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Kinematics of the particles subjected to rheonomic constraints

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Abstract

In this paper an idea of studying rheonomic problems for system of N particles as the "relative motion" is developed. Meaning of the "relative motion" should be assumed as the generalization of the classical relative motion. While the last one assumes transferred motion to be the rigid motion only (translation and rotation) the first one implies deformation as well. Deformation can be pure deformation and the deformation that changes the shape (the angles) of the surface. In that way acceleration of a representative particle consisted of three components: relative, transferred, and so cold "added" acceleration. It was shown that Coriolis acceleration is only the special case of "added" acceleration, i.e. in a case when transferred motion is rigid rotation only.

1 Introduction

In [1] motion of one particle subjected to one rheonomic constraint was studied. This motion was treated as relative motion where:
- motion of a particle along the constraint was assumed as relative motion,

- motion of a constraint was transferred motion.
In that way acceleration of a particle consisted of three components: relative, transferred, and so cold "added" acceleration. It was shown that Coriolis acceleration is only the special case of "added" acceleration. The aim of this paper is to generalize these results to the system of N particles, as well as to discuss them.

2 The preliminary considerations

System of N particles M_s, $(s = 1, ..., N)$ is studied. Position vectors of these particles in a Decartes coordinates are $\mathbf{r}_{(s)} = y^i_{(s)} \mathbf{G}_{(s)i} \quad (s = 1, N,\ i = 1, 2, 3)$. Motion of the system is described by $3N$ equations

$$y^i_{(s)} = y^i_{(s)}(t), \qquad (s = 1, .., N;\ i = 1, 2, 3) \tag{1}$$

These $3N$ equations can be considered as the equations of motion of the representative particle in $3N$ dimensional space $\mathcal{V}_{3N}$. Position vector, velocity and acceleration of this particle are defined in a following way:

$$\mathbf{R} = \left\{\mathbf{r}_{(1)}, ..., \mathbf{r}_{(N)}\right\} = \left\{(y^1_{(1)}, y^2_{(1)}, y^3_{(1)}), ..., (y^1_{(N)}, y^2_{(N)}, y^3_{(N)})\right\}, \tag{2}$$

$$\mathbf{V} = \left\{\dot{\mathbf{r}}_{(1)}, ..., \dot{\mathbf{r}}_{(N)}\right\} = \left\{(\dot{y}^1_{(1)}, \dot{y}^2_{(1)}, \dot{y}^3_{(1)}), ..., (\dot{y}^1_{(N)}, \dot{y}^2_{(N)} \cdot \dot{y}^3_{(N)})\right\}, \tag{3}$$

$$\mathbf{a} = \left\{\ddot{\mathbf{r}}_{(1)}, ..., \ddot{\mathbf{r}}_{(N)}\right\} = \left\{(\ddot{y}^1_{(1)}, \ddot{y}^2_{(1)}, \ddot{y}^3_{(1)}), ..., (\ddot{y}^1_{(N)}, \ddot{y}^2_{(N)} \cdot \ddot{y}^3_{(N)})\right\}. \tag{4a}$$

Base vectors are

$$\mathbf{G}^i_{(s)} = \mathbf{G}_{(s)i}. \tag{5}$$

"Scalar product" of two vectors $\mathbf{a} = \left\{\mathbf{a}_{(1)}, ..., \mathbf{a}_{(N)}\right\}$ and $\mathbf{b} = \left\{\mathbf{b}_{(1)}, ..., \mathbf{b}_{(N)}\right\}$ is

$$\mathbf{a} \cdot \mathbf{b} = \sum_{s=1}^{N} \mathbf{a}_{(s)} \cdot \mathbf{b}_{(s)} = \sum_{s=1}^{N} \sum_{i=1}^{3} a^i_{(s)} b^i_{(s)}. \tag{6}$$

"Vector product" of these vectors will be

$$\mathbf{a} \times \mathbf{b} = \left\{ \mathbf{a}_{(1)} \times \mathbf{b}_{(1)}, ..., \mathbf{a}_{(N)} \times \mathbf{b}_{(N)} \right\}, \tag{7}$$

where

$$\mathbf{a}_{(s)} \times \mathbf{b}_{(s)} = \varepsilon_{ijk} a^i_{(s)} b^j_{(s)} \mathbf{G}^k_{(s)} \quad (s = 1, ..., N; \quad i, j, k = 1, 2, 3).$$

