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Effect of uniform magnetic field on an unsteady flow due to an exponentially decay source between two infinite parallel disks

S. Chakraborty and A. K. Borkakati

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Abstract

An unsteady flow of a viscous incompressible electrically conducting fluid between two infinite parallel stationary disks composed of non-conducting material in presence of a uniform magnetic field applied transversely to the direction of flow is considered. The flow is due to the source whose strength decays exponentially. The aim of this study is to investigate the effect of magnetic Hartmann number and the decay factor on the laminar radial flow due to the source for different values of reduced Reynolds number. Solutions are obtained for the radial velocity and pressure distribution. The skin frictional different layers of the flow are calculated and their variations with magnetic Hartmann number and decay factor are shown graphically.

1 Introduction

Ostrach [1] first discussed the combined natural and forced flow of a viscous incompressible fluid through a rigid surface. Later on Grief et al. [2], Gupta et al. [3], Soundalgekar et al. [4] and many other authors studied the incompressible flow over a fixed flat plate. The parabolic velocity distribution of an incompressible radial flow between two parallel stationary disks was discussed by Livesey [5] using the integral approach. Savage [6] studied the velocity and pressure distribution of a down stream flow between two parallel disks. Later on similar types of problems were discussed by Peube [7], Chen et al. [8], and Gieger et al. [9].

Elkouh [10] obtained the solution for a flow due to a sinusoidal varying source of a zero mean value. His solutions are limited only for small values of reduced Reynolds number. Recently Gourla et al. [11] have discussed the flow due to an exponentially decay source between two infinite parallel disks. In their result, it has been shown that the effect of nonlinear inertia on the flow and the pressure are significant for all values of the decay factor of the source and the radial velocity increases with the increase of reduced Reynolds number.

In this paper, we have studied a laminar flow of a viscous incompressible fluid due to an exponentially decay source between two parallel stationary disks under the action of uniform magnetic field applied transversely to the direction of flow. Solutions are obtained for small values of reduced Reynolds number and large values of r (distance from the source line) which give the effects of linear and non-linear convective inertia on the flow and the pressure under the action of a uniform magnetic field. The results obtained are meant for simultaneous effect of inertia and magnetic field on fluid velocity, skin friction and the pressure, whose variations with respect to the field and decay factor of the source are presented graphically. The results observed here are in some cases significantly different from those in absence of the magnetic field (i.e. in ref. [11]) which in turn provides an understanding of the effects of decay factor and reduced Reynolds number on a viscous incompressible flow between two parallel disks under uniform transverse magnetic field.

2 Formulation of the problem

We have considered an unsteady axially symmetric flow of a viscous incompressible fluid between two parallel stationary infinite disks. A cylindrical polar coordinate system is considered such that the disks are situated at $z = \pm h$. The line source of the fluid is situated on the z axis at $r = 0$ whose strength varies according to $Q(t) = Q_0 e^{-nt}$; u and v are the velocity components along radial and z directions respectively. An uniform magnetic field of strength B_o is applied along z axis.

3 Governing equations and their solutions

The Navier-Stokes equations of this problem are

$$\partial u/\partial r + (u/r) + \partial v/\partial z = 0, \tag{1}$$

$$\begin{aligned}\rho\{(\partial u/\partial t) + u\,(\partial u/\partial r) + v\,(\partial u/\partial z)\} &= -\,(\partial p/\partial r) + \\ +\mu\,((\partial^2 u/\partial r^2) + (1/r)\,(\partial u/\partial r) &- u/r^2 + (\partial^2 u/\partial z^2)) - (\sigma u B_z^2),\end{aligned} \tag{2}$$

$$\begin{aligned}\rho\,((\partial v/\partial t) + u\,(\partial v/\partial r) + v\,(\partial v/\partial z)) &= -\,(\partial p/\partial z) + \\ +\mu\,((\partial^2 v/\partial r^2) + (1/r)\,(\partial v/\partial r) &+ (\partial^2 v/\partial z^2)).\end{aligned} \tag{3}$$

The boundary conditions of the problem are

$$u = 0, \quad v = 0, \quad \text{at} \quad z = \pm h, \tag{4}$$

$$\int_{-h}^{+h} 2\pi r\,(u dz) = Q_0 e^{-nt}, \tag{5}$$

where n is the decay factor of the source.

Introducing the following non-dimensional quantities

$$r' = (r/h), \quad z' = (z/h), \quad v' = (hv/v), \quad u' = (hu/v), \tag{6}$$
$$p' = \left(1/\rho\,(h/v)^2\,p\right), \quad n' = \left(h^2/v\right) n, \quad t' = \left(v/h^2\right) t,$$

equations (1)-(3) become (after removing the primes)

$$\partial u/\partial r + (u/r) + \partial v/\partial z = 0, \tag{7}$$

$$\partial u/\partial t + (u\partial u/\partial r) + v\,(\partial u/\partial z) - \partial^2 u/\partial r^2 - (1/r)\,\partial u/\partial r + \\ + (u/r^2) - \partial^2 u/\partial z^2 + (\partial p/\partial r + M^2 u) = 0, \tag{8}$$

$$\partial v/\partial t + (u\partial v/\partial r) + v\,(\partial v/\partial z) - (\partial^2 v/\partial r^2) - \\ - (1/r)\,\partial v/\partial r - \partial^2 v/\partial z^2 + (\partial p/\partial z) = 0. \tag{9}$$

where $(h2B_0^2\sigma/\rho v) = M$, Magnetic Hartmann number.

