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A general formulation for finite element analysis of flow through a porous deformable medium

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Abstract

We consider a coupled problem of deformation of porous solid, with flow of compressible fluid. We transform the governing equations of the problem to the corresponding finite element relations so that the nodal point variables in our general formulation are displacements of solid, fluid pore pressure, and relative velocity of fluid. According to this formulation we can include general boundary conditions for solid, relative velocity of fluid and fluid pressure. This is one of the main advantages of our formulation with respect to others given in literature. Numerical examples are solved by our general-purpose FE package PAK, and are taken from geomechanics and biomechanics. The results are compared with those available in the literature, demonstrating accuracy and generality of the presented procedure.

1 Introduction

Formulation of finite element solution procedures of the coupled problems which include deformation of solid and fluid flow through the porous medium, have been the subject of investigation of many authors [1]-[5]. These methods has been implemented to geomechanics and to biomechanics.

We give here a short description of various approach which start basically from the same fundamental equations for flow through porous deformable media. Some details are different in description of compressibility of solid material. Here we follow approach of Lewis [1]. In FE formulation some authors use displacements of solid and fluid pressure as the nodal point variables (u-p formulation), like Lewis [1], and Siriwardane [2], leading to symmetric system of equations; Simon [3] proposes two formulations: (1) displacements of solid and relative displacement of fluid - with symmetric system, and (2) u-p formulation - with nonsymmetric system; Gajo [4] uses displacements of solid and fluid, and pressure, with symmetric system of equations. In references [1]-[4] are considered small displacements, with materially nonlinearity of solid [1], [2], while in [5] a large displacement formulation is presented.

In our presentation we first give in some detail derivation of dynamic FE equations for linear material behavior, following reference [3], and reference [1] for the fluid continuity equation. This approach relies on the balance of linear momentum of solid and fluid, and continuity of fluid which takes into account compressibility of solid and of fluid. The nodal point variables are displacements of solid, fluid pressure and relative velocities of fluid.

The above formulation is suitable for general use, since we can implement all boundary conditions appearing in applications: boundary conditions for solid, fluid pressure and relative fluid velocity.

Finally, we generalize the linear dynamic system to materially nonlinear problems. This new system of equations have a standard incremental-iterative form. Further generalizations to include electrokinetic coupling are given in reference [6].

The paper is organized as follows. In the next section we summarize the fundamental equations of the coupled problem described above, and then, in Section 3 we derive the basic FE equations for linear and nonlinear dynamics. In Section 4 some typical numerical examples from geomechanics and biomechanics are presented, and finally we give some concluding remarks in Section 5.

2 Fundamental equations of flow through porous deformable medium

We present here the fundamental equations, supposing that displacements of solid are small, which represent the basis for the FE formulation in Section 3. The assumptions are that the solid is linear elastic, with compressible material of the skeleton , and that fluid is compressible. Also, we consider dynamic problem and take into account inertial forces of solid and fluid.

The balance equation of solid can be written in the form

$$(1-n)\,\mathbf{L}^T\sigma_s + (1-n)\,\rho_s\mathbf{b} + \mathbf{k}^{-1}n\mathbf{q} - (1-n)\,\rho_s\ddot{\mathbf{u}} = 0, \tag{1}$$

where σ_s is stress in the solid phase, n is porosity, $\mathbf{k}$ is permeability matrix, ρ_s density of solid, $\mathbf{b}$ body force per unit mass, $\mathbf{q}$ is relative velocity of fluid, and $\ddot{\mathbf{u}}$ is acceleration of solid material. The operator $\mathbf{L}^T$ is

$$\mathbf{L}^T = \begin{bmatrix} \frac{\partial}{\partial x_1} & 0 & 0 & \frac{\partial}{\partial x_2} & 0 & \frac{\partial}{\partial x_3} \\ 0 & \frac{\partial}{\partial x_2} & 0 & \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_3} & 0 \\ 0 & \frac{\partial}{\partial x_3} & 0 & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_1} & 0 \end{bmatrix}. \tag{2}$$