Let the motion of the system in study be constrained by l holonomic rheonomic constraints

$$f_\alpha(\mathbf{r}_{(1)}, ..., \mathbf{r}_{(N)}, t) = 0 \quad (\alpha = 1, ..., \quad l \leq 3N) \tag{8}$$

Existence of these constraints enables us to introduce $n = 3N - l$ generalized coordinates q^α

$$y^i_{(s)} = y^i_{(s)} \quad (q^1, ..., q^n, t), \tag{9}$$

if these coordinates satisfy the constraints equations identically:

$$f_\alpha(\mathbf{r}_{(1)}(q^1, ..., q^n, t), ..., \mathbf{r}_{(N)}(q^1, ..., q^n, t)) \equiv 0. \tag{10}$$

From the point of view of representative particle upper relations can be assumed as an n-dimensional manifold (surface) $\mathcal{V}_n$ emersed in $3N$ dimensional configuration space $\mathcal{V}_{3N}$. Presence of the time in the relations (9) means that this manifold changes in time, i.e. for any instant of time this will be different surface. Difference implies not only the change of the position of the manifold but it's deformation as well.

Partial derivative of the position vector $\mathbf{R}$ with respect to generalized coordinates q^α will describe n linearly independent tangent vectors to the manifold

$$\mathbf{g}_\alpha = \frac{\partial \mathbf{R}}{\partial q^\alpha} = \left\{ \frac{\partial \mathbf{r}_{(1)}}{\partial q^\alpha}, ..., \frac{\partial \mathbf{r}_{(N)}}{\partial q^\alpha} \right\}, \tag{11}$$

where

$$\frac{\partial \mathbf{r}_{(s)}}{\partial q^\alpha} = \frac{\partial y^i_{(s)}}{\partial q^\alpha} \mathbf{G}_{(s)}(i), \quad (s = 1, ..., N; \quad i = 1, 2, 3). \tag{12}$$

Each of the n vectors $\mathbf{g}_\alpha$ has $3N$ coordinates in $\mathcal{V}_{3N}$ configuration space.

According to (11) and (6) metric in $\mathcal{V}_n$ is introduced in a following way

$$g_{\alpha\beta} = \mathbf{g}_\alpha \cdot \mathbf{g}_\beta = \sum_{s=1}^{N} \frac{\partial \mathbf{r}_{(s)}}{\partial q^\alpha} \cdot \frac{\partial \mathbf{r}_{(s)}}{\partial q^\beta} = \sum_{s=1}^{N} \sum_{i=1}^{3} \frac{\partial y^i_{(s)}}{\partial q^\alpha} \cdot \frac{\partial y^i_{(s)}}{\partial q^\beta} = \sum_{s=1}^{N} q^{(s)}_{\alpha\beta}. \quad (13)$$

Projection of the base vector $\mathbf{g}_\alpha$ on the $y^i(s)$ coordinate axis is

$$\mathbf{g}_\alpha \cdot \mathbf{G}^i_{(s)} = \mathbf{g}^{(s)}_\alpha \cdot \mathbf{G}^i_{(s)} = \frac{\partial y^{(i)}_{(s)}}{\partial q^\alpha}, \quad (14)$$

and according to the definition of the projection of the vector onto the axis it will be

$$\frac{\partial y^i_{(s)}}{\partial q^\alpha} = l_{\alpha(s)} \cdot \cos\theta^i_{\alpha(s)},$$

where

$l_{\alpha(s)} = \sqrt{g^{(s)}_{\alpha\alpha}}$ - intensity of the component,

$\theta^i_{\alpha(s)}$ - angle between $\mathbf{g}^s_{(\alpha)}$ and $y^i_{(s)}$ coordinate.

3 Velocity and acceleration

Being aware of the fact that motion of representative particle consists of two components:
- motion of the particle along the manifold (relative motion),
- motion of the particle due to the motion of manifold itself (transferred motion),

our aim is to find out components of the velocity and acceleration of the both mentioned motions.

