THEORETICAL AND APPLIED MECHANICS
23, pp. 151-156, 1997, UDK 514.8

The metric which allows the pure radiation field

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Submitted 27 March 1997

Abstract

We investigate a diagonal Riemannian metric. The metric represented by four unknown functions, two of them depend on x^1 only and two depend on x^4 only. We compose the Einstein tensor. The functions composing the metric tensor are determined from the condition that the form of Einstein tensor correspond to the energy-momentum tensor of the pure radiation field. This is the special case of the electromagnetic field for which the electric and magnetic three-vectors are equal and perpendicular.

1 Introduction

In this paper we would like to investigate a diagonal Riemannian metric in order to find out whether it could represent some cosmological model. We propose a metric

$$ds^2 = \left(dx^1\right)^2 + p\xi\left(dx^2\right)^2 + q\psi\left(dx^3\right)^2 - \left(dx^4\right)^2 . \tag{1}$$

We suppose that functions p and q depend on coordinate x^1 only and functions ξ and ψ depend on x^4, the time, only. We present the metric tensor in the form of matrix

$$g_{\alpha\beta} = \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & p\xi & 0 & 0 \\ 0 & 0 & q\psi & 0 \\ 0 & 0 & 0 & -1 \end{Bmatrix}.$$

Note that both components g_{11} and g_{44} of the metric tensor are constant.

We shall calculate the Einstein tensor. It will be expressed by the functions appearing in the metric tensor. We shall determine these functions so that the form of the Einstein tensor corresponds to the form of some known energy-momentum tensor.

First, we calculate the Ricci tensor. We present the components of the Ricci tensor different from zero. In the following expressions prime denotes the differentiation with respect to x^1, and a dot denotes the differentiation with respect to x^4.

$$R_{11} = \frac{1}{4}\left\{\left[2\frac{p''}{p} - \frac{(p')^2}{p^2}\right] + \left[2\frac{q''}{q} - \frac{q'^2}{q^2}\right]\right\},$$

$$R_{14} = \frac{1}{4}\left[\frac{p'}{p}\frac{\dot\xi}{\xi} + \frac{q'}{q}\frac{\dot\psi}{\psi}\right],$$

$$R_{22} = -\frac{p\xi}{4}\left\{-\left[2\frac{p''}{p} - \frac{(p')^2}{p^2}\right] - \frac{p'}{p}\frac{q'}{q} + \frac{\dot\xi}{\xi}\frac{\dot\psi}{\psi} + \left[2\frac{\ddot\xi}{\xi} - \frac{\left(\dot\xi\right)^2}{\xi^2}\right]\right\},$$

$$R_{33} = \frac{-q\psi}{4}\left\{-\left[2\frac{q''}{q} - \frac{(q')^2}{q^2}\right] - \frac{p'}{p}\frac{q'}{q} + \frac{\dot\xi}{\xi}\frac{\dot\psi}{\psi} + \left[2\frac{\ddot\psi}{\psi} - \frac{\left(\dot\psi\right)^2}{\psi^2}\right]\right\},$$

$$R_{44} = \frac{1}{4}\left\{\left[2\frac{\ddot\xi}{\xi} - \frac{\left(\dot\xi\right)^2}{\xi^2}\right] - \left[2\frac{\ddot\psi}{\psi} - \frac{\left(\dot\psi\right)^2}{\psi^2}\right]\right\}$$

Next, we calculate the Ricci scalar curvature

$$R=\frac{1}{2}\left\{\left[\frac{p''}{p}-\frac{(p')^2}{p^2}\right]+\left[2\frac{q''}{q}-\frac{(q')^2}{q^2}\right]+\frac{p'}{p}\frac{q'}{q}-\right.$$

$$\left.-\frac{\dot{\xi}}{\xi}\frac{\dot{\psi}}{\psi}-\left[2\frac{\ddot{\xi}}{\xi}-\frac{(\dot{\xi})^2}{\xi^2}\right]-\left[2\frac{\ddot{\psi}}{\psi}-\frac{(\dot{\psi})^2}{\psi^2}\right]\right\}.$$

Now we compose the Einstein tensor. We present the components different from zero

$$G_{11}=\frac{1}{4}\left\{\left[2\frac{\ddot{\xi}}{\xi}-\frac{(\dot{\xi})^2}{\xi^2}\right]+\left[2\frac{\ddot{\psi}}{\psi}-\frac{(\dot{\psi})^2}{\psi^2}\right]+\frac{\dot{\xi}}{\xi}\frac{\dot{\psi}}{\psi}-\frac{p'}{p}\frac{q'}{q}\right\},$$

$$G_{14}=\frac{1}{4}\left[\frac{p'}{p}\frac{\dot{\xi}}{\xi}+\frac{q'}{q}\frac{\dot{\psi}}{\psi}\right],$$

$$G_{22}=-\frac{p\xi}{4}\left\{\left[2\frac{q''}{q}-\frac{(q')^2}{q^2}\right]-\left[2\frac{\ddot{\psi}}{\psi}-\frac{(\dot{\psi})^2}{\psi^2}\right]\right\},$$

$$G_{33}=-\frac{q\psi}{4}\left\{\left[2\frac{p''}{p}-\frac{(p')^2}{p^2}\right]-\left[2\frac{\ddot{\xi}}{\xi}-\frac{(\dot{\xi})^2}{\xi^2}\right]\right\},$$

$$G_{44}=\frac{1}{4}\left\{\left[2\frac{p''}{p}-\frac{(p')^2}{p^2}\right]+\left[2\frac{q''}{q}-\frac{(q')^2}{q^2}\right]+\frac{p'}{p}\frac{q'}{q}-\frac{\dot{\xi}}{\xi}\frac{\dot{\psi}}{\psi}\right\}.$$

In order to have a better insight in the form of the Einstein tensor we shall present it in the matrix form

$$G_{\alpha\beta}=\left\{\begin{matrix} G_{11} & 0 & 0 & G_{14} \\ 0 & G_{22} & 0 & 0 \\ 0 & 0 & G_{33} & 0 \\ G_{14} & 0 & 0 & G_{44} \end{matrix}\right\}.$$

To build up Einstein tensor which corresponds to the energy-momentum tensor of the "static" perfect fluid or the null electromagnetic field, the od koordinate x^1, a druge dve zavise samo od x^4. Zatim je odredjen Ajnštajnov tenzor. Nepoznate funkcije koje se pojavljuju u metričkom tenzoru odredjene su tako da oblik Ajnštajnovog tenzora odgovara tenzoru energije-impulsa polja čiste radijacije. To je specijalan slučaj elektromagnetnog polja kada su vektori električnog i magnetnog polja jednakog intenziteta i upravni jedan na drugog.