THEORETICAL AND APPLIED MECHANICS
25, pp. 61-75, 1999, UDK 517.933

On the Newtonian forces having Mayer's potential

M. M. Lukačević and V. M. Čović

Submitted 16 September 1998

Abstract

One proves that the Newtonian force acting on the particle, if it is derivable from the corresponding Mayer's potential, has, in the most general case, the form of the Lorentz force. The necessary and sufficient conditions providing the existence of the mentioned Mayer's potential, obtained in the case of the holonomic rheonomic dynamical system by the authors in [2], reduce, in the case of the system consisting of a free particle, which is considered in this paper, to the equations having the form of the Maxwell's equations. One further proves that the Coriolis force, which can be considered as the Newtonian force in the dynamics of relative motion, is derivable from the corresponding Mayer's potential, which one determines, only in the case of a constant angular velocity of the moving frame. Finally, the obtained result is illustrated by an example.

1 Introduction

Starting from the fact that the only generalized Newtonian forces which can aspire to be derivable from the corresponding Mayer's potential are the forces linear with respect to the generalized velocities, i.e. the forces having the form

$$Q_i = b_{ij}\dot{q}^j + b_{i0},$$
$$b_{ij} = b_{ij}\left(q^k, t\right), \quad b_{i0} = b_{i0}\left(q^k, t\right), \tag{1}$$

and that the potential itself, if it exists, must also be linear with respect to the generalized velocities (see [1])

$$V = -A_i\dot{q}^j - A_0,$$
$$A_i = A_i\left(q^k, t\right), \quad A_0 = A_0\left(q^k, t\right), \tag{2}$$

in our paper [2] we proved that, besides (1), the following necessary and sufficient conditions have to be satisfied

$$b_{ij} = -b_{ji}, \tag{3}$$

$$\frac{\partial b_{ij}}{\partial q^k} + \frac{\partial b_{jk}}{\partial q^i} = \frac{\partial b_{ik}}{\partial q^j}, \quad \begin{array}{c} i = 1, 2, ..., n-2, \\ j = i+1, ..., n-1, \\ k = j+1, ..., n, \end{array} \tag{4}$$

and

$$\frac{\partial b_{ij}}{\partial t} + \frac{\partial b_{j0}}{\partial q^i} = \frac{\partial b_{i0}}{\partial q^j}, \quad \begin{array}{c} i = 1, 2, ...n-1, \\ j = i+1, ..., n, \end{array} \tag{5}$$

to provide that the generalized forces Q_i are derivable from the potential (2).

In the above relations the indices take the values from 1 to n, n being the number of the system's degrees of freedom. By q^i we denote the Lagrangian coordinates of the system, by $\dot{q}^i$ - the corresponding generalized velocities, and by t - time. The repeated indices mean summation.

It should be noticed that in the case of the system consisting of a free particle acted on by the Newtonian force, the conditions (1), (3), (4) and (5) present also the conditions which provide the conservativity in the sense of Santilli of this force [2-4].

2 General case of the Newtonian force which is derivable from the Mayer's potential

Let us consider the Newtonian force

$$\mathbf{F} = \mathbf{F}(\mathbf{r}, \mathbf{v}, t), \tag{6}$$

acting on the particle whose radius vector and velocity are denoted by $\mathbf{r}$ and $\mathbf{v}$ respectively. We begin by proving the following theorem concerning the force (6):

Theorem 1 *Newtonian force given by (6) can be derived from the corresponding Mayer's potential if, and only if, it can be presented in the form*

$$\mathbf{F} = \mathbf{v} \times rot\ \mathbf{A} - \frac{\partial \mathbf{A}}{\partial t} + grad\ A_0, \tag{7}$$

where

$$\mathbf{A} = \{A_i(q^1, q^2, q^3, t)\}, \quad i = 1, 2, 3,$$

$$A_0 = A_0(q^1, q^2, q^3, t),$$

with the functions A_i, A_0, as well as their first and second partial derivatives, defined and continuous in some domain of $q^1, q^2, q^3,\ t$.