According to (3) and (11) velocity vector is

$$\mathbf{V} = \frac{\partial \mathbf{R}}{\partial q^\alpha} \dot{q}^\alpha + \frac{\partial \mathbf{R}}{\partial t} = \mathbf{g}_\alpha \cdot \dot{q}^\alpha + \frac{\partial \mathbf{R}}{\partial t}. \quad (15)$$

In a case of seleconomic systems ($\partial \mathbf{R}/\partial t = 0$) absolute velocity is $\mathbf{V} = \mathbf{g}_\alpha \dot{q}^\alpha$ defining relative velocity in the rheonomic case

$$\mathbf{V}_r = \mathbf{g}_\alpha \dot{q}^\alpha = \left\{ \mathbf{g}_\alpha^{(1)} \dot{q}^\alpha, ..., \mathbf{g}_\alpha^{(N)} \dot{q}^\alpha \right\} =$$

$$= \left\{ (\mathbf{g}_1^{(1)} \dot{q}^1 + \mathbf{g}_2^{(1)} \dot{q}^2 + \cdots + \mathbf{g}_n^{(1)} \dot{q}^n), ..., (\mathbf{g}_1^{(N)} \dot{q}^1 + . + \mathbf{g}_n^{(N)} \dot{q}^n) \right\}. \tag{16}$$

Transferred velocity, i.e. velocity of the point of manifold which in given instant of time coincides with representative particle is calculated under the condition that all $\dot{q}^\alpha = 0$

$$\mathbf{V}_{tr} = \frac{\partial \mathbf{R}}{\partial t} = \left\{ \frac{\partial \mathbf{r}_{(1)}}{\partial t}, ..., \frac{\partial \mathbf{r}_{(N)}}{\partial t} \right\}. \tag{17}$$

Relation (17) introduces the vector $\mathbf{g}_o = \partial \mathbf{R}/\partial t$ which contrary to $\mathbf{g}_\alpha$ need not be tangent to $\mathcal{V}_n$. This vector consists of $3N$ components

$$\mathbf{g}_o = \left\{ (\frac{\partial y^1_{(1)}}{\partial t}, \frac{\partial y^2_{(1)}}{\partial t}, \frac{\partial y^3_{(1)}}{\partial t}), ..., (\frac{\partial y^1_{(N)}}{\partial t}, \frac{\partial y^2_{(N)}}{\partial t}, \frac{\partial y^3_{(N)}}{\partial t}) \right\}. \tag{18}$$

Acceleration of the representative particle, according to (4) is

$$\mathbf{a} = \frac{\partial^2 \mathbf{R}}{\partial q^\alpha \partial q^\beta} \dot{q}^\alpha \dot{q}^\beta + \frac{\partial \mathbf{R}}{\partial q^\alpha} \ddot{q}^\alpha + \frac{\partial^2 \mathbf{R}}{\partial t^2} + 2 \frac{\partial^2 \mathbf{R}}{\partial q^\alpha \partial t} \dot{q}^\alpha, \tag{19}$$

$$\mathbf{a} = \frac{\partial \mathbf{g}_\alpha}{\partial q^\beta} \dot{q}^\alpha \dot{q}^\beta + \mathbf{q}_\alpha \ddot{q}^\alpha + \frac{\partial \mathbf{g}_o}{\partial t} + 2 \frac{\partial \mathbf{g}_\alpha}{\partial t} \dot{q}^\alpha. \tag{20}$$

Relative acceleration, under the same condition applied for velocity ($\partial \mathbf{R}/\partial t = 0$) is

$$\mathbf{a}_r = \frac{\partial^2 \mathbf{R}}{\partial q^\alpha \partial q^\beta} \dot{q}^\alpha \dot{q}^\beta + \frac{\partial \mathbf{R}}{\partial q^\alpha} \ddot{q}^\alpha = \frac{\partial \mathbf{g}_\alpha}{\partial q^\beta} \dot{q}^\alpha \dot{q}^\beta + \mathbf{g}_\alpha \ddot{q}^\alpha. \tag{21}$$

