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Minimum weight design of statically indeterminate beams with normal stress constraint

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Abstract

An optimization problem of linear elastic statically indeterminate beams is discussed. The cross-section of the beam is variable and determined for the equal stress criterion. The problem is solved with the aid of Castiglianos second theorem. Some optimization conditions are deduced. A few illustrating examples are given.

1 Introduction

In the minimum weight design of beams two general types of problems are dealt with, i.e., those concerning the determination of the shape of a beam subject to various external actions under conditions that stress [1], [2] or strain [3], [4], [5] state of a body being prescribed. The second type of the problem prevails in investigations within structural design. In design with stress constraints only statically determinate problems were dealt with and the shape of the cross-sectional area is simple (rectangular or circular). The optimum design is usually found by applying technics of classical methods of calculus of variations [3], [4] or optimal control theory [6], [7], or by the method of finite differences, finite elements [8] etc. It would be of interest to determine optimal shape of statically indeterminate beam and to use methods that do not appear to have been discussed previously in the structural mechanics literature for application in optimum design.

The present paper is concerned with shape optimization corresponding to fully stressed design of statically indeterminate beams. The cross-sectional dimensions are varying in an attempt to maintain a constant maximum stress at all cross-sections and in the ideal case it is equal to the maximum allowable bending stress σ_d .This can be written in the form

$$\sigma_{\max}(z) = \frac{|M(z)|}{W_x(z)} \leq \sigma_d, \tag{1}$$

where z denotes the beam coordinate, $M(z)$ - bending moment and $W_x(z)$ - section modulus of the cross-sectional area.

2 Basic consideration and problem solution

We consider a horizontal elastic statically indeterminate beam (continuous beam, propped cantilever beam or fixed beam) subjected to transverse load. The degree of statical indeterminacy is "s". The form of cross-section is given and the area is variable, i.e., $A = A(z)$. The objective of this paper is to determine characteristic dimension of cross-section $c = c(z)$, Fig. 1, so that condition (1) holds.

The first step is the choice of statical redundances X_j, $j = 1, .., s$. All other reactions of the beam should be determinate from equilibrium equations in terms of load and redundant reactions.

The beam shown in Fig. 2 has "n" portions and for each of them the bending moment at any cross-section $M_i(z_i)$, $i = 1, .., n$ depends on redundant reactions X_j. For that reason we shall instead of $M_i(z_i)$

write $M_i\left(X_1, X_2, .., X_s, z_i\right)$.

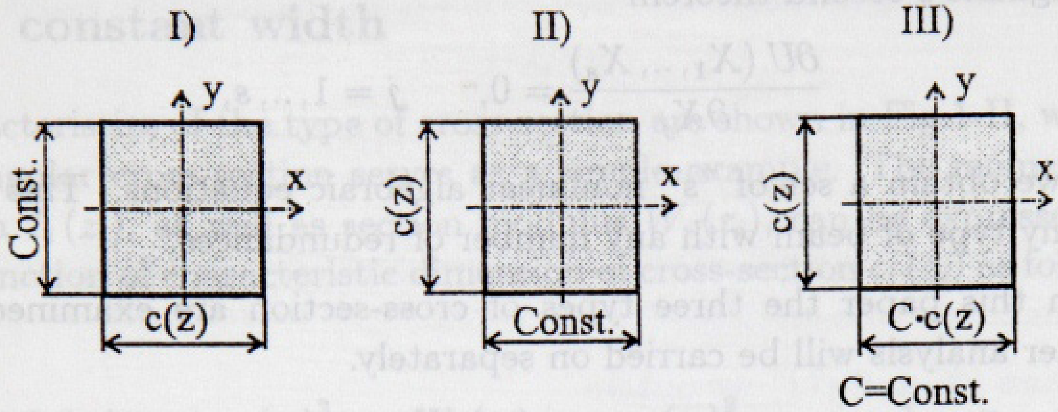

Fig. 1. Cross-section with varying dimensions.

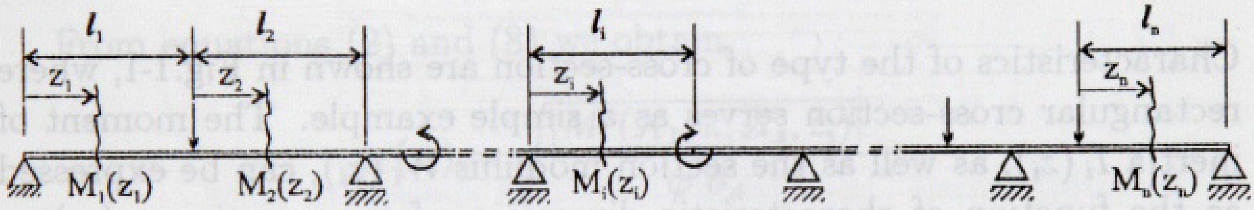

Fig. 2. Load configuration for the beam.

For each cross-section of the beam must hold the equations

$$\frac{\left|M_i\left(X_1, .., X_s, z_i\right)\right|}{W_i\left(z_i\right)}=\sigma_d, \qquad i=1, .., n, \tag{2}$$

where $W_i\left(z_i\right)$ denotes section modulus of cross-section for " i " portion of the beam.