Remark 1 *The form (7) has, e.g., the Lorentz force, for which, if we take for simplicity that the speed of light $c = 1$,*

$$\mathbf{A} = e\mathbf{B}, \quad A_0 = -e\varphi, \tag{8}$$

where e denotes the charge of the particle, and where $\mathbf{B}$ and φ are the potentials of the electromagnetic field in which the charged particle moves.

To prove the theorem, let us suppose that for the generalized forces

$$Q_i = \mathbf{F} \cdot \frac{\partial \mathbf{r}}{\partial q^i}, \quad i = 1, 2, 3, \tag{9}$$

which correspond to the Newtonian force (6), the necessary and sufficient conditions for the existence of Mayer's potential, given by (1), (3), (4) and (5), are satisfied. Then we have the relations (see [1,2])

$$Q_i = \frac{d}{dt}\frac{\partial V}{\partial \dot{q}^i} - \frac{\partial V}{\partial q^i}, \tag{10}$$

which, using (2), lead to

$$Q_i = \left(\frac{\partial A_j}{\partial q^i} - \frac{\partial A_i}{\partial q^j}\right)\dot{q}^j - \frac{\partial A_i}{\partial t} + \frac{\partial A_0}{\partial q^i}, \tag{11}$$

wherefrom, by comparing with (1), we obtain

$$b_{ij} = \frac{\partial A_j}{\partial q^i} - \frac{\partial A_i}{\partial q^j}, \tag{12}$$

$$b_{i0} = \frac{\partial A_0}{\partial q^i} - \frac{\partial A_i}{\partial t}. \tag{13}$$

Having now in mind that in the case considered the indices i, j, just like all the other indices which will be used throughout the paper, take the values $1, 2, 3$, we can write

$$\frac{\partial A_j}{\partial q^i} - \frac{\partial A_i}{\partial q^j} = \delta_{ij}^{kr}\frac{\partial A_r}{\partial q^k}.$$

Further, as the Kronecker's delta symbol of the fourth order, δ_{ij}^{kr}, can be expressed, using the Ricci's skewsymmetric tensors ε^{krs} and ε_{ijs}, in the form

$$\delta_{ij}^{kr} = \varepsilon^{krs}\varepsilon_{ijs},$$

we have

$$\left(\frac{\partial A_j}{\partial q^i} - \frac{\partial A_i}{\partial q^j}\right)\dot{q}^j = \varepsilon_{ijs}\dot{q}^j h^s, \tag{14}$$

where we put

$$h^s = \varepsilon^{skr}\frac{\partial A_r}{\partial q^k}. \tag{15}$$

Using (14), for the force (11) we obtain

$$Q_i = \varepsilon_{ijs}\dot{q}^j h^s - \frac{\partial A_i}{\partial t} + \frac{\partial A_0}{\partial q^i},$$

or, expressed in vector notation,

$$\mathbf{F} = \mathbf{v}\times\mathbf{h} - \frac{\partial \mathbf{A}}{\partial t} + grad\ A_0, \tag{16}$$

where, in accordance with (15),

$$\mathbf{h} = \left\{\varepsilon^{skr}\frac{\partial A_r}{\partial q^k}\right\} = rot\ \mathbf{A}, \tag{15'}$$

so that (16) gets the form (7), and the Theorem 1 is proved.

The condition (4), in the case considered, i.e. for $n = 3$, gets the form

$$\frac{\partial b_{12}}{\partial q^3} + \frac{\partial b_{23}}{\partial q^1} = \frac{\partial b_{13}}{\partial q^2},$$

which, in virtue of (15) and (12), leads to

$$div\ \mathbf{h} = 0. \tag{17}$$

Similarly, for $n = 3$, (5) reduces to the relations

$$\frac{\partial b_{12}}{\partial t} + \frac{\partial b_{20}}{\partial q^1} = \frac{\partial b_{10}}{\partial q^2},$$

$$\frac{\partial b_{13}}{\partial t} + \frac{\partial b_{30}}{\partial q^1} = \frac{\partial b_{10}}{\partial q^3},$$