The same is valid for transferred acceleration ($\dot{q}^\alpha = 0$)

$$\mathbf{a}_{tr} = \frac{\partial^2 \mathbf{R}}{\partial t^2} = \frac{\partial \mathbf{g}_o}{\partial t}. \tag{22}$$

Comparing (20) to (21) and (22) following relation can be obtained:

$$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_{tr} + 2\frac{\partial \mathbf{g}_\alpha}{\partial t}\dot{q}^\alpha. \tag{23}$$

Last member on the right hand side of (23) will be named "added" acceleration. It is the consequence of the complexity of the motion, and it will be the object of our further study. In a case of rigid rotation of the manifold it should reduce to Coriolis acceleration.

Due to the fact that $\frac{\partial}{\partial q^\alpha}(\cdot)\dot{q}^\alpha$ and $\frac{\partial}{\partial t}$ are commutative, added acceleration can be given as:

$$\mathbf{a}_{ad} = 2\frac{\partial \mathbf{g}_o}{\partial q^\alpha}\dot{q}^\alpha, \tag{24}$$

or

$$\mathbf{a}_{ad} = 2\frac{\partial \mathbf{g}_\alpha}{\partial t^\alpha}\dot{q}^\alpha. \tag{25}$$

4 Analysis of the added acceleration

As it is said in Section 1. motion of the manifold $\mathcal{V}_n$ consists of rigid motion and deformation. Existence of the added acceleration is the consequence, of both motion and we want to derive parts of this acceleration due to each of these motions.

With this idea in mind we shall represent base vector $\mathbf{g}_\alpha$ in a way:

$$\mathbf{g}_\alpha = \left\{ l_{\alpha(1)}\mathbf{e}^1_\alpha, ..., l_{\alpha(N)}\mathbf{e}^N_\alpha \right\}, \tag{26}$$

where

$l_{\alpha(s)}$ is introduced in (15),

$\mathbf{e}^s_\alpha$ is the unit vector in $\mathbf{g}_{\alpha(s)}$ direction.

Partial derivative with respect to time of the $\mathbf{g}_{\alpha(s)}$ components is

$$\frac{\partial \mathbf{g}_{\alpha(s)}}{\partial t} = \frac{\partial l_{\alpha(s)}}{\partial t}\mathbf{e}^s_\alpha + l_{\alpha(s)}\frac{\partial \mathbf{e}^s_\alpha}{\partial t}, \tag{27}$$

At this instant we introduce the vector

$$\boldsymbol{\omega}_\alpha = \left\{\boldsymbol{\omega}_{\alpha(1)}, ..., \boldsymbol{\omega}_{\alpha(N)}\right\}. \tag{28}$$

This vector is the angular velocity of the transferred motion and will be assumed to be normal to $\mathbf{e}_\alpha(\mathbf{g}_\alpha)$:

$$\boldsymbol{\omega}_\alpha \cdot \mathbf{g}_\alpha = \boldsymbol{\omega}_{\alpha(1)} \cdot \mathbf{g}_{\alpha(1)} + ... + \boldsymbol{\omega}_{\alpha(N)} \cdot \mathbf{g}_{\alpha(N)} = 0. \tag{29}$$

In the other words angular velocity $\boldsymbol{\omega}_\alpha$ is the angular velocity of the rotation of the $\mathbf{g}_\alpha$ vector. Introduction of the angular velocity enables us to write relation (27) in a fashion:

$$\frac{\partial \mathbf{g}_{\alpha(s)}}{\partial t} = \frac{\partial l_{\alpha(s)}}{\partial t}\mathbf{e}^s_\alpha + l_{\alpha(s)}\boldsymbol{\omega}_{\alpha(s)} \times \mathbf{e}^s_\alpha, \tag{30}$$

$$\frac{\partial \mathbf{g}_{\alpha(s)}}{\partial t} = \frac{\partial}{\partial t}(\ln \cdot l_{\alpha(s)}) \cdot \mathbf{g}_{\alpha(s)} + \boldsymbol{\omega}_{\alpha(s)} \times \mathbf{g}_{\alpha(s)}. \tag{31}$$