The beam behaves linearly and there exists a positive definite strain energy which may be expressed as a quadric function of both the loads and the statical redundances X_j

$$U=\frac{1}{2E} \sum_{i=1}^{n} \int_{0}^{l_i} \frac{M_i\left(X_1, .., X_s, z_i\right)^2}{I_i\left(z_i\right)} dz_i, \tag{3}$$

where the integration is carried out over the entire length of the beam. E is modulus of elasticity and $I_i\left(z_i\right)$ moment of inertia with respect to the neutral axis x.

The unknown redundant reactions can be obtained by the use of Castigliano's second theorem

$$\frac{\partial U\left(X_1,..,X_s\right)}{\partial X_j}=0, \qquad j=1,..,s, \tag{4}$$

and we obtain a set of "s" nonlinear algebraic equations. This holds for any type of beam with any number of redundances.

In this paper the three types of cross-section are examined and further analysis will be carried on separately.

2.1 Type of cross-section with varying width and constant height

Characteristics of the type of cross-section are shown in Fig.1-I, where rectangular cross-section serves as a simple example. The moment of inertia $I_i\left(z_i\right)$, as well as the section modulus $W_i\left(z_i\right)$, can be expressed as the function of characteristic dimension of cross-section $c_i\left(z_i\right)$ as follows

$$I_i\left(z_i\right)=\psi\, c_i\left(z_i\right), \qquad W_i\left(z_i\right)=\varphi\, c_i\left(z_i\right), \qquad i=1,..,n, \tag{5}$$

where ψ and φ are constants depending on the shape of cross-section.

From equations (2) and (5) we obtain

$$c_i\left(z_i\right)=\frac{\left|M_i\left(X_1,..,X_s,z_i\right)\right|}{\varphi\,\sigma_d}$$

$$I_i\left(z_i\right)=\frac{\psi}{\varphi\,\sigma_d}\left|M_i\left(X_1,..,X_s,z_i\right)\right|, \qquad i=1,..,n. \tag{6}$$

We can also substitute (6) into (3) and thus the strain energy becomes

$$U=\frac{\varphi\,\sigma_d}{2E\psi}\sum_{i=1}^{n}\int_0^{l_i}\left|M_i\left(X_1,..,X_s,z_i\right)\right|dz_i. \tag{7}$$

Applying (4) to (7) we obtain statical redundances, and then from (6) the characteristic dimension $c_i\left(z_i\right)$.

2.2 Type of cross-section with varying height and constant width

Characteristics of the type of cross-section are shown in Fig.1-II, where rectangular cross-section serves as a simple example. The moment of inertia $I_i(z_i)$, as well as section modulus $W_i(z_i)$, can be expressed as the function of characteristic dimension of cross-section $c_i(z_i)$ as follows

$$I_i(z_i) = \psi\, c_i(z_i)^3, \quad W_i(z_i) = \varphi\, c_i(z_i)^2, \quad i = 1,..,n, \tag{8}$$

where ψ and φ are also constants depending on the shape of cross-section.

From equations (2) and (8) we obtain

$$c_i(z_i) = \sqrt{\frac{|M_i(X_1,..,X_s,z_i)|}{\varphi\,\sigma_d}},$$
$$I_i(z_i) = \frac{\psi}{\left(\sqrt{\varphi.\sigma_d}\right)^3}\left(\sqrt{|M_i(X_1,..,X_s,z_i)|}\right)^3, \quad i = 1,..,n. \tag{9}$$

By substituting (9) equation (3) becomes

$$U = \frac{\left(\sqrt{\varphi\,\sigma_d}\right)^3}{2E\psi}\sum_{i=1}^{n}\int_0^{l_i}\sqrt{|M_i(X_1,..,X_s,z_i)|}dz_i. \tag{10}$$

Following the same procedure as in previous case we get statical redundances X_j ,and then from (9) characteristic dimension $c_i(z_i)$.

2.3 Type of cross-section with varying width and height

All cross-sections are similar and equally oriented. Characteristics of the type of cross-section are shown in Fig.1-III, where rectangular cross-section serves as a simple example. The moment of inertia $I_i(z_i)$, as well as section modulus $W_i(z_i)$, can be expressed as the function of characteristic dimension of cross-section $c_i(z_i)$ as follows

$$I_i(z_i) = \psi\, c_i(z_i)^4, \qquad W_i(z_i) = \varphi\, c_i(z_i)^3, \quad i = 1, .., n. \tag{11}$$

Notice that ψ and φ are non-dimensional constants depending on the shape of cross-section.

From (2) and (11) we obtain

$$c_i(z_i) = \sqrt[3]{\frac{|M_i(X_1, .., X_s, z_i)|}{\varphi\, \sigma_d}}, \qquad I_i(z_i) = \frac{\psi}{\left(\sqrt[3]{\varphi\, \sigma_d}\right)^4}\left(\sqrt[3]{|M_i(X_1, .., X_s, z_i)|}\right)^4, \quad i = 1, .., n. \tag{12}$$

By substituting (12) into (3) the strain energy becomes

$$U = \frac{\left(\sqrt[3]{\varphi\, \sigma_d}\right)^4}{2E\psi} \sum_{i=1}^{n} \int_0^{l_i} \left(\sqrt[3]{M_i(X_1, .., X_s, z_i)}\right)^2 dz_i. \tag{13}$$

Following the same procedure as in previous cases we get statical redundances X_j, and then from(12) characteristic dimension $c_i(z_i)$. This method of solution can be easily applied to complicated structures for all three types of cross-section and given shape of cross-sectional area and various loads. The problems that occur are mathematical and due to complicated integration of equations (7), (10) and (13), determination of real solutions of nonlinear algebraic equations (4) and operations with the functions that are absolute values of other functions. Let us now study some illustrating examples.