$$\frac{\partial b_{23}}{\partial t} + \frac{\partial b_{30}}{\partial q^2} = \frac{\partial b_{20}}{\partial q^3},$$

which, introducing the vector

$$\mathbf{G} = -\frac{\partial \mathbf{A}}{\partial t} + grad\ A_0, \tag{18}$$

and having in mind (12),(13) and (15), can be written as

$$rot\ \mathbf{G} = -\frac{\partial \mathbf{h}}{\partial t}. \tag{19}$$

We notice that the relations (17) and (19), in the case of Lorentz force, which can be written in the form

$$\mathbf{F}_L = e\,(\mathbf{v}\times\mathbf{H}+\mathbf{E}),$$

obtained from (7) using (8) and introducing the vectors

$$\mathbf{H} = rot\ \mathbf{B}, \qquad \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} - grad\ \varphi,$$

represent the well-known Maxwell's equations

$$div\ \mathbf{H} = 0, \qquad rot\ \mathbf{E} = -\frac{\partial \mathbf{H}}{\partial t}.$$

Finally, Mayer's potential (2), from which the force (7) can be derived, expressed using vectorial notation, reads

$$V = -\mathbf{A}\cdot\mathbf{v} - A_0, \tag{20}$$

wherefrom, having in mind (8), we obtain the well-known potential corresponding to the Lorentz force

$$V_L = -e\mathbf{B}\cdot\mathbf{v} + e\varphi.$$

The Theorem 1 states that the relation (7) gives the most general form of the Newtonian force which can be derived from the corresponding Mayer's potential. But reducing the given Newtonian force to this form, which is possible only in the case if the conditions (1), (3), (4) and (5) are satisfied, needs that both the vector and scalar potentials, $\mathbf{A}$ and A_0, i.e. the corresponding Mayer's potential (20), must be determined first. That is why we shall now establish one another formulation of the Theorem 1, the formulation which may appear more convenient in some cases when we have to examine whether the conditions providing the existence of the Mayer's potential are satisfied for the given Newtonian force. This formulation we give in the form of the following theorem:

Theorem 2 *Newtonian force, given by (6), can be derived from the corresponding Mayer's potential if, and only if, it can be presented in the form*

$$\mathbf{F} = \mathbf{v} \times \mathbf{h} + \mathbf{G}, \tag{21}$$

where the vectors $\mathbf{h}$ *and* $\mathbf{G}$ *satisfy the relations (17) and (19), i.e., the relations*

$$div\ \mathbf{h} = 0,$$

and

$$rot\ \mathbf{G} = -\frac{\partial \mathbf{h}}{\partial t}.$$

The proof of the Theorem 2 becomes evident if we remember that, as a consequence of the conditions (1) and (3), the force (6) must have the form (21), where

$$\mathbf{h} = \{b_{23}, b_{31}, b_{12}\},$$

and

$$\mathbf{G} = \{b_{10}, b_{20}, b_{30}\},$$

and that the conditions (4) and (5), as it was already demonstrated, lead to the equations (17) and (19).

If the conditions from the Theorem 2 are satisfied, Mayer's potential is given, as it was proved in [2], by

$$V = -\dot{q}^2 \left[\int_{c_1}^{q^1} b_{12} dq^1\right] - \dot{q}^3 \left[\int_{c_1}^{q^1} b_{13} dq^1 + \int_{c_2}^{q^2} b_{23}\left(q^1{=}c_1, q^2, q^3\right) dq^2\right] - \left[\int_{c_1}^{q^1} b_{10} dq^1 + \int_{c_2}^{q^2} b_{20}\left(q^1{=}c_1, q^2, q^3\right) dq^2 + \int_{c_3}^{q^3} b_{23}\left(q^1{=}c_1, q^2{=}c_2, q^3\right) dq^3\right], \tag{22}$$

where c_1, c_2 and c_3 are any constant values of q^1, q^2 and q^3 respectively, belonging to the relevant domain of q^1, q^2, q^3.