Balance equation of the fluid phase is

$$-n\nabla p + n\rho_f\mathbf{b} - \mathbf{k}^{-1}n\mathbf{q} - n\rho_f\dot{\mathbf{v}}_f = 0 \tag{3}$$

where p is pore fluid pressure, ρ_f density of fluid and $\dot{\mathbf{v}}_f$ is acceleration of fluid. This equation is also known as generalized Darcy's law. The both equilibrium equations are written per unit volume of the mixture. Combining equations (1) and (3) we obtain

$$\mathbf{L}^T\sigma + \rho\mathbf{b} - \rho\ddot{\mathbf{u}} - \rho_f\dot{\mathbf{q}} = \mathbf{0}, \tag{4}$$

where σ is total stress which can be expressed in terms of σ_s and p as

$$\sigma = (1-n)\,\sigma_s - n\mathbf{m}p, \tag{5}$$

and $\rho = (1-n)\,\rho_s + n\rho_f$ is density of mixture. Here $\mathbf{m}$ is a constant vector defined as $\mathbf{m}^T = \{1\ 1\ 1\ 0\ 0\ 0\}$ to take into account that the pressure contributes to normal stresses only. Also, we have taken into account that the pressure has the compressive character. In further analysis we will use the effective stress σ'defined as

$$\sigma' = \sigma + \mathbf{m}p, \tag{6}$$

which is relevant for the constitutive relations of solid. Using definition of relative velocity $\mathbf{q}$ as volume of solid passing in unit time through unit area of the mixture, i.e.

$$\mathbf{q} = \mathbf{n}\,(\mathbf{v}_f - \dot{\mathbf{u}}), \tag{7}$$

we transform (3) in a form

$$-\nabla p + \rho_f\mathbf{b} - \mathbf{k}^{-1}\mathbf{q} - \rho_f\ddot{\mathbf{u}} - \frac{\rho_f}{n}\dot{\mathbf{q}} = \mathbf{0}. \tag{8}$$

The next fundamental equation is constitutive relation of solid,

$$\sigma' = \mathbf{C}^E\,(\mathbf{e} - \mathbf{e}_p), \tag{9}$$

where $\mathbf{C}^E$ is elastic constitutive matrix of the solid skeleton, $\mathbf{e}$ is total strain, and $\mathbf{e}_p$ is deformation of solid material due pressure [1],

$$\mathbf{e}_p = -\frac{\mathbf{m}}{3K_s}p, \tag{10}$$

Here K_s is the bulk modulus of the solid skeleton which can be expressed in terms of the Young's modulus E and Poisson's ratio ν as $K_s = E/[3\,(1-2\nu)]$.

Now we write the continuity equation for fluid [1]

$$Q_v + Q_p + Q_\rho + Q_s + \nabla^T(\rho_f \mathbf{q}) = 0. \quad (11)$$

The partial contributions in this equation are as follows:

a) due to total volumetric strain rate:

$$Q_v = \rho_f \frac{\partial \mathbf{e}_v}{\partial t} = \rho_f \mathbf{m}^T \frac{\partial \mathbf{e}}{\partial \mathbf{t}}, \quad (12)$$

b) due to compressibility of grain volume:

$$Q_p = \rho_f \frac{1-n}{K_s} \frac{\partial p}{\partial t}, \quad (13)$$

c) due to compressibility of fluid:

$$Q_p = \frac{n\rho_f}{K_f} \frac{\partial p}{\partial t}, \quad (14)$$

where K_f represents the fluid bulk modulus,

d) due to compressibility of solid skeleton under action of the effective stress σ:

$$Q_s = -\frac{\rho_f}{3K_s} \mathbf{m}^T \frac{\partial \sigma'}{\partial t} \quad (15)$$

Using the elastic constitutive law and expressions (12)-(15). we can write the continuity equation (11) in the form

$$\nabla^T \mathbf{q} \left(\mathbf{m}^T - \frac{\mathbf{m}^T \mathbf{C}^E}{3K_s} \right) \dot{\mathbf{e}} + \left(\frac{1-n}{K_s} + \frac{n}{K_f} - \frac{\mathbf{m}^T \mathbf{C}^E \mathbf{m}}{9K_s^2} \right) \dot{p} = 0. \quad (16)$$

3 Finite element equations

In this section we transform the fundamental relations of Section 2 into the finite element equations.

First, by employing principle of virtual work we can write the equation (4) as

$$\int_V \delta \mathbf{e}^T \sigma dV + \int_V \delta \mathbf{u}^T \rho \ddot{\mathbf{u}} dV + \int_V \rho_f \mathbf{u}^T \rho_f \dot{\mathbf{q}} dV = \int_V \delta \mathbf{u}^T \rho \mathbf{b} dV + \int_V \delta \mathbf{u}^T \mathbf{t} dA, \tag{17}$$

where $\delta\mathbf{e}$ and $\delta\mathbf{u}$ are strain and displacements variations,$\mathbf{t}$ is traction on the boundary A, and V is finite element volume. Supposing that material is elastic and using (6), (9) and (10) we write (17) in a form

$$\begin{aligned} &\int_V \delta \mathbf{e}^T \mathbf{C}^E \mathbf{e} dV + \int_V \delta \mathbf{e}^T \left(\frac{\mathbf{C}^E \mathbf{m}}{3K_s} - \mathbf{m} \right) p dV + \int_V \delta \mathbf{u}^T \rho \ddot{\mathbf{u}} dV + \\ &\qquad + \int_V \delta \mathbf{u}^T \rho_f \dot{\mathbf{q}} dV = \int_V \delta \mathbf{u}^T \rho \mathbf{b} dV + \int_A \delta \mathbf{u}^T \mathbf{t} dA. \end{aligned} \tag{18}$$