3 Examples

As an example the technique is applied to determine the optimal design of propped cantilever beam. The centrally-loaded beam is simply supported at the end $z_1 = 0$ and built in at the end $z_2 = 1$, Fig. 3. All three types of cross-section are examined, as shown in Fig. 3.

$$M_1(z_1) = Xz_1, \quad M_2(z_2) = X(1 + z_2) - Fz_2. \tag{14}$$

Because of different form of the strain energy for each type of cross-section we in fact have three separate examples.

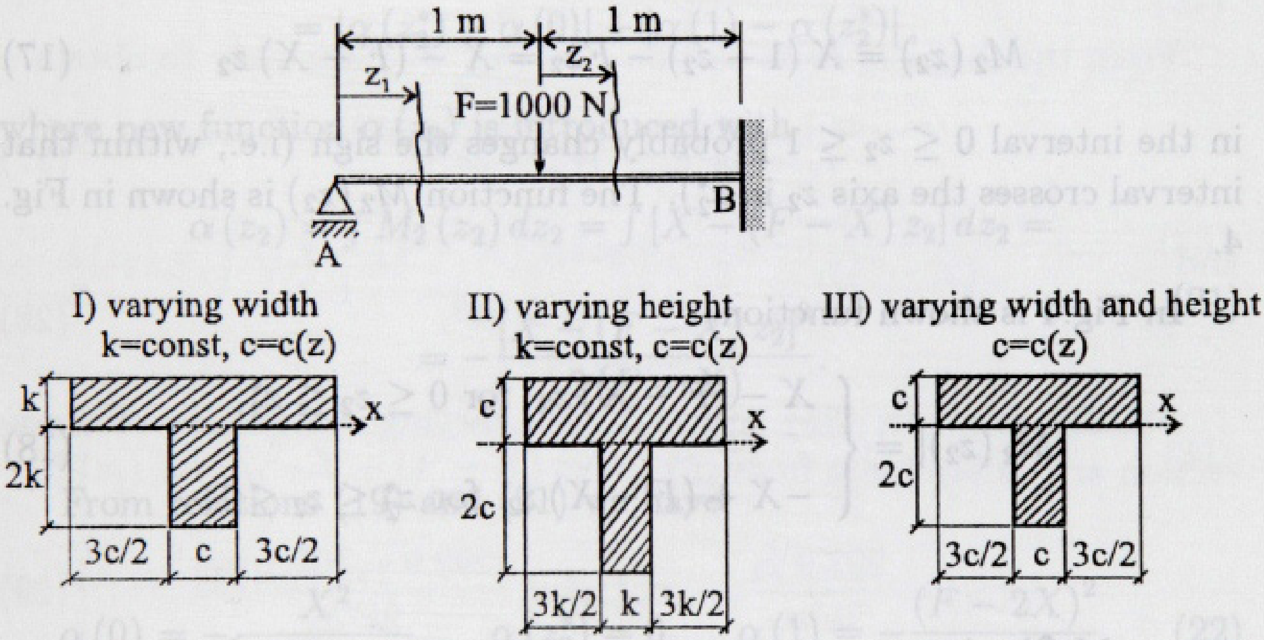

Fig. 3. The beam and three variants of cross-section.

3.1 Solution for the cross-section with varying width

It is obvious that X is positive. According to (7) which consists of two integrations, and with respect to (14), the strain energy as a function in terms of statical redundant X is

$$U=\frac{\varphi\,\sigma_d}{2E\psi}\left[\int_0^1 |M_1(X,z_1)|\,dz_1+\int_0^1 |M_2(X,z_2)|\,dz_2\right]= \tag{15}$$
$$=\frac{\varphi\,\sigma_d}{2E\psi}\left[X\int_0^1 z_1 dz_1\right]+\int_0^1 |X(1+z_2)-Fz_2|\,dz_2.$$

There is mathematical difficulty with solving the second integral

$$I_2=\int_0^1 |X(1+z_2)-Fz_2|\,dz_2, \tag{16}$$

because the linear function

$$M_2(z_2)=X(1+z_2)-Fz_2=X-(F-X)z_2 \tag{17}$$

in the interval $0\leq z_2\leq 1$ probably changes the sign (i.e., within that interval crosses the axis z_2 in z_2^*). The function $M_2(z_2)$ is shown in Fig. 4.