The corresponding boundary conditions are

$$u = 0, \quad v = 0, \quad \text{at} \quad z = \pm 1, \tag{10}$$

$$\int_{-1}^{+1} (udz) = 2\,(\mathrm{Re}/r)\, e^{-nt}, \tag{11}$$

where $\{Q_0/(4\pi vh)\} = \mathrm{Re}$, (Reynolds number), the controlling factor of the source.

The following expansions are considered

$$u = (\mathrm{Re}/r)\left[f_0'(z,t) + \overset{*}{\mathrm{Re}}\,(f_1'(z,t)) + \left(\overset{*}{\mathrm{Re}}\right)^2 (f_2'(z,t)) + ...\right], \tag{12}$$

$$v = \left[2\left(\overset{*}{\mathrm{Re}}\right)^2 \{f_1(z,t)\} + 4\left(\overset{*}{\mathrm{Re}}\right)^3 \{f_2(z,t)\} + ...\right], \tag{13}$$

$$p = \left[K(z,t)\,\mathrm{Re}\left\{K_0(z,t)\log(r)\right\} + \left(\overset{*}{\mathrm{Re}}\right)\left\{K_1(z,t)\right\} + ...\right], \tag{14}$$

where $\overset{*}{\mathrm{Re}} = (\mathrm{Re}/r^2)$, reduced Reynolds number.

These expansions are valid for small values of $\overset{*}{\mathrm{Re}}$ and large values of r i.e. at a large distance from the source line and satisfy the equation of continuity. The primes denote partial differentiation with respect to z only.

The corresponding boundary conditions in terms of $f(z,t)$ and $f'(z,t)$, are given as

$$f_i(\pm 1, t) = 0, \quad \text{where} \quad i = 1, 2, \tag{15}$$

$$f_i'(\pm 1, t) = 0, \quad \text{where} \quad i = 0, 1, 2, \tag{16}$$

and

$$f_0(1,t) - f_0(-1,t) = 2e^{-nt}, \tag{17}$$

For streamline flow, we choose

$$f_0(1,t) = e^{-nt}, \tag{18}$$

and

$$f_0(-1,t) = -e^{-nt}, \tag{19}$$

so that the equation (16) is satisfied.

Using the expansions (11)-(13) and equating the coefficients of like powers of r, we have the equations:

$$\left(\partial^3 f_0/\partial z^3\right) - \partial^2 f_0/(\partial z \partial t) + M^2(\partial f_0/\partial z) = K_0(z,t), \tag{20}$$

$$(\partial k_0/\partial z) = 0, \tag{21}$$

$$(\partial^3 f_1/\partial z^3) - \{\partial^2 f_1/(\partial z \partial t)\} - M^2 (\partial f_1/\partial z) = -\left\{2k_1(z,t) + (\partial f_0/\partial z)^2\right\}, \tag{22}$$

$$(\partial k_1/\partial z) = 0, \tag{23}$$

Also, $(\partial k/\partial z) = 0$; This implies that $k(z,t) = k(t)$.

The equations (17) and (18) suggest

$$f_i(z,t) = C_i(z)\, e^{-(i+1)nt}, \tag{24}$$

and

$$k_i P_i e^{-(i+1)nt}, \tag{25}$$

where $i = 0, 1$.

Substituting (23) and (24), equations (19)-(22) reduce to

$$C_0'''(z) + \alpha C_0'(z) = P_0, \tag{26}$$

$$C_1'''(z) + \beta C_1'(z) = -\left(2P_1 + (f_0'(z))^2\right), \tag{27}$$

where

$$\alpha = \left(n - M^2\right), \quad \text{and} \quad \beta = \left(2n - M^2\right). \tag{28}$$