Partial derivative of the base vector, according to (15) will be

$$\frac{\partial \mathbf{g}_{\alpha(s)}}{\partial t} = \frac{\partial l_{\alpha(s)}}{\partial t}\cos\theta^i_{\alpha(s)}\mathbf{G}^s_i + l_{\alpha(s)}\frac{\partial}{\partial t}(\cos\theta^i_{\alpha(s)}) \cdot \mathbf{G}^s_i. \tag{32}$$

In order to be compared to (31) upper relation can be written

$$\frac{\partial \mathbf{g}_{\alpha(s)}}{\partial t} = \frac{\partial(\ln l_{\alpha(s)})}{\partial t}\frac{\partial y^i_{(s)}}{\partial t}\mathbf{G}^{(s)}_i + l_{\alpha(s)}\frac{\partial}{\partial t}(\frac{1}{l_{\alpha(s)}} \cdot \frac{\partial y^i_{(s)}}{\partial q^\alpha})\mathbf{G}^{(s)}_i. \tag{33}$$

Comparing (33) to (31) we obtain

$$\boldsymbol{\omega}_{\alpha(s)} \times \mathbf{g}_{\alpha(s)} = l_{\alpha(s)}\frac{\partial}{\partial t}(\frac{1}{l_{\alpha(s)}}\frac{\partial y^i_{(s)}}{\partial q^\alpha})\mathbf{G}^{(s)}_i. \tag{34}$$

According to the definition of the "vector product" upper relation can be written in a way:

$$\varepsilon_{ijk}\omega^j_{\alpha(s)}\frac{\partial y^k_{(s)}}{\partial q^\alpha} = l_{\alpha(s)}\frac{\partial}{\partial t}(\frac{1}{l_{\alpha(s)}} \cdot \frac{\partial y^i_{(s)}}{\partial q^\alpha}). \tag{35}$$

This relation along with (29) enables us to write the components of the $\boldsymbol{\omega}_\alpha$ in a form:

$$\omega^i_{\alpha(s)} = \varepsilon^i_{jk}\Big(\frac{1}{l_{\alpha(s)}} \cdot \frac{\partial y^j_{(s)}}{\partial q^\alpha}\Big) \cdot \frac{\partial}{\partial t}\Big(\frac{1}{l_{\alpha(s)}} \cdot \frac{\partial y^k_{(s)}}{\partial q^\alpha}\Big). \tag{36}$$

This relation can be also written as:

$$\omega^i_{\alpha(s)} = \varepsilon^i_{jk} \cos\theta^j_{\alpha(s)} \frac{\partial}{\partial t}(\cos\theta^k_{\alpha(s)}). \tag{37}$$

5 Case when added acceleration reduces to Coriolis acceleration

We shall consider the case when the manifold $\mathcal{V}_n$ is rigid and want to derive added acceleration and angular velocity in that particular case.

Condition of rigidity of the $V/_n$ is:

$$\frac{\partial g_{\alpha\beta(s)}}{\partial t} = 0. \tag{38}$$

For $\alpha = \beta$ this condition turns to:

$$\frac{\partial l_{\alpha(s)}}{\partial t} = 0. \tag{39}$$

Condition (38) according to the definition (13) and to the condition (39) turns to

$$(\boldsymbol{\omega}_{\alpha(s)} \times \mathbf{g}_{\alpha(s)}) \cdot \mathbf{g}_{\beta(s)} + (\boldsymbol{\omega}_{\beta(s)} \times \mathbf{g}_{\beta(s)}) \cdot \mathbf{g}_{\alpha(s)} = 0. \tag{40}$$

Upper relation can be given in a fashion

$$(\boldsymbol{\omega}_{\alpha(s)} - \boldsymbol{\omega}_{\beta(s)})(\mathbf{g}_{\alpha(s)} \times \mathbf{g}_{\beta(s)}) = 0. \tag{41}$$

This relation, because of (29) is satisfied only for

$$\boldsymbol{\omega}_{\alpha(s)} = \boldsymbol{\omega}_{\beta(s)} = \boldsymbol{\omega}_{(s)}. \tag{42}$$

In that case added acceleration becomes

$$\mathbf{a}_{ad}^{(s)} = 2\frac{\partial \mathbf{g}_{\alpha(s)}}{\partial t}\dot{q}^{\alpha} = 2\boldsymbol{\omega}_{(s)} \times \mathbf{g}_{\alpha(s)} \cdot \dot{q}^{\alpha} = 2\boldsymbol{\omega}_{(s)} \times \mathbf{V}_{r(s)} = \mathbf{a}_{cor}. \tag{43}$$

Relation (43) was the result we expected, which means that added acceleration in a case of rigid motion of the manifold, i.e. in a case when transferred motion is rigid rotation only, reduces to Coriolis acceleration.