In Fig.4 is shown function

$$|M_2(z_2)|=\begin{cases} X-(F-X)z_2 & \text{for } 0\leq z_2\leq z_2^*,\\ -X+(F-X)z_2 & \text{for } z_2^*\leq z_2\leq 1.\end{cases} \tag{18}$$

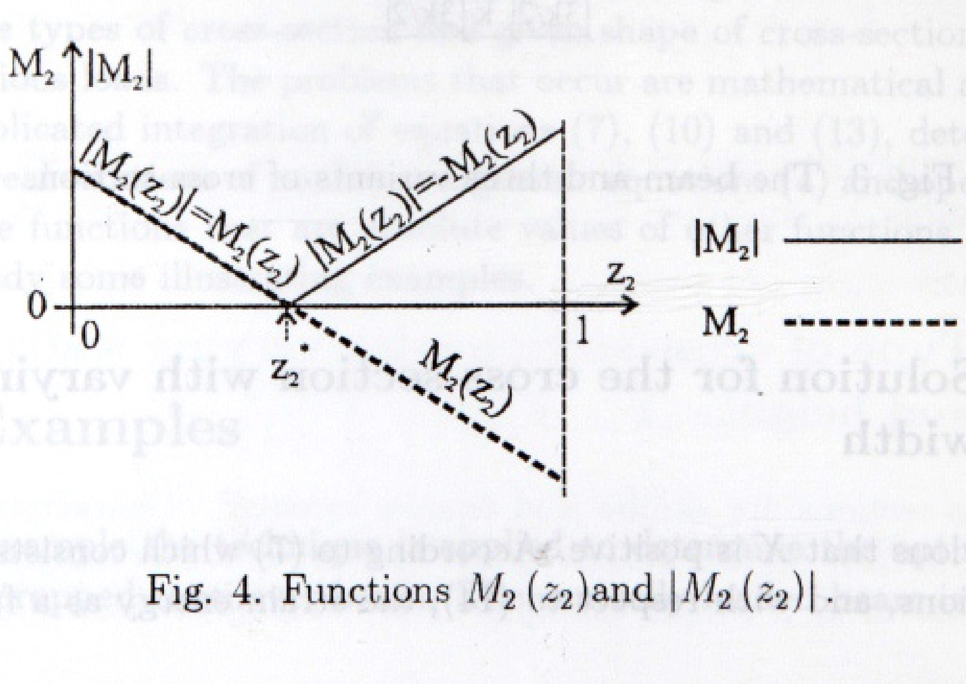

Fig. 4. Functions $M_2(z_2)$ and $|M_2(z_2)|$.

For given F the position z_2^* can be solved from (17) and condition that $M_2(z_2^*) = 0$, and so we obtain

$$z_2^* = \frac{X}{F - X}. \tag{19}$$

Mentioned difficulty can be overcome by the use of characteristic of determinate integral that holds

$$\begin{gathered} I_2 = \int_0^1 |M_2(z_2)|\, dz_2 = \int_0^{z_2^*} |M_2(z_2)|\, dz_2 + \int_{z_2^*}^1 |M_2(z_2)|\, dz_2 = \\ = |\alpha(z_2^*) - \alpha(0)| + |\alpha(1) - \alpha(z_2^*)|, \end{gathered} \tag{20}$$

where new function $\alpha(z_2)$ is introduced with

$$\begin{gathered} \alpha(z_2) = \int M_2(z_2)\, dz_2 = \int [X - (F - X) z_2]\, dz_2 = \\ = -\frac{[X - (F - X) z_2]^2}{2(F - X)}. \end{gathered} \tag{21}$$

From relations (19) and (21) we have

$$\alpha(0) = -\frac{X^2}{2(F - X)}, \quad \alpha(z_2^*) = 0, \quad \alpha(1) = -\frac{(F - 2X)^2}{2(F - X)}. \tag{22}$$

Then, introducing (22) in (20) we get

$$I_2 = \frac{X^2}{2(F - X)} + \frac{(F - 2X)^2}{2(F - X)} = \frac{X^2 + (F - 2X)^2}{2(F - X)}. \tag{23}$$

Because of (15), (16) and (23) analytical form of strain energy function is

$$U(X) = \frac{\varphi \sigma_d}{4E\psi}\left[X + \frac{X^2 + (F - 2X)^2}{F - X}\right]. \tag{24}$$

According to Castigliano's second theorem we can determine the real solution of quadratic equation

$$\frac{dU\left(X\right)}{dX}=0\Rightarrow\frac{2X^2-4FX+F^2}{\left(F-X\right)^2}=0\Rightarrow 2X^2-4FX+F^2=0,$$

$$F=1000\ [N], \tag{25}$$

for which the strain energy (24) is minimum. The solution is

$$X=F-\frac{\sqrt{2}}{2}F=292.9\ [N], \tag{26}$$

From the shape of cross-section (see Fig. 3-I) it is easy to obtain

$$I_x=4k^3c, \quad W_x=2k^2c, \tag{27}$$

and

$$\psi=4k^3,\ \varphi=2k^2. \tag{28}$$

Finally, for (14), (26) and (27), the varying dimensions of cross-section are determinate from (6).

$$c_1\left(z_1\right)=\frac{292.9\ z_1}{2k^2\sigma_d}, \quad c_2\left(z_2\right)=\frac{|292.9-707.1\ z_2|}{2k^2\sigma_d}. \tag{29}$$

3.2 Solution for the cross-section with varying height

According to (10) which consists of two integrations, and with respect to (14), the strain energy as a function in terms of statical redundant X is

$$U=\frac{\left(\sqrt{\varphi\ \sigma_d}\right)^3}{2E\psi}\left[\int_0^1\sqrt{X\ z_1}dz_1+\int_0^1\sqrt{|X-\left(F-X\right)z_2|}dz_2\right]. \tag{30}$$