Next, we multiply equation (8) by the interpolation matrix $\mathbf{H}_q$ for the relative velocity of fluid $\mathbf{q}$ and integrate over the finite element volume according to the Galerkin method. The resulting equation is

$$\begin{aligned} &-\int_V \mathbf{H}_q^T \nabla p dV + \int_V \mathbf{H}_q^T \rho_f \mathbf{b} dV - \int_V \mathbf{H}_q^T \mathbf{k}^{-1} \mathbf{q} dV - \\ &\qquad - \int_V \mathbf{H}_q^T \rho_f \ddot{\mathbf{u}} dV - \int_V \mathbf{H}_q^T \frac{\rho_f}{n} \dot{\mathbf{q}} dV = \mathbf{0}. \end{aligned} \tag{19}$$

Finally, we multiply the continuity equation (16) by the interpolation matrix $\mathbf{H}_p^T$ for pressure (which is vector-column) and obtain

$$\begin{aligned} &\int_V \mathbf{H}_p^T \nabla^T \mathbf{q} dV + \int_V \mathbf{H}_p^T \left(\mathbf{m}^T - \frac{\mathbf{m}^T \mathbf{C}^E}{3K_s} \right) \dot{\mathbf{e}} dV + \\ &\qquad + \int_V \mathbf{H}_p^T \left(\frac{1-n}{K_s} + \frac{n}{K_f} - \frac{\mathbf{m}^T \mathbf{C}^E \mathbf{m}}{9K_s^2} \right) \dot{p} dV = \mathbf{0}. \end{aligned} \tag{20}$$

We note that (usually) in practical applications interpolation functions for displacements $\mathbf{H}_u$ and for relative velocities $\mathbf{H}_q$ are quadratic, while $\mathbf{H}_p$ for pressure is linear.

We employ the standard procedure of integration over the element volume in equations (18)-(20) and use of the Gauss theorem. The resulting FE system equations is

$$\begin{bmatrix} \mathbf{m}_{uu} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{m}_{qu} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{p}} \\ \ddot{\mathbf{q}} \end{Bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{c}_{uq} \\ \mathbf{c}_{pu} & \mathbf{c}_{pp} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{c}_{qq} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{p}} \\ \dot{\mathbf{q}} \end{Bmatrix} + \tag{21}$$

$$+ \begin{bmatrix} \mathbf{k}_{uu} & \mathbf{k}_{up} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{k}_{pq} \\ \mathbf{0} & \mathbf{k}_{qp} & \mathbf{k}_{qq} \end{bmatrix} \begin{Bmatrix} \underline{\mathbf{u}} \\ \underline{\mathbf{p}} \\ \underline{\mathbf{q}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_u \\ \mathbf{f}_p \\ \mathbf{f}_q \end{Bmatrix}.$$

The matrices and vectors in this equation are

$$\mathbf{m}_{uu} = \int_V \mathbf{H}_u^T \rho \mathbf{H}_u dV, \qquad \mathbf{m}_{qu} = \int_V \mathbf{H}_q^T \rho_f \mathbf{H}_u dV,$$

$$\mathbf{c}_{uq} = \mathbf{m}_{qu}^T = \int_V \mathbf{H}_u^T \rho_f \mathbf{H}_q dV, \quad \mathbf{c}_{pu} = -\int_V \mathbf{H}_p^T \left(\mathbf{m}^T - \frac{\mathbf{m}^T \mathbf{C}^E}{3K_s} \right) \mathbf{B} dV,$$

$$\mathbf{c}_{pp} = -\int_V \mathbf{H}_p^T \left(\frac{1-n}{K_s} + \frac{n}{K_f} - \frac{\mathbf{m}^T \mathbf{C}^E \mathbf{m}}{9K_s^2} \right) \mathbf{H}_p dV, \quad \mathbf{c}_{qq} = \int_V \mathbf{H}_p^T \frac{\rho_f}{n} \mathbf{H}_q dV,$$