In order to divide parts of the added acceleration due to the rigid rotation and to the deformation which implies changing of the shape of the manifold we should assume to be:

$$\boldsymbol{\omega}_{\alpha(s)} = \boldsymbol{\omega}'_{\alpha(s)} + \boldsymbol{\omega}_{(s)}, \qquad (\alpha = 1, ..., n; \qquad s = 1, ..., N),$$

$\boldsymbol{\omega}'_{\alpha(s)}$ - angular velocity of the transferred motion which causes deformation of the manifold

$\boldsymbol{\omega}_{(s)}$ - angular velocity of the transferred motion representing rigid rotation of the manifold.

6 An example

Two particles, M_1 and M_2 are connected by extensible rod of the length $b(t)$. Both particles belong to the vertical plane which rotates about z axes with $\dot{\theta}(t)$ angular velocity. Particle M_1 belongs to the xy plane too. Added acceleration and angular velocity of the transferred motion should be calculated.

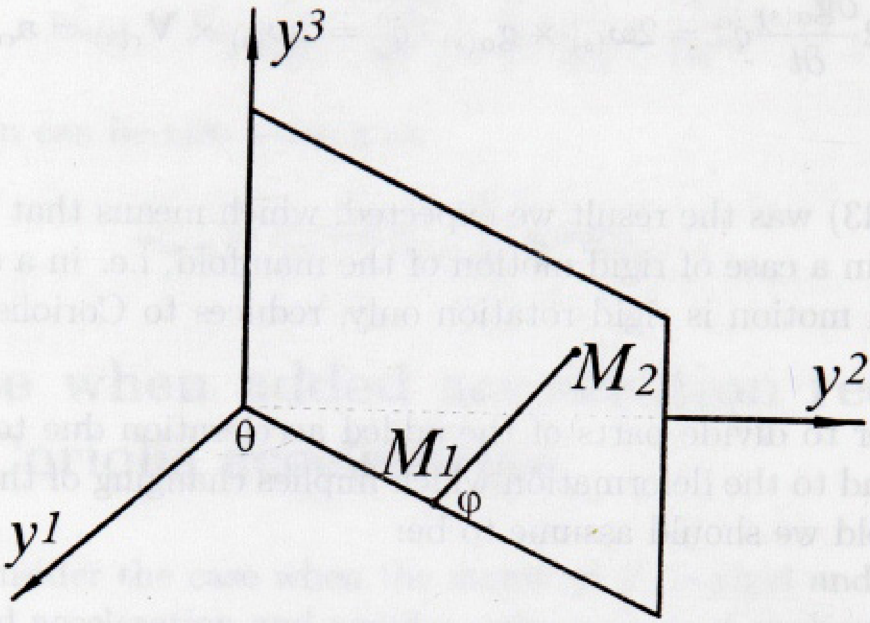

Fig. 1. System under consideration.

We introduce generalized coordinates $q^1 = \rho$, $q^2 = \varphi$, see Fig. 1, and note:

$$y^1_{(1)} = \rho\cos\theta, \qquad y^1_{(2)} = (\rho + b\cos\varphi)\cos\theta,$$

$$y^2_{(1)} = \rho\sin\theta, \qquad y^2_{(2)} = (\rho + b\cos\varphi)\sin\theta$$