Because of the function $M_2(z_2)$, (see relations (17), (18) and Fig. 4), the second integral in (30) can be calculated from

$$I_2 = \int_0^1 |M_2(z_2)|\, dz_2 = \int_0^1 \sqrt{|X - (F-X)\, z_2|}dz_2 =$$

$$= \int_0^{z_2^*} \sqrt{X - (F-X)\, z_2}dz_2 + \int_{z_2^*}^1 \sqrt{-X + (F-X)\, z_2}dz_2. \quad (31)$$

In the same manner as in previous case position of z_2^*, can be solved from (17) and condition that $M_2(z_2^*) = 0$, and so we obtain

$$z_2^* = \frac{X}{F-X}. \quad (32)$$

Let us introduce functions $\alpha(z_2)$ and $\beta(z_2)$ as

$$\alpha(z_2) = \int \sqrt{X - (F-X)\, z_2}dz_2 = -\frac{2}{3}\frac{[X - (F-X)\, z_2]^{\frac{3}{2}}}{(F-X)}, \quad (33)$$

$$\beta(z_2) = \int \sqrt{-X + (F-X)\, z_2}dz_2 = \frac{2}{3}\frac{[-X + (F-X)\, z_2]^{\frac{3}{2}}}{(F-X)}, \quad (34)$$

so that for (32) we get

$$\alpha(0) = -\frac{2}{3}\frac{X^{\frac{3}{2}}}{F-X}, \quad \alpha(z_2^*) = \beta(z_2^*) = 0,$$

$$\beta(1) = \frac{2}{3}\frac{(F-2X)^{\frac{3}{2}}}{F-X}. \quad (35)$$

Integral I_2 for (31), (33), (34) and (35) becomes

$$I_2 = |\alpha(z_2^*) - \alpha(0)| + |\beta(1) - \beta(z_2^*)| =$$

$$= \frac{2}{3}\frac{X^{\frac{3}{2}}}{F-X} + \frac{2}{3}\frac{(F-2X)^{\frac{3}{2}}}{F-X}. \quad (36)$$

Because of (30), (31) and (36) analytical form of strain energy function is

$$U(X) = \frac{\left(\sqrt{\varphi}\,\sigma_d\right)^3}{3E\psi}\left[\sqrt{X} + \frac{X^{\frac{3}{2}} + (F-2X)^{\frac{3}{2}}}{F-X}\right]. \tag{37}$$

From (37) a nonlinear algebraic equation is found

$$\frac{dU(X)}{dX} = 0,$$

$$\Rightarrow$$

$$\frac{1}{2\sqrt{X}} + \frac{\frac{3}{2}\sqrt{X} - 3\sqrt{F-2X}}{F-X} + \frac{\left(\sqrt{X}\right)^3 + \left(\sqrt{F-2X}\right)^3}{(F-X)^2} = 0, \tag{38}$$

$$F = 1000\ [N].$$

The numerical solutions of (38), obtained by the use of computer software are

$$X^{I} = 111.4\ [N], \quad X^{II} = 242.3\ [N]. \tag{39}$$

The second solution is the real one, because the strain energy is a minimum. For the first solution the strain energy is a maximum. For linear structures the redundant quantities X_j have such values as to make the strain energy a minimum, provided that there are no displacements corresponding to the redundances. The solution for statical redundant is therefore

$$X = X^{II} = 242.3\ [N]. \tag{40}$$

From the shape of cross-section (see Fig. 3-II) it is easy to obtain

$$I_x = 4kc^3, \quad W_x = 2kc^2, \tag{41}$$

and

$$\psi = 4k, \quad \varphi = 2k. \tag{42}$$

Finally, for (14), (40) and (42), the varying dimensions of cross-section are determinate from (9).

$$c_1(z_1) = \sqrt{\frac{242.3\ z_1}{2k\sigma_d}}, \quad c_2(z_2) = \sqrt{\frac{|242.3 - 757.7\ z_2|}{2k\sigma_d}}. \tag{43}$$

3.3 Solution for the cross-section with varying width and height

According to (13) and (14) the strain energy function can be obtained from the formula

$$U = \frac{\left(\sqrt[3]{\varphi\ \sigma_d}\right)^4}{2E\psi}\left[\int_0^1\left(\sqrt[3]{X\ z_1}\right)^2 dz_1 + \int_0^1\left(\sqrt[3]{X\left(1+z_2\right) - Fz_2}\right)^2 dz_2\right], \tag{44}$$

where, after the integration, we have the function of statical redundant X in the form

$$U(X) = \frac{3\left(\sqrt[3]{\varphi\ \sigma_d}\right)^4}{10E\psi}\left[\left(\sqrt[3]{X}\right)^2 + \frac{\left(\sqrt[3]{2X-F}\right)^5 - \left(\sqrt[3]{X}\right)^5}{X-F}\right],$$

$$F = 1000\ [N]. \tag{45}$$

According to (4), the derivative of (45) gives the nonlinear algebraic equation in terms of X

$$\frac{2\left(X-F\right)^2}{3^3\sqrt{X}} + \left(X-F\right)\left[\frac{10}{3}\left(\sqrt[3]{2X-F}\right)^2 - \tfrac{5}{3}\left(\sqrt[3]{X}\right)^2\right] - \left(\sqrt[3]{2X-F}\right)^5 + \left(\sqrt[3]{X}\right)^5 = 0. \tag{46}$$

