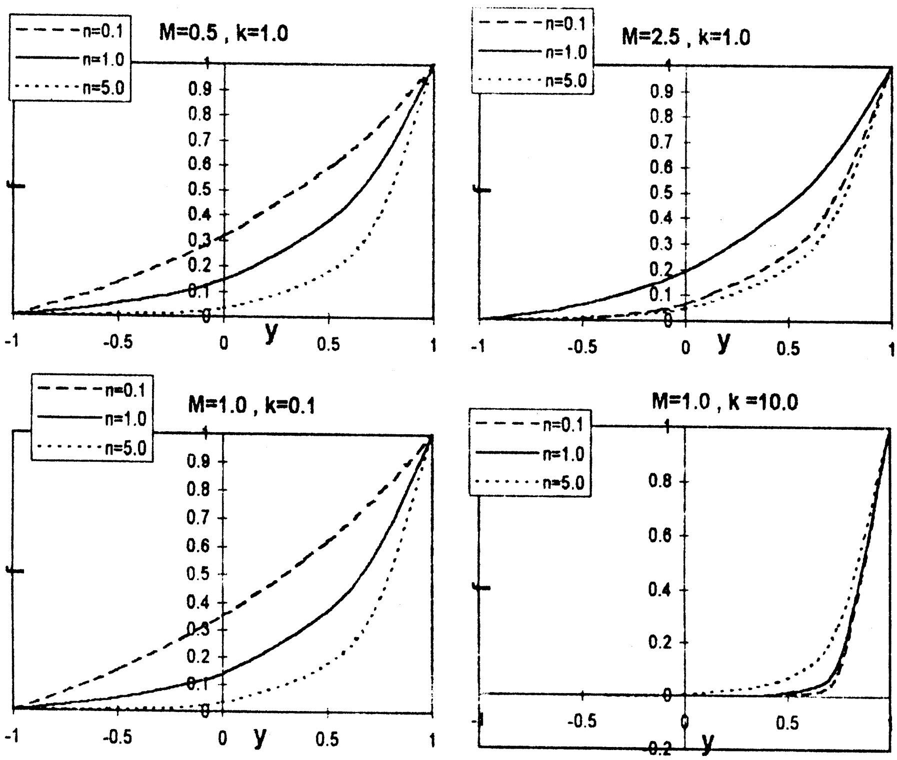

Fig.1. Fluid velocity distribution $(f\ Vs\ y) : \lambda = .1$, $Rt = .1$, $C = 0.1$, $\eta = 0.1$, $d = 0.1$, $\Pr = 0.71$, $E = 0.1$, $R = 0.5$