3 The case of Coriolis force

We further consider the Coriolis force

$$\mathbf{F}^{in}_{Cor} = -2m\boldsymbol{\omega} \times \mathbf{u}, \tag{23}$$

where m is the mass of the particle M which moves with respect to the frame of reference $A\xi\eta\zeta$ whose motion relative to the Newtonian base is prescribed by the velocity $\mathbf{v}_A = \mathbf{v}_A(t)$ of the origin A and by the angular velocity $\boldsymbol{\omega} = \boldsymbol{\omega}(t)$, while $\mathbf{u}$ denotes the relative velocity of M with respect to the moving frame $A\xi\eta\zeta$.

Remembering that

$$\mathbf{u} = \mathbf{v} - \mathbf{v}_A - \boldsymbol{\omega} \times \boldsymbol{\rho}, \tag{24}$$

where $\mathbf{v}$ denotes the absolute velocity of M, and where $\boldsymbol{\rho} = \overrightarrow{AM}$, the force (23) can be written in the form

$$\mathbf{F}^{in}_{Cor} = \mathbf{v} \times 2m\boldsymbol{\omega} + 2m\boldsymbol{\omega} \times \mathbf{v}_A + 2m(\boldsymbol{\omega} \cdot \boldsymbol{\rho})\boldsymbol{\omega} - 2m\omega^2\boldsymbol{\rho}, \tag{25}$$

or, in tensor notation,

$$C_i = 2m\varepsilon_{ijk}\omega^k \dot{q}^j + 2m\varepsilon_{ijk}\omega^j v^k_A + 2m\omega_k\rho^k\omega_i - 2m\omega^2\rho_i, \tag{25'}$$

where we introduced the denotations for the covariant and contravariant coordinates of the vectors appearing in (25) as follows

$$C_i = \mathbf{F}^{in}_{Cor} \cdot \mathbf{g}_i, \quad \omega_i = \boldsymbol{\omega} \cdot \mathbf{g}_i, \quad \rho_i = \boldsymbol{\rho} \cdot \mathbf{g}_i \quad v_{Ai} = \mathbf{v}_A \cdot \mathbf{g}_i,$$

$$\omega^i = g^{ij}\omega_j, \quad \rho^i = g^{ij}\rho_j, \quad v^i_A = g^{ij}v_{Aj},$$

where $\mathbf{g}_i = \partial\mathbf{r}/\partial q^i$ are the basis vectors in the system of coordinates q^i, while g^{ij} are the contravariant coordinates of the fundamental tensor $g_{ij} = \mathbf{g}_i \cdot \mathbf{g}_j$.

By comparing (25') with (1), we find that, in the case of Coriolis force,

$$b_{ij} = 2m\varepsilon_{ijk}\omega^k, \qquad b_{i0} = 2m\varepsilon_{ijk}\omega^j v^k_A + 2m\omega_k\omega_i\rho^k - 2m\omega^2\rho_i,$$

wherefrom we obtain

$$\frac{\partial b_{ij}}{\partial q^k} = 2m\Gamma^r_{ik}\varepsilon_{rjs}\omega^s + 2m\Gamma^r_{jk}\varepsilon_{irs}\omega^s,$$

$$\frac{\partial b_{ij}}{\partial t} = 2m\varepsilon_{ijk}\frac{\partial \omega^k}{\partial t},$$

$$\frac{\partial b_{i0}}{\partial q^j} = 2m\Gamma^r_{ij}\varepsilon_{rks}\omega^r v^s_A + 2m\omega_i\omega_j + 2m\Gamma^r_{ij}\omega_k\rho^k\omega_r - 2m\omega^2 g_{ij} - 2m\omega^2\Gamma^r_{ij}\rho_r,$$

where Γ^r_{ij} are the Christoffel symbols with respect to the metric g_{ij}. It is now easy to verify that the conditions (3) and (4) are always satisfied, while the condition (5) leads to

$$2m\varepsilon_{ijk}\frac{\partial \omega^k}{\partial t} = 0,$$

i.e.

$$\frac{\partial \omega^k}{\partial t} = 0,$$

wherefrom, remembering that we start with $\boldsymbol{\omega} = \boldsymbol{\omega}(t)$, we conclude that the Coriolis force can be obtained from the corresponding Mayer's potential if, and only if,

$$\boldsymbol{\omega} = const. \tag{26}$$

In order to determine this potential, we shall first seek, supposing that (26) holds, what are, in the case considered, the vector $\mathbf{A} = \{A_i\}$ and the scalar function A_0.