$$y^3_{(1)} = 0, \qquad y^3_{(2)} = b\sin\varphi,$$

$$\mathbf{g}_\alpha = \frac{\partial \mathbf{R}}{\partial q^\alpha}, \quad \rightarrow \quad \begin{array}{l} \mathbf{g}_1 = \{(\cos\theta, \sin\theta, 0), (\cos\theta, \sin\theta, 0)\}, \\ \mathbf{g}_2 = \{(0,0,0), (-b\sin\varphi\cos\theta, -b\sin\varphi\sin\theta, b\cos\varphi\}, \end{array}$$

$$\mathbf{a}_{ad} = 2\frac{\partial \mathbf{g}_\alpha}{\partial t}\dot{q}^\alpha, \rightarrow$$

$$\begin{aligned}\mathbf{a}_{ad} = & \Big\{\Big[-\sin\theta\dot{\theta}\dot{\rho}, \cos\theta\dot{\theta}\dot{\rho}, 0\Big]; \\ & \Big[(-\sin\theta\dot{\theta}\dot{\rho} + b\sin\theta\sin\varphi\dot{\varphi}\dot{\theta} - \dot{b}\cos\theta\sin\varphi\dot{\varphi}), \\ & (\cos\theta\dot{\theta}\dot{\rho} - b\cos\theta\sin\varphi\dot{\theta}\dot{\varphi} - \dot{b}\sin\varphi\dot{\varphi}), \big(\dot{b}\cos\varphi\dot{\varphi}\big)\Big]\Big\},\end{aligned}$$

$$\boldsymbol{\omega}_{\alpha(s)} = \varepsilon^i_{jk}\frac{1}{l_{\alpha(s)}} \cdot \frac{\partial y^j_{(s)}}{\partial \rho^\alpha} \cdot \frac{\partial}{\partial t}\left(\frac{1}{l_{\alpha(s)}} \cdot \frac{\partial y^k_{(s)}}{\partial q^\alpha}\right)\mathbf{G}^{(s)}, \rightarrow$$

$$\boldsymbol{\omega}_1 = \{(0,0,\theta); (0,0,\theta)\},$$

$$\boldsymbol{\omega}_2 = \Big\{(0,0,0), (-\sin\varphi\cos\varphi\cos\theta, \theta, \sin\varphi\cos\varphi\cos\theta\theta, -\sin^2\varphi\theta)\Big\}.$$

7 Conclusion

This paper develops an idea of studying rheonomic problems as the "relative motion". Meaning of the "relative motion" should be assumed as the generalization of the classical relative motion. While the last one assumes transferred motion to be the rigid motion only (translation and rotation) the first one implies deformation as well. Deformation can be pure deformation and the deformation that changes the shape (the angles) of the surface.

Added acceleration of the "relative motion" is:

$$\mathbf{a}_{ad} = \mathbf{a}'_{ad} + \mathbf{a}''_{ad} + \mathbf{a}_{cor},$$

$$\mathbf{a}_{ad}^{\prime(s)} = 2\sum_{\alpha=1}^{n} \frac{\partial l_n l_{\alpha(s)}}{\partial t} \mathbf{g}_{\alpha(s)} \cdot \dot{q}^{\alpha} \text{ (pure deformation)},$$

$$\mathbf{a}_{ad}^{\prime\prime(s)} = 2\sum_{\alpha=1}^{n} \boldsymbol{\omega}_{\alpha}^{\prime(s)} \times \mathbf{g}_{\alpha(s)} \cdot \dot{q}^{\alpha} \text{ (changing of the shape)},$$

$$\mathbf{a}_{cor}^{(s)} = 2\boldsymbol{\omega}_{(s)} \times \mathbf{V}_{r(s)} \text{ (Coriolis acceleration)}.$$

References

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Kinematika reonomnih sistema

U ovom radu predlaže se interpretacija kretanja reonomnog sistema kao uopštenje složenog kretanja. Razmatra se kretanje reprezentativne tačke po $n-$dimenzijskoj mnogostrukosti $\mathcal{V}_n$ koja zbog reonomnosti veze može da menja položaj, oblik i veličinu. Relativno kretanje je kretanje tačke u odnosu na $\mathcal{V}_n$, prenosno kretanje same mnogostrukosti. Izvode se formule za relativno, prenosno i "dodatno" ubrzanje. Poslednje se, u slučaju krute mnogostrukosti, podudara sa ubrzanjem Koriolisa.