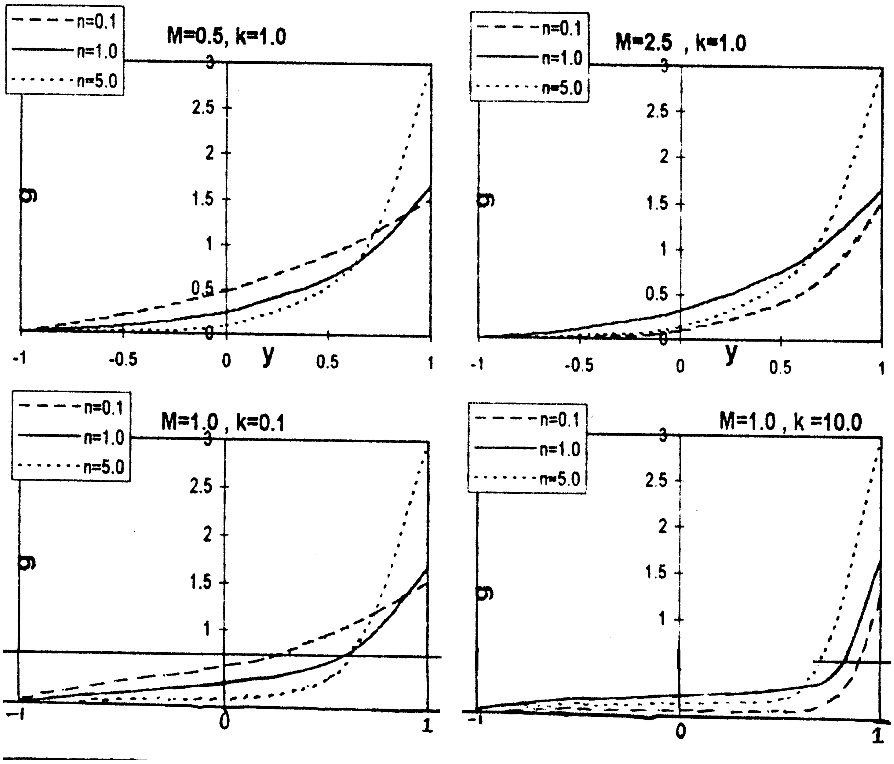

Fig.2. Particle velocity distribution $(g\ Vs\ y)$: $\lambda = .1$, $Rt = .1$, $C = 0.1$, $\eta = 0.1$, $d = 0.1$, $\Pr = 0.71$, $E = 0.1$, $R = 0.5$

References

[1] J.N.Kapur and J.B.Sukhia, ZAMM, 44, 6 (1964), 268.

[2] K.C.Bachi, Ind.Jour.Mech.Math., III, 2, (1965), 108.

[3] R.K.Gupta and S.C.Gupta, Flow of viscous liquid through a channel with arbitrary time varying pressure gradient, ZAMP, 27, (1978), 119-125.

[4] P.K.Purkait, MHD transient flow of second order Rivlin-Ericksen fluid down an inclined channel, Ph.D. Thesis, Agra University Chapter II, (ii), (1984).

[5] S.Lahiri and G.K.Ganguly, Ind.Jour.Theo.Phys., 34, (1986), 71.

[6] S.S.Sisodie and M.Gupta. Unsteady flow of a dusty viscous flow through a circular and coaxial circular ducts., Ind.Jour.Pure and Appl.Math's, 4 (1), (1986), 21-24.

[7] Lal, Banwari and A.K.Johri, MHD transient flow of second order Rivlin-Ericksen fluid down an inclined porous channel, Ind.Jour.Theo.Phys., 38, 4, (1990), 271-282.

[8] P.R.Sengupta and P.Supriye, Proc.Math.Soc.BHU, 7, (1991), 41.

[9] A.K.Singh and N.P.Singh, MHD flow and heat transfer of a dusty visco-elastic liquid down an inclined channel in porous medium, 43, 4, (1995), 193-302.

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MHD tečenje i prenos toplote prašnjavog slojevitog viskoelastičnog fluida niz nagnuti tunel u poroznoj sredini sa promenljivom viskoznoscu

UDK 532.522; 537.312.7

Proučava se laminarno konvektivno tečenje nestišljivog elektroprovodnog slojevitog viskoelastičnog fluida drugog reda u poroznoj sredini niz paralelni pločasti kanal nagnut pod uglom θ u odnosu na horizontalnu površ u prisustvu uniformnog poprečnog magnetnog polja. Ploče se održavaju na temperaturuma koje opadaju tokom vremena. Za fluidne osobine gustinu i viskoznost predpostavlja se da se menjaju kroz različite slojeve unutar tunela. Rešenja su dobijena korišćenjem perturbacione metode za fluidnu brzinu, brzinu čestica i temperaturnu distribuciju fluida unutar tunela. Njihove promene u tunelu su prikazane

grafički za razne vrednosti stratifikacionog faktora pri konstantnim vrednostima parametra magnetnog polja i parametra poroznosti. Izrazi za viskozno povlačenje, brzinu promene prenosa toplote na pločama, fluks za fluid i česticu su takodje dobijeni.