$$\mathbf{k}_{uu} = \int_V \mathbf{B}^T \mathbf{C}^E \mathbf{B} dV, \qquad \mathbf{k}_{up} = \mathbf{c}_{pu}^T = \int_V \mathbf{B}^T \left(\frac{\mathbf{C}^E \mathbf{m}}{3K_s} - \mathbf{m} \right) \mathbf{H}_p dV, \tag{22}$$

$$\mathbf{k}_{pq} = \int_V \mathbf{H}_{p,x}^T \mathbf{H}_q dV, \qquad \mathbf{k}_{qp} = \int_V \mathbf{H}_q^T \mathbf{H}_{p,x} dV,$$

$$\mathbf{k}_{qq} = \int_V \mathbf{H}_q^T k^{-1} \mathbf{H}_q dV, \qquad \mathbf{f}_u = \int_V \mathbf{H}_u^T \rho \mathbf{b} dV + \int_A \mathbf{H}_u^T \mathbf{t} dA,$$

$$\mathbf{f}_p = \int_A \mathbf{H}_p^T \mathbf{n}^T \mathbf{q} dA, \qquad \mathbf{f}_q = \int_V \mathbf{H}_q^T \rho_f \mathbf{b} dV.$$

In these expression $\mathbf{n}$ is the normal vector to the boundary and $\mathbf{B}$ is the strain-displacement transformation matrix.

As we can see from (22) the nodal point variables are: displacements of solid $\mathbf{u}$, relative velocities $\mathbf{q}$ and pressures p. Boundary conditions include: general boundary conditions for the solid, relative velocities and surface pressures.

The system of equations (21) is nonsymmetric in general. In case when inertial forces are neglected the system becomes symmetric. A standard Newmark's method can be employed for time integration of the system (22), as it is done in [8].

In case of nonlinear behavior, as we have in case of elastic-plastic deformation, matrix $\mathbf{k}_{uu}$ and vector $\mathbf{f}_u$ must be corrected. In the expression for $\mathbf{k}_{uu}$ the elastic matrix $\mathbf{C}^E$ should be replaced by the tangent constitutive matrix $\mathbf{C}^{(i-1)}$, where "i" is the equilibrium iteration number [9]. The force vector $\mathbf{f}_u$ is

$$\mathbf{f}_u = \mathbf{f}_u\,(eq.22) - \int\limits_V \mathbf{B}^T \sigma^{(i-1)} dV. \tag{23}$$

Therefore we have now a nonlinear dynamic system of equations (22), which can be transformed into linear algebraic system with unknowns $\mathbf{u}$,p,$\mathbf{q}$, at end of time step, by a standard procedure [9],[10].

4 Numerical examples

We give several typical examples from geomechanics and from biomechanics demonstrate that our results agree with those available in cited references.

Example 1. One-dimensional elastic-plastic consolidation. We consider one-dimensional consolidation of Cam-clay elastic-plastic material, according to data given in references [1] and [2], Fig. 1a. The material is subjected to constant pressure p_o, which is also taken to be the initial pore pressure.