Equation (46) has the following two real solutions

$$X^{I} = 28.8 \ [N], \quad X^{II} = 271.6 \ [N] \quad \text{for} \quad F = 1000 \ [N]. \tag{47}$$

From (45) we can see clearly that the solution X^I should be rejected, because the strain energy is a maximum. The solution for statical redundant is

$$X = X^{II} = 271.6 \ [N], \tag{48}$$

for it makes the strain energy a minimum.

From the shape of cross-section (see Fig. 3-III) it is easy to obtain

$$I_x = 4c^4, \quad W_x = 2c^3, \tag{49}$$

and

$$\psi = 4, \quad \varphi = 2. \tag{50}$$

Finally, for (14), (48) and (50), varying dimensions of cross-section are determinate from (12):

$$c_1(z_1) = \sqrt[3]{\frac{271.6 \ z_1}{2\sigma_d}}, \quad c_2(z_2) = \sqrt[3]{\frac{|271.6 - 728.4 \ z_2|}{2\sigma_d}}. \tag{51}$$

3.4 Comparison of obtained results

The obtained results are presented in diagrammatic form in Fig. 5. We shall assume that all three types of cross-section are made of the same material, having equal allowable bending stress $\sigma_d = 10^6 \ N/m^2$. Let us compare the volume of the optimal beam of type III (Fig. 3-III) with the volume of the beam with constant cross-section of the same shape.

For the beam with constant cross-section the redundant is $X = 5F/16 = 312.5 \ [N]$. And the constant characteristic dimension of the cross-section, calculated from the condition that $W_x = M_{\max}/\sigma_d$, is

$$c_0 = 0.05723571 \ [m]. \tag{52}$$

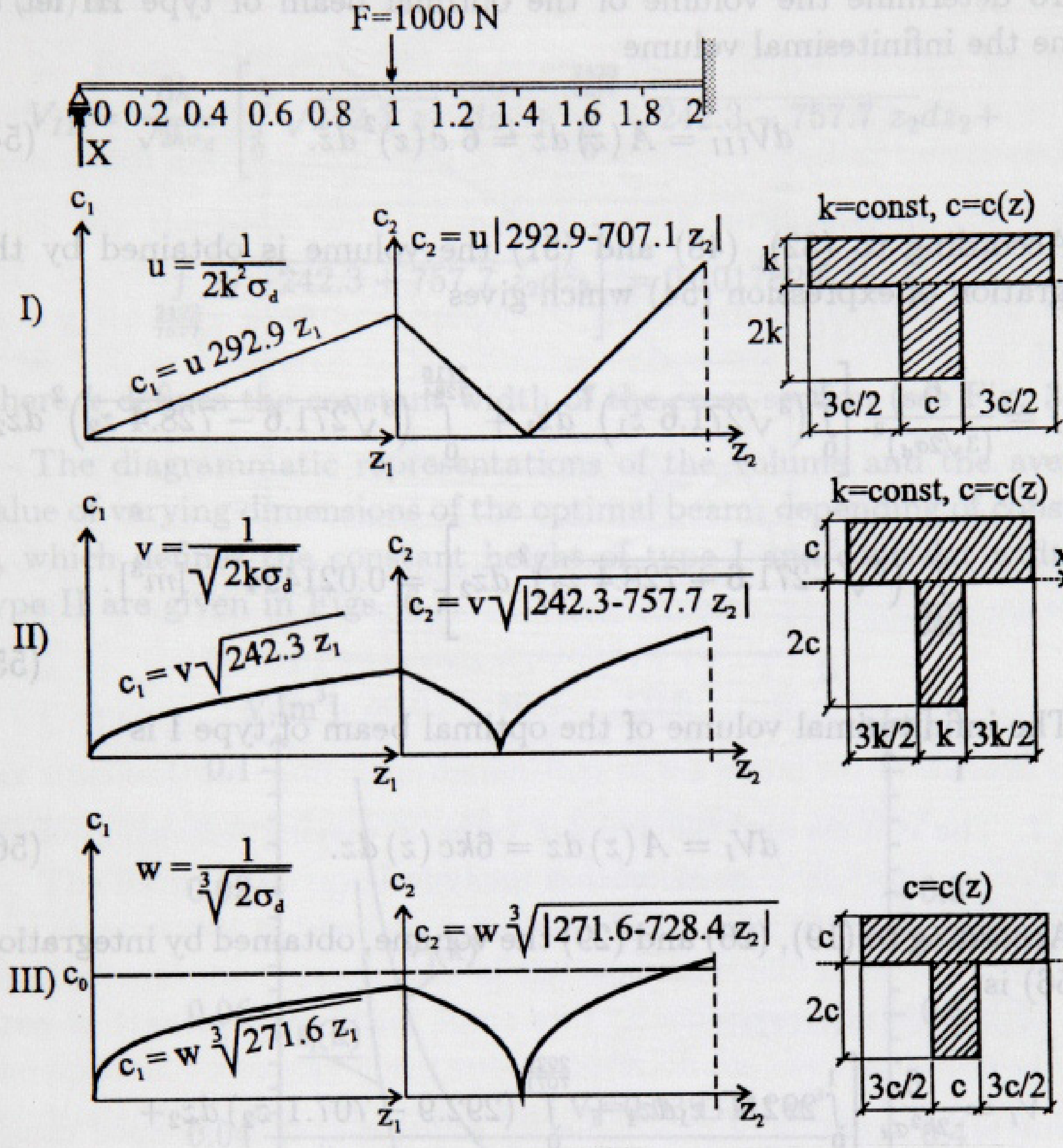

Fig. 5. Varying dimensions for the cross-section.