The corresponding boundary conditions are

$$C_0(\pm 0) = \pm 1, \quad C_0'(\pm 1) = 0, \tag{29}$$

$$C_1(\pm 1) = 0, \quad C_1'(\pm 1) = 0, \tag{30}$$

and the solutions of equations (19)-(22) subject to the boundary conditions (29) and (30), are obtained as

$$f_0(z,t) = \left(\left(z\alpha\cos\sqrt{\alpha} - \sin\sqrt{\alpha}z\right)/A\right)e^{-nt}, \tag{31}$$

$$K_0(t) = \left(\left(\alpha^{3/2}\cos\sqrt{\alpha}\right)/A\right)e^{-nt}, \tag{32}$$

$$\begin{aligned} f_1(z,t) = \Big[& G\sin\left(\sqrt{\beta}z\right) - \left(2P_1\sqrt{\beta}\right)z - \\ & -\left(\alpha/\left(2A^2\beta\right)\right)\left(2\cos^2\sqrt{\alpha}+1\right)z + \left(\sin 2\sqrt{\alpha}z\right)/\left(4\sqrt{\alpha}\left(\beta-4\alpha\right)\right) - \\ & -\left(2\cos\sqrt{\alpha}\sin\sqrt{\alpha}z\right)/\left(\sqrt{\alpha}\left(\beta-\alpha\right)\right)\Big]e^{-2nt}, \end{aligned} \tag{33}$$

$$\begin{aligned} k_1(t) = \Bigg[& \beta/\left(2A^2\right)\left(1/B\right)\left\{\frac{\left(3\left(5\alpha-\beta\right)\sqrt{\alpha}\sqrt{\beta}\cos\sqrt{\beta}\sin 2\sqrt{\alpha}\right)}{\left(4\left(\beta-4\alpha\right)\left(\beta-\alpha\right)\right)} - \right. \\ & \left. -\left[\cos 2\sqrt{\alpha}/\left(2\left(\beta-4\alpha\right)\right) - 2\cos^2\sqrt{\alpha}/\left(\beta-\alpha\right)\right]\alpha\sin\sqrt{\beta}\right\} - \\ & -\alpha/\left(4A^2\right)\left(2\cos^2\sqrt{\alpha}+1\right)\Bigg]e^{-2nt}, \end{aligned} \tag{34}$$

where

$$B = \left(\sin\sqrt{\beta} - \sqrt{\beta}\cos\sqrt{\beta}\right), \quad A = \left(\sqrt{\alpha}\cos\sqrt{\alpha} - \sin\sqrt{\alpha}\right), \tag{35}$$

$$\begin{aligned} G = 1/\sin\sqrt{\beta}\Big\{ & \alpha/\left(2\beta A^2\right)\left(2\cos^2\sqrt{\alpha}+1\right) + \\ & +\sqrt{\alpha}/A^2\frac{\left(3\left(5\alpha-3\right)\sin 2\sqrt{\alpha}\right)}{\left(4\left(\beta-4\alpha\right)\left(\beta-\alpha\right)\right)} + 2P_1\beta\Big\} \end{aligned} \tag{36}$$

4 Results and discussion

We define radial velocity (u^*) and pressure (p^*) as

$$u^* = \left[f_0'(z,t) + \overset{*}{\mathrm{Re}}\{f_1'(z,t)\} + \left(\overset{*}{\mathrm{Re}}\right)^2 \{f_2'(z,t)\} + ... \right], \tag{37}$$

$$p^* = \left[K_0(t)\log(r) + \left(\overset{*}{\mathrm{Re}}\right) K_1(t) + ... \right], \tag{38}$$

where

$$u^* = u(r/\mathrm{Re}), \quad \text{and} \quad p^*\{p(r,t) - p(R,t)\}/\mathrm{Re}, \tag{39}$$

Here we assume that $\{p(R,t)\}$ is a known pressure at some cross-section in the flow domain at $r = R$ (see ref. [11]).

The shear stresses (i.e. the skin friction) at the disks are defined as

$$\tau = -\mu(\partial u/\partial z)_{z=\pm h}, \tag{40}$$

Substituting the parameter given in (6), the shear stresses are given by

$$\tau^* = -\left(f_0''(\pm1,t) + \overset{*}{\mathrm{Re}}(f_1''(\pm1,t)) \right) + \left(\overset{*}{\mathrm{Re}}\right)^2 (f_2''(\pm1,t)) + ..., \tag{41}$$

where

$$\tau^* = \left[\tau/\left\{ (\mu Q_0)/\left(4\pi r h^3\right) \right\} z = \pm1 \right], \tag{42}$$

From the relations (12)-(14), it is clear that we have only linear effect of convective inertia on the flow and the pressure at $\overset{*}{\mathrm{Re}} \cong 0$, while the non-linear effects of convective inertia are observed for finite values of $\overset{*}{\mathrm{Re}}$. Therefore, we have calculated u^*, τ^* and p^* for the values of reduced Reynolds number $\overset{*}{\mathrm{Re}} = 0.001$ to 0.75. Their variations with magnetic field parameter ($M = 0.0$ to 4.0) and the decay factor of the source ($n = 0.0$ to 10.0) are shown graphically in the figures (1-7). The results obtained here are in presence of uniform transversed magnetic field which are in some cases significantly different from the results obtained by Gourla et. al. in absence of the field (ref. [11]).

The perturbation technique is used for small values of $\overset{*}{\mathrm{Re}}$ $(= \mathrm{Re}/r^2)$, therefore the results are valid for a large distance from the source at z-axis while $M = 0$ in the ordinate axis means in absence of the magnetic field. Following are the observations drawn from the figs. (1-7).