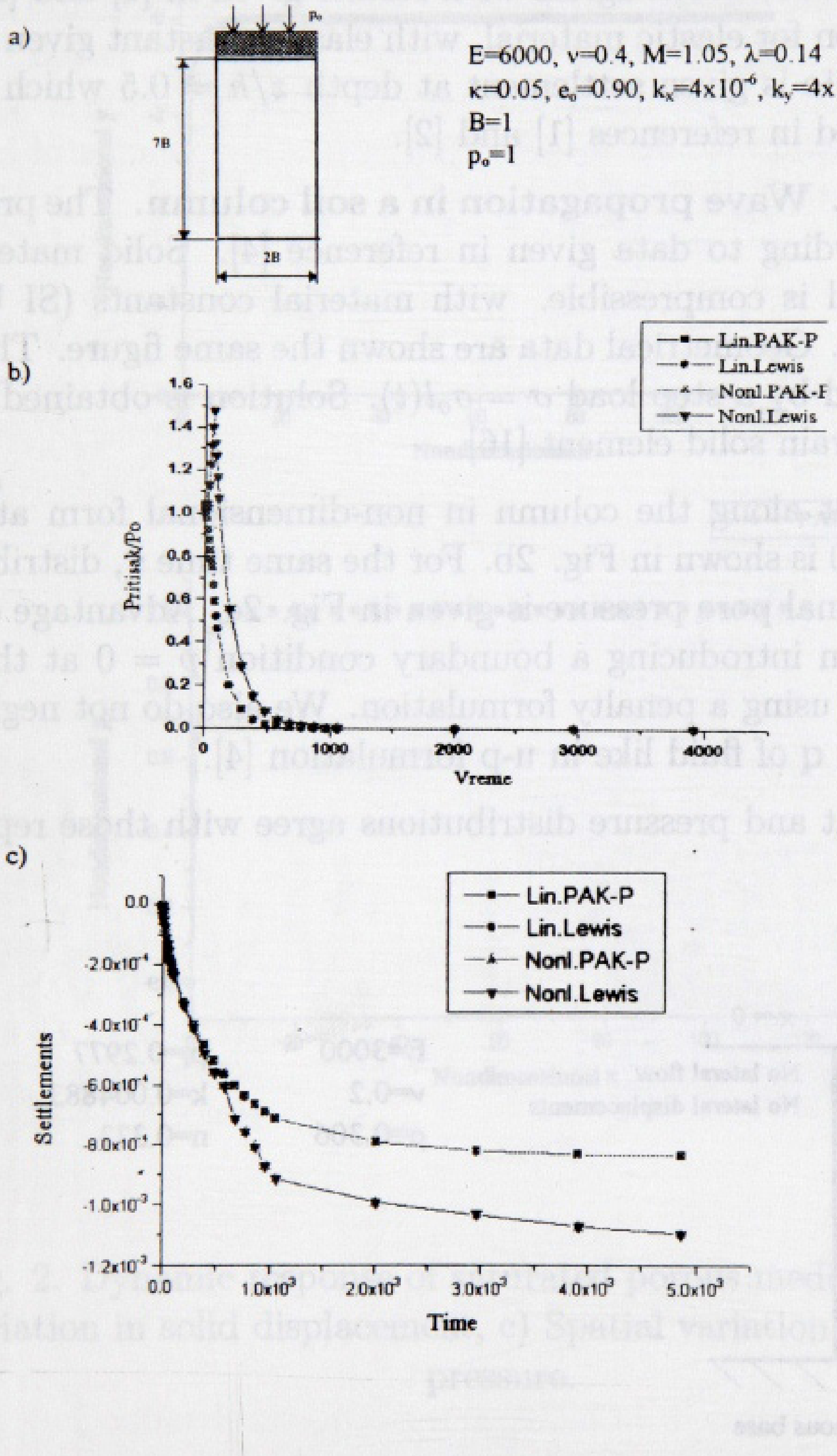


Fig. 1. One dimensional consolidation: a) Material and geometrical data, b) Dissipation of pore water pressure with time at depth $z/h = 0.3$, c) Settlement at depth $z/h = 0.5$.

We solve the example by using incremental procedure and implicit stress integration of the Cam-clay material model [9], [11] within our FE package PAK [8]. Change of ratio p/p_o with time at depth $z/h = 0.3$ is shown in Fig. 1b, which agrees with results given in [1] and [2]. We also give solution for elastic material, with elastic constant given in the figure. In Fig. 1c is given settlement at depth $z/h = 0.5$ which is the some as reported in references [1] and [2].

Example 2. Wave propagation in a soil column. The problem is defined according to data given in reference [4]. Solid material is elastic and fluid is compressible. with material constants (SI Units) given in Fig. 2a. Geometrical data are shown the same figure. The free surface is loaded by a step load $\sigma = \sigma_o l(t)$. Solution is obtained using 9-node plane strain solid element [16].

Displacement along the column in non-dimensional form at time $\tau = t/(\rho k) = 20$ is shown in Fig. 2b. For the same time τ, distribution of non-dimensional pore pressure is given in Fig. 2c. Advantage of our formulation is in introducing a boundary condition $p = 0$ at the top surface without using a penalty formulation. We also do not neglected relative velocity $\mathbf{q}$ of fluid like in u-p formulation [4].

Displacement and pressure distributions agree with those reported in reference [4].