We begin by the fact, which is not difficult to verify, that $2m\omega^k$, which appears in the first member on the right-hand side of the equation (25'), can be written in the form

$$2m\omega^k = \varepsilon^{ksp}\varepsilon_{jrp}m\omega^j\delta^r_s, \tag{27}$$

where δ^r_s is the Kronecker delta symbol.

Introducing further the vector

$$K_p = \varepsilon_{jrp}m\omega^j\rho^r, \tag{28}$$

and having in mind that

$$\nabla_s K_p = \varepsilon_{jrp} m\omega^j \delta^r_s,$$

where ∇_s denotes covariant differentiation with respect to q^s, for (27) we obtain

$$2m\omega^k = G\varepsilon^{ksp}\nabla_s K_p = \varepsilon^{ksp}\frac{\partial K_p}{\partial q^s}, \tag{29}$$

or, expressed in vector notation,

$$2m\boldsymbol{\omega} = rot\ \mathbf{K}, \tag{30}$$

where, according to (28),

$$\mathbf{K} = m\boldsymbol{\omega}\times\boldsymbol{\rho}. \tag{31}$$

Now, using (29) and introducing the vector

$$D_i = m\varepsilon_{ijk}\omega^j v^k_A + 2m\omega_k\omega_i\rho^k - 2m\omega^2\rho_i, \tag{32}$$

which obviously can be expressed as the partial derivative

$$D_i = \frac{\partial\Phi}{\partial q^i},$$

where

$$\Phi = m\varepsilon_{rjk}\omega^j v^k_A\rho^r + m\left(\omega_k\rho^k\right)^2 - m\omega^2\rho^2, \tag{33}$$

for (25') we obtain

$$C_i = \varepsilon_{ijk}\varepsilon^{ksp}\frac{\partial K_p}{\partial q^s}\dot{q}^j + m\varepsilon_{ijk}\omega^j v^k_A + \frac{\partial\Phi}{\partial q^i},$$

or, in vector notation,

$$\mathbf{F}^{in}_{Cor} = \mathbf{v}\times rot\ \mathbf{K} + m\boldsymbol{\omega}\times\mathbf{v}_A + grad\ \Phi. \tag{34}$$

If we now remember that

$$\boldsymbol{\rho} = \mathbf{r} - \mathbf{r}(t), \quad \mathbf{r}_A = \overrightarrow{OA},$$

wherefrom

$$\frac{\partial \boldsymbol{\rho}}{\partial t} = -\mathbf{v}_A,$$

we obtain

$$m\boldsymbol{\omega} \times \mathbf{v}_A = -m\boldsymbol{\omega} \times \frac{\partial \boldsymbol{\rho}}{\partial t},$$

i.e., in virtue of (31),

$$m\boldsymbol{\omega} \times \mathbf{v}_A = -\frac{\partial \mathbf{K}}{\partial t},$$

and the force (34) finally gets the form

$$\mathbf{F}_{Cor}^{in} = \mathbf{v} \times rot\ \mathbf{K} - \frac{\partial \mathbf{K}}{\partial t} + grad\ \Phi, \tag{35}$$

from which, after comparing with (7), we conclude that in the case of Coriolis force, under the condition (26), we have

$$\mathbf{A} = \mathbf{K} = m\boldsymbol{\omega} \times \boldsymbol{\rho}, \qquad A_0 = \Phi. \tag{36}$$