Radial velocity (u^*)

Effect of $\overset{*}{\mathrm{Re}}$: With the increase of $\overset{*}{\mathrm{Re}}$, radial velocity u^*(i) increases near the middle of the channel $\{z \to 0$, figs. (1 and 2)$\}$ and (ii) decreases near the disks ($z \to \pm 1$; figs. 1 and 2) at constant M and n. Results are similar in nature to those in absence of the field (ref. [11]).

Effect of M: For constant n and $\overset{*}{\mathrm{Re}}$, u^* first decreases slowly with the increase of M, then increases with the increase of M (fig. 3). The value of M from where u^* increases also increases with the rise of n.

Effect of n: The variations of u^* with n at constant values of $\overset{*}{\mathrm{Re}}$ and M are almost opposite to those in absence of the field (ref. [11]). At constant $\overset{*}{\mathrm{Re}}$ and M, u^* decreases appreciably with the increase of n (fig. 3).

Skin friction (τ^*)

(a) Results at the upper plate ($z = +1$, figs. 4 and 5):

Effect of $\overset{*}{\mathrm{Re}}$: The effects of $\overset{*}{\mathrm{Re}}$ on τ^* is significant only for small values of n (≤ 2.0) where τ^* decreases with the increase of $\overset{*}{\mathrm{Re}}$ at constant values of n and M.

Effect of M: At constant n and $\overset{*}{\mathrm{Re}}$ with the increase of M, τ^* first increases slowly then decreases steadily. The value of M from where τ^* decreases also increases with the rise of n. At higher n the rate of increase of τ^* is high.

Effect of n: τ^* decreases with the increase of n at constant $\overset{*}{\mathrm{Re}}$ and M.

The effects of $\overset{*}{\text{Re}}$ and n on τ^* are similar in nature to those in ref. [11].

(b) Results at the lower plate for τ^* at constant values of $\overset{*}{\text{Re}}$, M and nt, are opposite to those obtained in (a).

Fig.(1), Radial velocity distribution for Re* = 0.001 & 0.75 at M = 0.1 & 2.5 , n =1.0 , t = 1.0 :