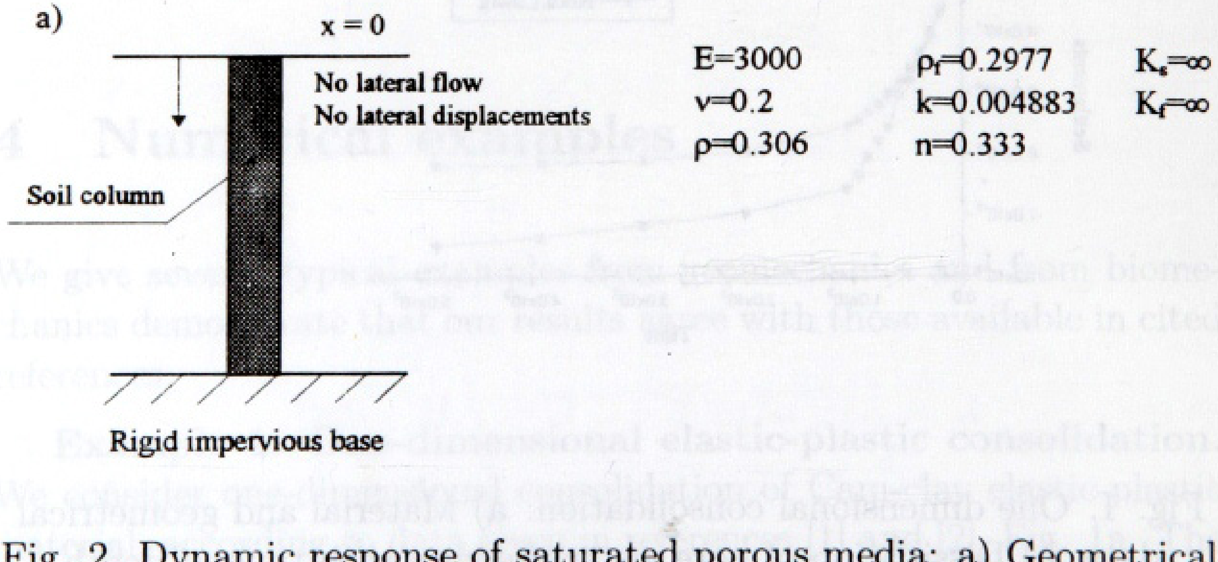
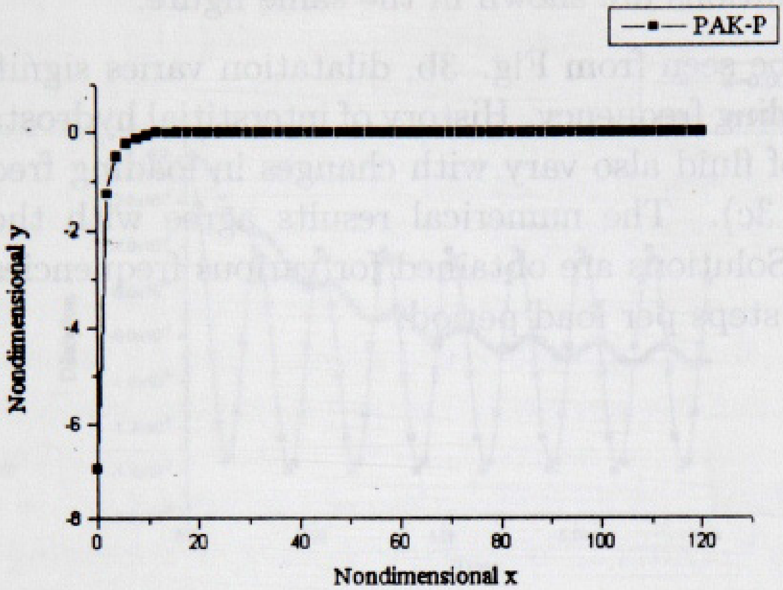


Fig. 2. Dynamic response of saturated porous media: a) Geometrical and material data,

b)



c)