In Fig. 5-III the constant dimension c_0 (presented with a broken line) is given in the same scale as the corresponding varying dimension c_i of the optimal beam of type III.

For (52) the volume of the beam with constant cross-sectional area is

$$V_0 = A_0 l = 6\ c_0^2 l = 0.03931112\ \left[m^3\right]. \tag{53}$$

where $A_0 = 6\ c_0^2$ denotes the corresponding constant cross-sectional area and $l = 2\ [m]$ is the length of the beam.

To determine the volume of the optimal beam of type III let us define the infinitesimal volume

$$dV_{III} = A(z)\,dz = 6\;c(z)^2\,dz. \tag{54}$$

According to (32), (48) and (51) the volume is obtained by the integration of expression (54) which gives

$$V_{III} = \frac{6}{\left(3\sqrt{2\sigma_d}\right)^2}\left[\int_0^1 \left(\sqrt[3]{271.6\;z_1}\right)^2 dz_1 + \int_0^{\frac{2716}{7284}} \left(\sqrt[3]{271.6 - 728.4\;z_2}\right)^2 dz_2 + \right.$$

$$\left.\int_{\frac{2716}{7284}}^{1} \left(\sqrt[3]{-271.6 + 728.4\;z_2}\right)^2 dz_2\right] = 0.021494 \quad [m^3]. \tag{55}$$

The infinitesimal volume of the optimal beam of type I is

$$dV_I = A(z)\,dz = 6kc(z)\,dz. \tag{56}$$

According to (19), (26) and (29) the volume, obtained by integration of (56) is

$$V_I = \frac{6}{2k^2\sigma_d}\left[\int_0^1 292.9\;\;z_1 dz_1 + \int_0^{\frac{2929}{7071}} (292.9 - 707.1\;z_2)\,dz_2 + \right.$$

$$\left.\int_{\frac{2929}{7071}}^{1} (-292.9 + 707.1\;z_2)\,dz_2\right] = \frac{9.85272}{k\;10^4}, \tag{57}$$

where k defines the constant height of the cross-section (see Fig. 3-I).

The infinitesimal volume of the optimal beam of type II has form

$$dV_{II} = A(z)\,dz = 6kc(z)\,dz. \tag{58}$$

According to (32), (40) and (43) the volume, obtained by integration of (58) is

$$V_{II} = \frac{6k}{\sqrt{2k\sigma_d}}\left[\int_0^1 \sqrt{242.3\, z_1}\, dz_1 + \int_0^{\frac{2423}{7577}} \sqrt{242.3 - 757.7\, z_2} dz_2 + \right. \tag{59}$$

$$\left. \int_{\frac{2423}{7577}}^{1} \sqrt{-242.3 + 757.7\, z_2} dz_2 \right] = 0.1017828\sqrt{k},$$

where k defines the constant width of the cross-section (see Fig. 3-II).

The diagrammatic representations of the volume and the average value of varying dimensions of the optimal beam, depending of constant k, which defines the constant height of type I and constant width of type II are given in Figs. 6,7.