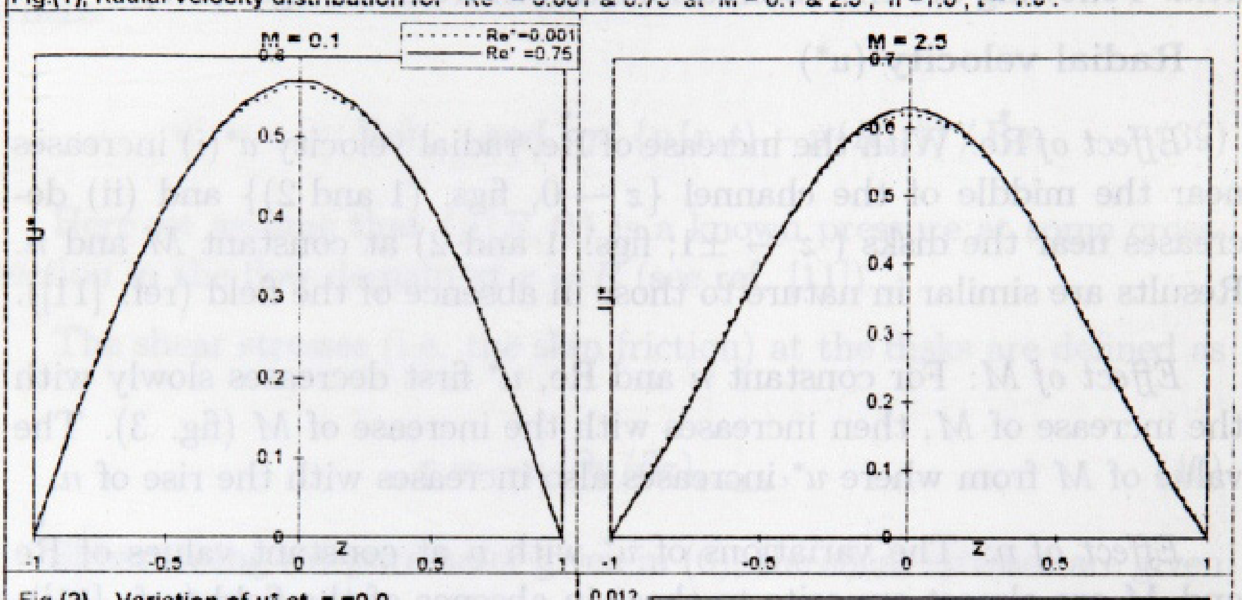
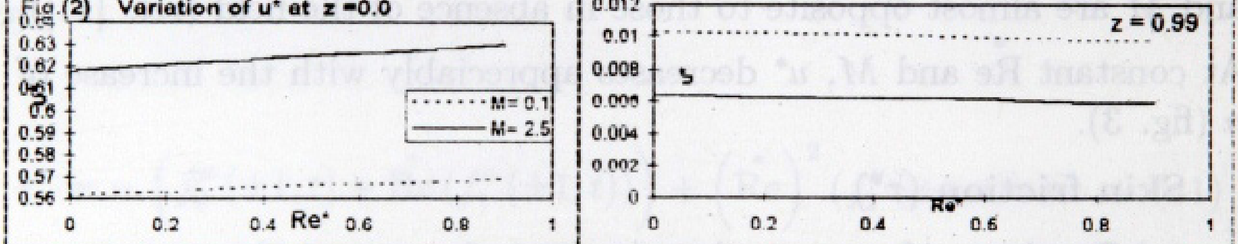
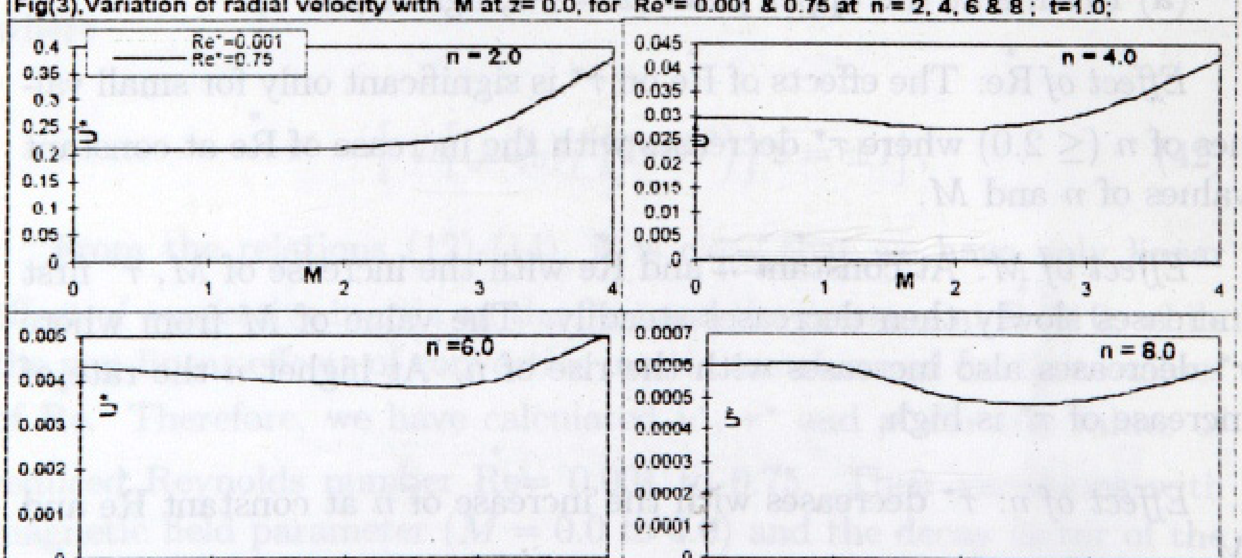


Fig.(2) Variation of u* at z =0.0



Fig(3),Variation of radial velocity with M at z= 0.0, for Re*= 0.001 & 0.75 at n = 2, 4, 6 & 8 ; t=1.0;



Fig(4), Variation of skin friction with M at z = 1.0, for Re* = 0.001 & 0.75 and t = 1.0