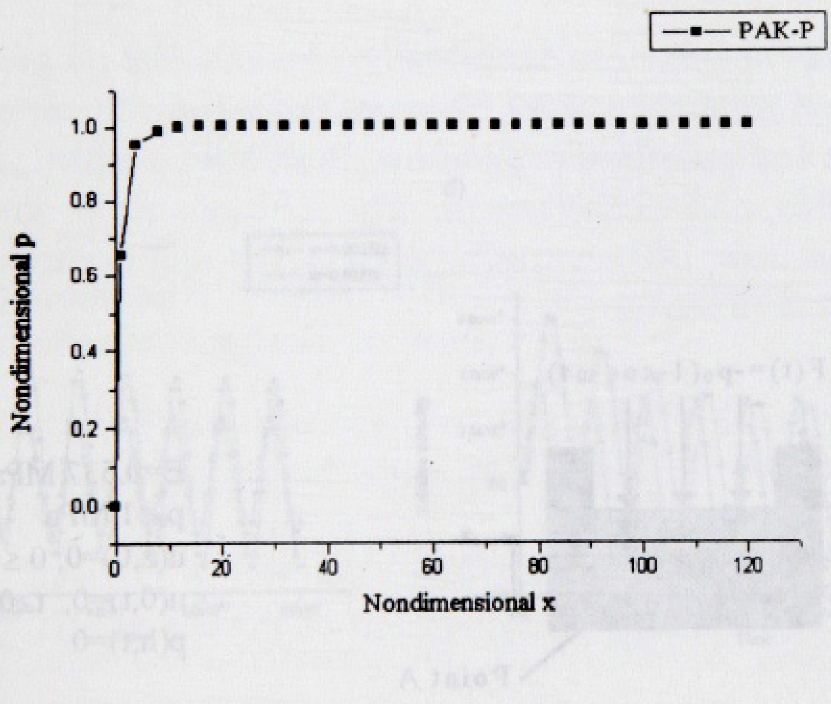


Fig. 2. Dynamic response of saturated porous media: b) Spatial variation in solid displacement, c) Spatial variation in pore fluid pressure.

Example 3. Dynamic behavior of a cartilage model under cyclic compressive loading. Cylindrical plug of articular cartilage is constrained in a confining chamber and is subjected to a cyclic compressive load via a porous filter, as shown in Fig. 3a. The initial and boundary conditions are shown in the same figure.

As it can be seen from Fig. 3b, dilatation varies significantly with changes in loading frequency. History of interstitial hydrostatic pressure and velocity of fluid also vary with changes in loading frequency (Fig. 3b and Fig. 3c). The numerical results agree with those given in reference [7]. Solutions are obtained for various frequencies of load, by using 10 time steps per load period.

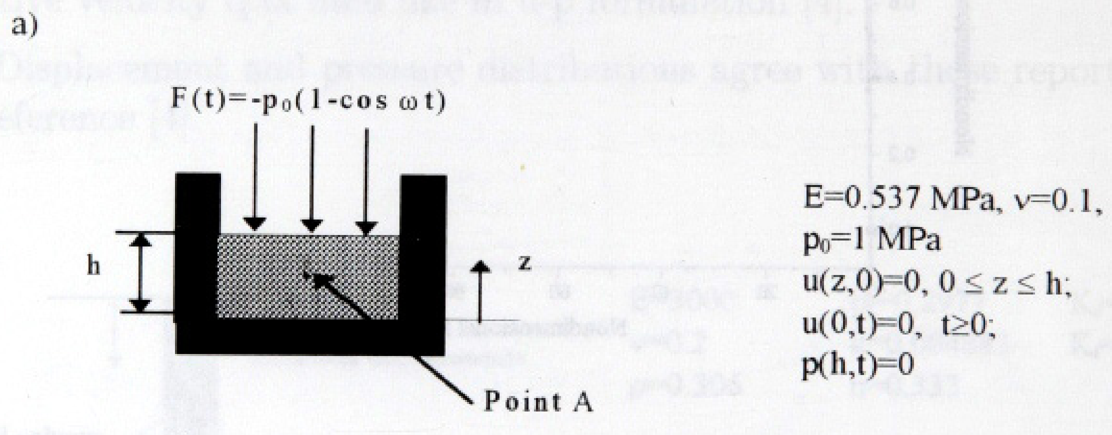


Fig. 4. Confined compression problem of articular cartilage: a) geometrical and material data and initial and boundary conditions.