The potential which corresponds to the force (35), in virtue of (20) and (36), has the form

$$V_{Cor} = -(m\boldsymbol{\omega} \times \boldsymbol{\rho}) \cdot \mathbf{v} - \Phi,$$

i.e., using (33),

$$V_{Cor} = -(m\boldsymbol{\omega} \times \boldsymbol{\rho}) \cdot \mathbf{v} - (m\boldsymbol{\omega} \times \mathbf{v}_A) \cdot \boldsymbol{\rho} - m(\boldsymbol{\omega} \times \boldsymbol{\rho})^2 + m\omega^2\rho^2. \tag{37}$$

It is now of interest to find the form of the potential (37) in which the relative velocity of the particle, $\mathbf{u}$, will appear instead of its absolute velocity, $\mathbf{v}$. If we take into account that

$$\mathbf{v} = \mathbf{u} + \mathbf{v}_A + \boldsymbol{\omega} \times \boldsymbol{\rho},$$

and that

$$(\boldsymbol{\omega} \times \boldsymbol{\rho}) \cdot (\boldsymbol{\omega} \times \boldsymbol{\rho}) = \omega^2\rho^2 - (\boldsymbol{\omega} \cdot \boldsymbol{\rho})^2,$$

from (37) we easily obtain the potential in the form

$$V_{Cor} = -(m\boldsymbol{\omega} \times \boldsymbol{\rho}) \cdot \mathbf{u}, \tag{38}$$

which is appropriate if the Coriolis force is, as usually, given by (23).

4 An Example

Determine the Mayer's potential in the case of Coriolis force acting on the particle M having mass m, which moves relative to the Earth.

We consider the Earth as a sphere which rotates with the constant angular velocity ω about its axis, which, in this problem, we can take as being at rest.

The origin of the Cartesian frame $A\xi\eta\zeta$, which moves together with Earth, we choose at the point A on the Earth northern hemispheres's surface, the point near which we take that the particle M moves. We denote the latitude of this point by φ, the axis $A\zeta$ we take upwards along the line of apparent gravity in A (Fig. 1). The axis $A\xi$ is tangent to the meridian and directed to the south, while the axis $A\eta$ is tangent to the parallel and directed to the east. The plane $\xi = 0$ is not strictly the tangent plane to the sphere, but the deviation is slight for points near to A.

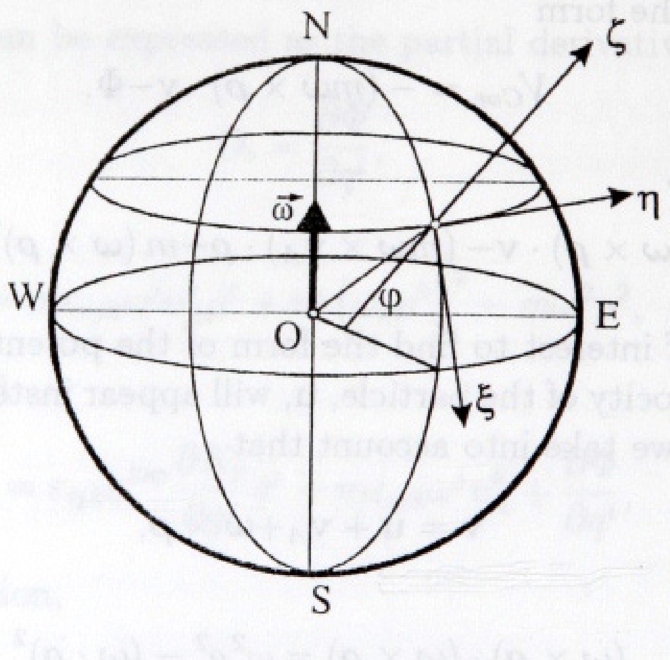

Fig. 1. System under consideration.