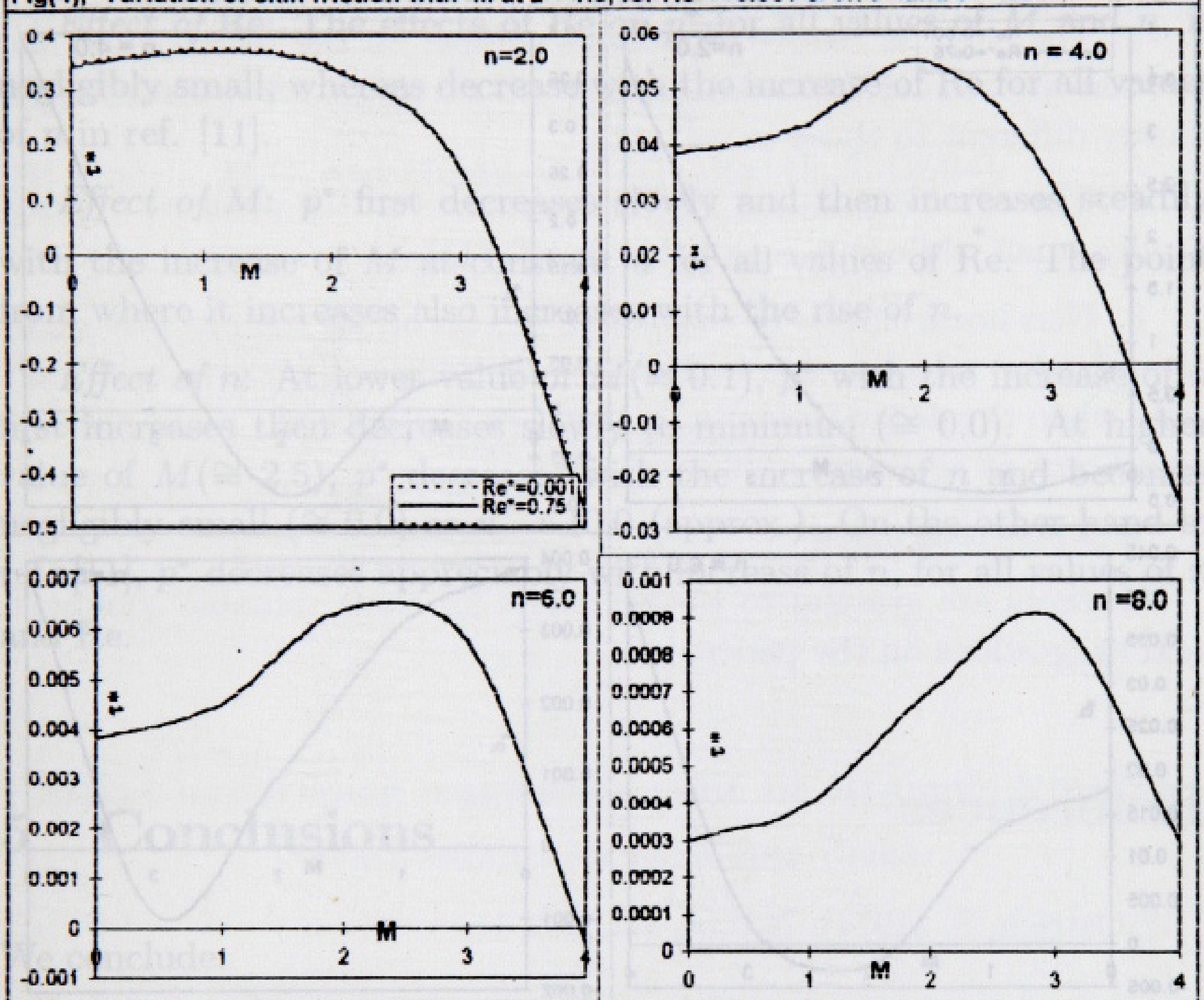


Fig (5),Variation of skin friction with n at z = 1.0 , for Re* =0.001 & 0.75; t =1.0

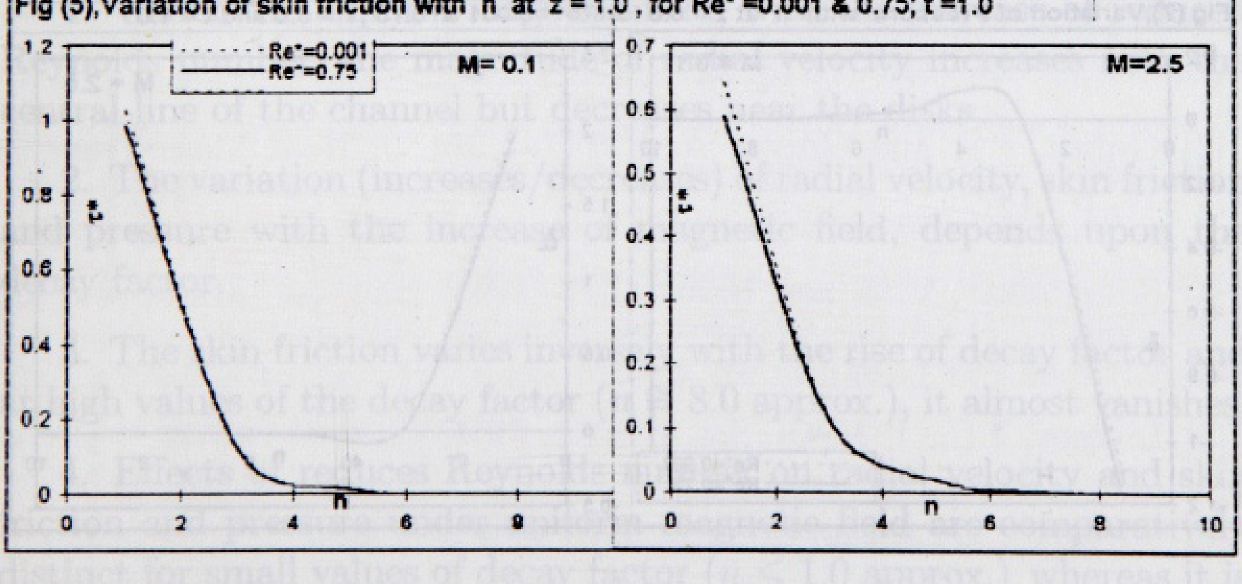


Fig (6),Variation of Pressure with M at z = 0.0, for Re* =0.001 & 0.75, r = 5.0 and t = 1.0