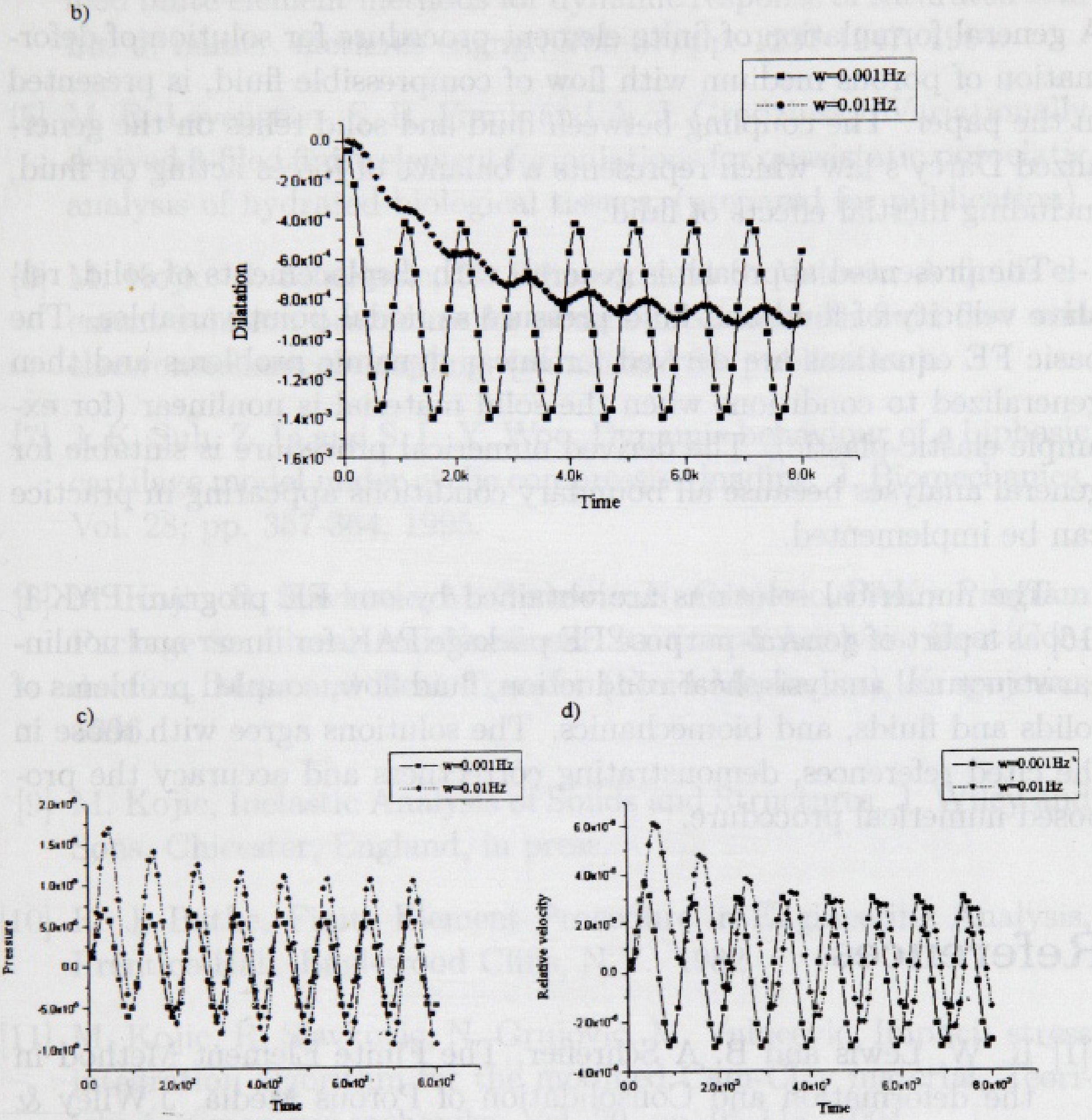


Fig. 4. Confined compression problem of articular cartilage: b) Temporal changes in the dilatation of the cartilage (point A of Fig 3a), c) Temporal changes in hydrostatic pressure (point A of Fig 3a), d) Temporal changes relative fluid velocity (point A of Fig 3a).

5 Conclusions

A general formulation of finite element procedure for solution of deformation of porous medium with flow of compressible fluid, is presented in the paper. The coupling between fluid and solid relies on the generalized Darcy's law which represents a balance of forces acting on fluid, including inertial effects of fluid.

The presented approach is general, with displacements of solid, relative velocity of fluid and fluid pressure as nodal point variables. The basic FE equations are derived for linear dynamic problems and then generalized to conditions when the solid material is nonlinear (for example elastic-plastic). The derived numerical procedure is suitable for general analyses because all boundary conditions appearing in practice can be implemented.

The numerical solutions are obtained by our FE program PAK-P [16] as a part of general-purpose FE package PAK for linear and nonlinear structural analysis, heat conduction, fluid flow, coupled problems of solids and fluids, and biomechanics. The solutions agree with those in the cited references, demonstrating correctness and accuracy the proposed numerical procedure.

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Opšta formulacija za analizu metodom konačnih elemenata problema stujanja fluida kroz poroznu deformabilnu sredinu

Posmatramo spregnuti problem deformisanja poroznog solida, sa strujanjem stišljivog fluida. Osnovne jednačine transformišemo na odgovarajuće relacije metode konačnih elemenata, tako da su veličine u čvorovima, u našoj opštoj formulaciji, pomeranja solida, pritisak fluida i relativna brzina fluida. Prema ovoj formulaciji možemo da uključimo opšte granične uslove za solid, relativne brzine i pritisak fluida. Ova mogućnost predstavlja glavnu prednost naše u odnosu na druge formulacije u literaturi. Numerički primeri su rešeni korišćenjem našeg MKE paketa opšte namene PAK, a uzeti su iz oblasti geomehanike i biomehanike. Rezultati su poredjeni sa raspoloživim iz literature, pokazujući tačnost i opštost izloženog postupka.