Having now in mind that the coordinates of the vectors ω and $\mathbf{u}$

with respect to the frame $A\xi\eta\zeta$ are

$$\left.\begin{aligned} \boldsymbol{\omega} &= \{-\omega\cos\varphi, 0, \omega\sin\varphi\}, \\ \mathbf{u} &= \{\dot{\xi}, \dot{\eta}, \dot{\zeta}\} \end{aligned}\right\} \tag{39}$$

from (23) we easily determine the coordinates of the Coriolis force in this case

$$\left.\begin{aligned} C_\xi &= C_1 = 2m\omega\dot{\eta}\sin\varphi, \\ C_\eta &= C_2 = -2m\omega\dot{\xi}\sin\varphi - 2m\omega\dot{\zeta}\cos\varphi, \\ C_\zeta &= C_3 = 2m\omega\dot{\eta}\cos\varphi, \end{aligned}\right\} \tag{40}$$

wherefrom, using the denotations

$$\xi = q^1, \quad \eta = q^2, \quad \zeta = q^3,$$

and then comparing (40) with (1), we find

$$\left.\begin{aligned} b_{12} = 2m\omega\sin\varphi, \quad b_{13} = 0, \quad b_{23} = -2m\omega\cos\varphi \\ b_{10} = b_{20} = b_{30} = 0. \end{aligned}\right\} \tag{41}$$

The relation (22), in which we take $c_1 = c_2 = 0$, leads now to the Mayer's potential

$$V_{Cor} = -2m\omega\xi\dot{\eta}\sin\varphi + 2m\omega\eta\dot{\zeta}\cos\varphi. \tag{42}$$

The Mayer's potential corresponding to the Coriolis force (40) can also be obtained, of course, directly from (38). In this case, using (39) and having in mind that the coordinates of the vector $\boldsymbol{\rho}$ are

$$\boldsymbol{\rho} = \{\xi, \eta, \zeta\},$$

we find the expression

$$V_{Cor}^{(1)} = m\omega\eta\dot{\zeta}\sin\varphi - m\omega\left(\xi\sin\varphi + \zeta\cos\varphi\right)\dot{\eta} + m\omega\eta\dot{\zeta}\cos\varphi, \tag{43}$$

which differs from (42) by the term

$$\frac{d}{dt}\left(m\omega\eta\xi\sin\varphi - m\omega\eta\zeta\cos\varphi\right).$$

This term, being a total derivative with respect to time of a function depending on the coordinates ξ, η, ζ, has no effect, as it is well-known, in the forces (40).

Finally, it is worthy to notice that the condition (26), which holds in this example, presents one of the two mutually independent conditions under which, in the case of a system of particles, the Mayer's potential corresponding to the Coriolis forces of the system, exists [2]. In the case of the system consisting of a free particle, the sole condition providing the existence of the Mayer's potential from which the Coriolis force acting on the particle can be derived is given by (26).

References

[1] L. A. Pars, A treatise on analytical dynamics, Heinemann, London, 1965.

[2] V. M. Čović, and M. M. Lukačević, On the existence of Mayer's potential, ASME Journal of Appl. Mech., 64, (1997).

[3] R. M. Santilli, Foundations of theoretical mechanics, I, Springer-Verlag, New York, 1978.

[4] R. M. Santilli, Foundations of theoretical mechanics, II, Springer-Verlag, New York, 1983.

M. M. Lukačević
V. M. Čović

University of Belgrade
Faculty of Mechanical Engineering
27. Marta 80
11000 Belgrade
Yugoslavia

O njutnovskim silama koje imaju Majerov potencijal

Dokazuje se da njutnovska sila koja dejstvuje na materijalnu tačku, ukoliko može biti izvedena iz odgovarajućeg Majerovog potencijala, ima, u najopštijem slučaju, oblik Lorencove sile. Pri tome se potrebni i dovoljni uslovi koji obezbedjuju egzistenciju pomenutog Majerovog potencijala, a koje su autori za slučaj holonomnog reonomnog dinamičkog sistema dobili u svome radu [2], svode u slučaju sistema koji se sastoji od jedne slobodne materijalne tačke, koji se u ovom radu razmatra, na jednačine koje imaju oblik Maksvelovih jednačina.

Dalje se dokazuje da Koriolisova sila, koja se u dinamici relativnog kretanja formalno razmatra kao njutnovska, može biti izvedena iz odgovarajućeg Majerovog potencijala, koji se pri tome i odredjuje, samo u slučaju da je ugaona brzina pokretnog koordinatnog sistema konstantna. Dobijeni rezultat je ilustrovan jednim primerom.