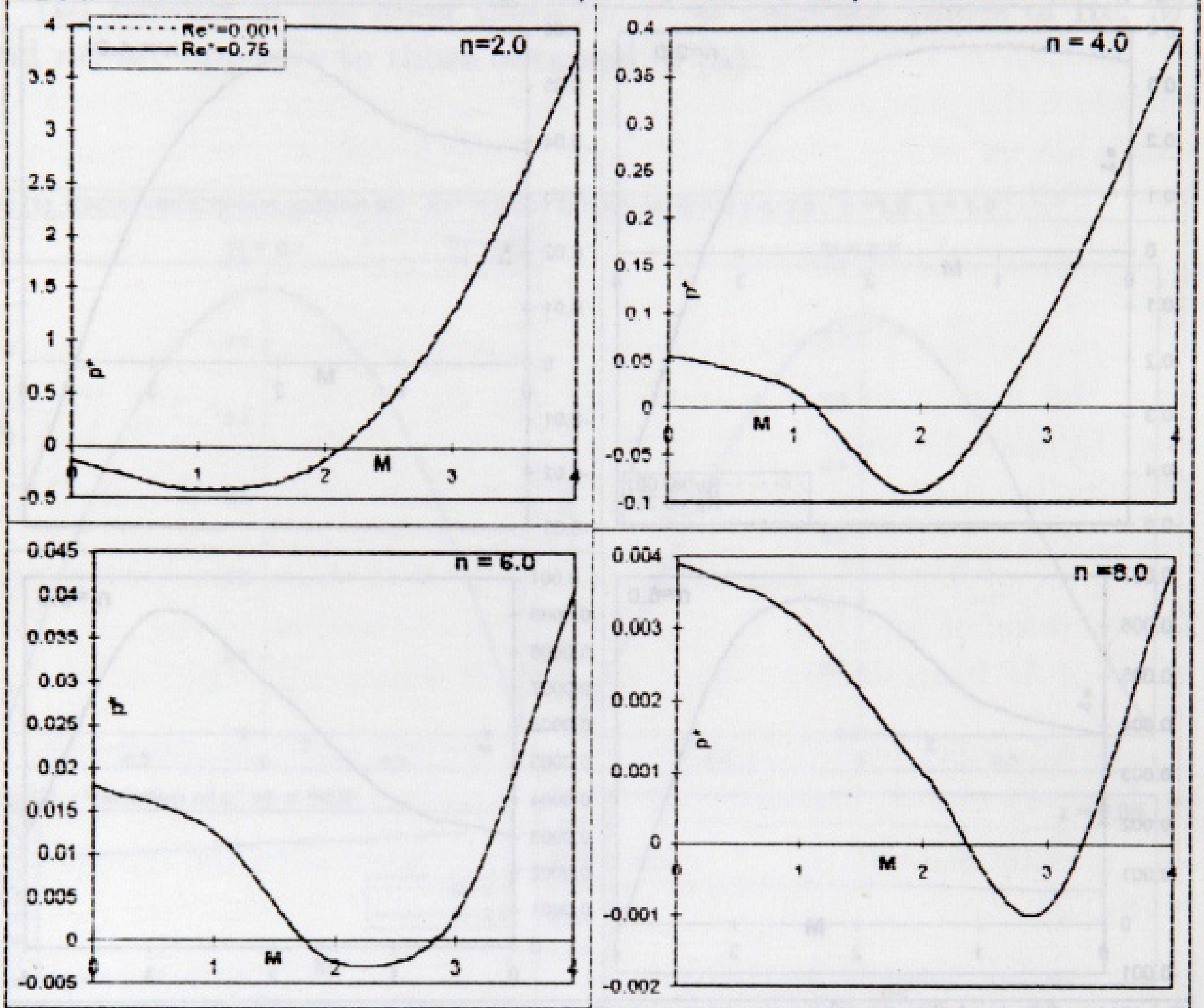
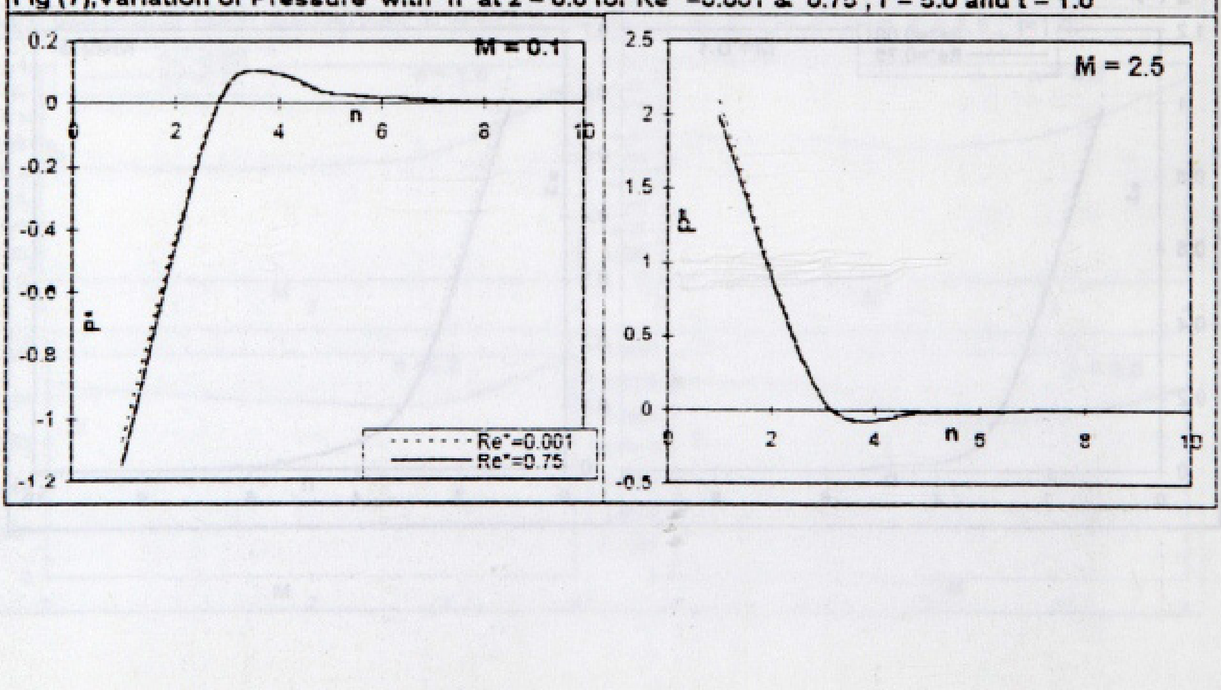