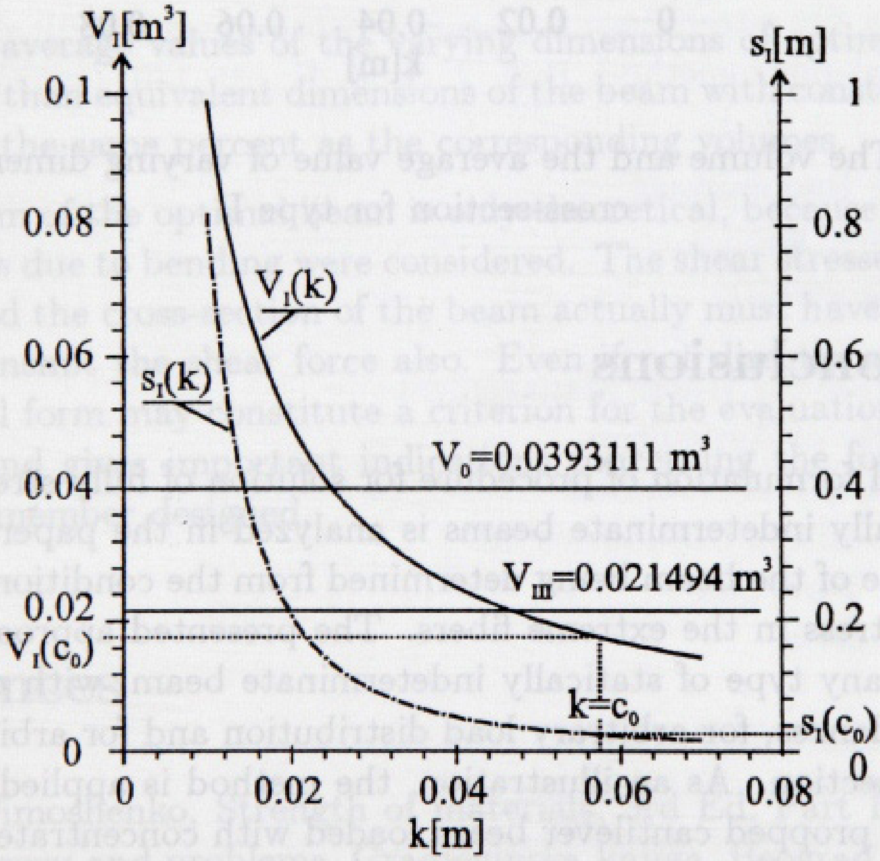

Fig. 6. The volume and the average value of varying dimension of the cross-section for type I.

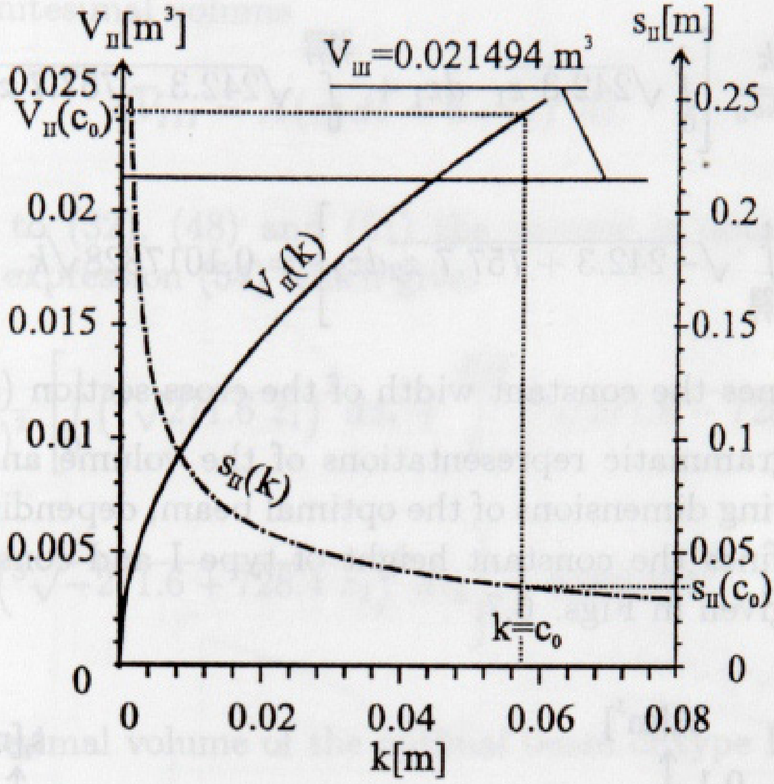

Fig. 7. The volume and the average value of varying dimension of the cross-section for type II.

4 Conclusions

A general formulation of procedure for solution of fully stressed design of statically indeterminate beams is analyzed in the paper. The optimal shape of the beam being determined from the condition of uniform normal stress in the extreme fibers. The presented approach is applicable to any type of statically indeterminate beam, with any number of redundances, for arbitrary load distribution and for arbitrary shape of cross-section. As an illustration, the method is applied to optimal design of propped cantilever beam loaded with concentrated force and for three types of cross-section (Fig. 3). By the comparison of the results we showed the following:

1. The procedure has proven itself as both a simple and effective method for solving the class of problems considered. It is conceptually

simple and easy to implement.

2. From (53) and (55) it is seen that the percentage saving of material through optimum design for the cross-section with varying both width and height is 45.3 % as compared to constant cross-section.

3. For the beam with cross-section with varying width the volume (57) depends of the constant k that defines the height. The volume saving through optimum design for cross-section with varying width is 56.2 % as compared to the constant cross-section with equal height (Fig. 6).

4. For the beam with cross-section with varying height the volume (59) depends of the constant k that defines the width. The volume saving through optimum design for cross-section with varying height is 38.1 % as compared to the constant cross-section with equal width (Fig. 7).

5. The average values of the varying dimensions of optimal beams are smaller than equivalent dimensions of the beam with constant cross-section for the same percent as the corresponding volumes.

The form of the optimal beam is only theoretical, because only normal stresses due to bending were considered. The shear stresses are also present, and the cross-section of the beam actually must have sufficient area to transmit the shear force also. Even if not directly applicable, the optimal form may constitute a criterion for the evaluation of other solutions and gives important indications concerning the form of the structural member designed.

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Projektovanje statički neodredjenih greda minimalne težine uz ograničenje na normalni napon

U radu se razmatra problem optimizacije oblika za linearno elastične statički neodredjene grede. Poprečni presek grede je promenljiv i odredjen iz kriterijuma jednakih napona. Problem je rešen pomoću druge Kastiljanove teoreme. Izvedeni su neki uslovi optimizacije. Dato je nekoliko ilustrativnih primera.