Fig (7),Variation of Pressure with n at z = 0.0 for Re* =0.001 & 0.75 ; r = 5.0 and t = 1.0



Pressure (p^*), (Figs. 6 and 7)

Effect of $\overset{*}{\text{Re}}$: The effects of $\overset{*}{\text{Re}}$ on p^* for all values of M and n, is negligibly small, whereas decrease with the increase of $\overset{*}{\text{Re}}$ for all values of n in ref. [11].

Effect of M: p^* first decreases slowly and then increases steadily with the increase of M at constant n for all values of $\overset{*}{\text{Re}}$. The point from where it increases also increases with the rise of n.

Effect of n: At lower value of $M(\cong 0.1)$, p^* with the increase of n first increases then decreases slowly to minimum ($\cong 0.0$). At higher value of $M(\cong 2.5)$, p^* decreases with the increase of n and becomes negligibly small ($\cong 0.0$) as $n \to 10.0$ (approx.). On the other hand in ref. [11], p^* decreases appreciably with increase of n, for all values of n and $\overset{*}{\text{Re}}$.

5 Conclusions

We conclude:

1. Under the uniform magnetic field, with the increase of reduced Reynolds number, the magnitude of radial velocity increases near the central line of the channel but decreases near the disks.

2. The variation (increases/decreases) of radial velocity, skin friction and pressure with the increase of magnetic field, depends upon the decay factor.

3. The skin friction varies inversely with the rise of decay factor and at high values of the decay factor ($n \cong 8.0$ approx.), it almost vanishes.

4. Effects of reduces Reynolds number on radial velocity and skin friction and pressure under uniform magnetic field are comparatively distinct for small values of decay factor ($n \leq 1.0$ approx.) whereas it is clearly distinct for all values of n in ref. [11].

5. With the increase of magnetic field, pressure decreases for smaller range of magnetic field, but increases for the higher range of it depending upon the decay factor.

6. The effect of reduced Reynolds number on the velocity and skin friction are similar whereas the effect of decay parameter are significantly different to those in ref. [11].

7. The effect of reduced Reynolds number on the pressure are negligibly small while it is reasonable in ref. [11].

8. Non-linear effects of convective inertia on all the physical quantities (radial velocity, skin friction and the pressure) become significant only when the decay factor (n) is small ($n \leq 1$ approx.) while they are significant at all values of n in ref. [11].

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S. Chakraborty
A. K. Borkakati
Department of Mathematical Sciences
Tezpur University, Tezpur
P.O. Tezpur, Pin 784001
Assam
India

Efekat uniformnog magnetnog polja na strujanje izmedju dva beskonačna paralelna diska

U radu se razmatra nestacionarno strujanje viskoznog nekompresibilnog električno provodljivog fluida izmedju dva beskonačna paralelna stacionarna diska napravljena od neprovodljivog materijala u prisustvu uniformnog magnetnog polja koje deluje upravno na pravac strujanja. Strujanje nastaje zbog izvora čija jačina opada eksponencijalno. Cilj studije ja da se istraži efekat Hartmanovog broja i faktora strujanja na laminarno radijalno strujanje za različite vrednosti redukovanbog Rejnoldsovog broja. Dobijena rešenja za radijalnu brzinu i raspored pritiska prikazana su grafički.