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Variation of the lift coefficient of an arbitrary oscillating airfoil

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Abstract

Complex potential of the flow around airfoil is split into part which corresponds to steady constant circulation flow and part for unsteady flow. Steady potential flow is further divided into three component each proportional to lagging, flapping and pitching motion. Unsteady potential is used to simulate free vortex generation and free vortex convection.

Computation is performed by the conformal mapping of the exterior of airfoil contour to the interior of the circle, while trailing edge of the airfoil is mapped to the intersection of the mapping circle and real axis in the transformed plane. Unsteady flow around circle with presence of the singularities is solved in the circle plane and mapped back in the airfoil plane. Thomson's reflection principle is used to preserve circular shape in the transformed plane.

Newly generated free vortices are set in the vicinity of the airfoil trailing edge, while their intensity had to satisfy Kelvin theorem and unsteady form of Kutta-Jukovsky condition. Unsteady forces are determined by the integration of unsteady form of Bernoulli equation. Obtained results are compared with available wind tunnel tests.

1 Introduction

Conformal mapping is well developed technique for the solution of potential flow problems [1] - [4]. In this paper we developed procedure motivated by the works [2], [3] and by the book from Sedov [7]. Mapping of the points outside of the circle is done by the same series by which mapping to the circle is done. There are many procedures by which is possible to calculate corresponding points outside airfoil contour and circle. Accuracy of these procedures is not greater than direct application of power series. Coordinates are chosen fixed to airfoil contour, so that only one mapping to the unit circle is necessary.

Changed circulation around the airfoil should be compensated by the immediate vortex shedding from the trailing edge in order to keep total circulation unchanged according to Kelvin theorem. Position and the strength of the shaded vortex is essential question. Some motivations about this question can be found in [5] or [6].

Unsteady Kutta condition is basically connected to viscous flow, but, for the inviscid flow it is demanded that loading of the trailing edge is equal on the upper and on the lower side. That connects tangential velocity difference at the trailing edge with circulation changes.

Once generated free vortices travel with surrounding fluid velocity without changing its strength. During calculation number of free vortices is increased, so suitable data structure is needed to support such needs.

Unsteady force is calculated by the integration of the unsteady Bernoulli equation in the form derived as in [7], which is also called generalized Blasius theorem.

2 Problem statement

For the inviscid and incompressible flow continuity equation is the same for steady and unsteady flows

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0, \tag{1}$$

where φ is velocity potential, and x and y are coordinates of the arbitrary point in the flow field. We are not going to solve this equation directly. Instead of that we shall map known solution for the circle to the specified airfoil shape. Impermeability of the airfoil contour is expressed in the form

$$\mathbf{V}_R \cdot \mathbf{n} = 0, \qquad (2)$$

$$\mathbf{V}_R = \mathbf{V}_A - \mathbf{V}_T,$$

where $\mathbf{V}_R$ is the velocity in the body fixed coordinate system, $\mathbf{V}_A$ is the absolute velocity of the fluid flow, and $\mathbf{V}_T$ is the transport velocity of the body fixed coordinate system. With $\mathbf{n}$ is assigned normal to the arbitrary point of the airfoil contour. Transport velocity of the point m on the airfoil contour is expressed with

$$\mathbf{V}_T = \left(U_o(t) - \Omega(t)\, y_m\right)\mathbf{i} + \left(V_o(t) + \Omega(t)\, x_m\right)\mathbf{j}, \qquad (3)$$

where U_o and V_o are linear velocities of the body fixed coordinate system origin, and Ω is angular velocity of the body fixed coordinate system.

Unsteady Kutta condition can be expressed in the form

$$\frac{\partial \Gamma}{\partial t} = \frac{V_{t_1}^2 - V_{t_N}^2}{2}, \qquad (4)$$

where V_{t_1} and V_{t_N} are tangential velocities at the lower and upper side at the trailing edge of the airfoil, and Γ is the circulation around airfoil contour. This equation is nonlinear because unknown strength, free vortex influence quantities on both side of this equations, and hence whole problem is nonlinear. The only way to solve such problem is to iterate over the unknown free vortex strength until Eq. (4) is not approximately satisfied, at each time instance.

We have to satisfy one equation more. That's Kelvin theorem which for unchanged quantity of fluid states that circulation for the inviscid ideal flow doesn't change with time

$$\frac{\partial \Gamma}{\partial t} = 0. \qquad (5)$$

3 Conformal mapping

The mapping is conformal when the angles between lines are preserved during mapping. More interesting mappings are those for which exists some points at which mapping function isn't conformal. Using of such mappings allow us to map circle to the cuspid shapes. Mapping of the airfoil contour will be performed in few steps.

First step is to map airfoil shape to a near circle by the application of the Karman-Treftz transformation

$$\frac{z - z_o}{z - z_1} = \left(\frac{\zeta_1 - \beta z_o}{\zeta_1 - \beta z_1}\right)^{\frac{1}{\beta}}, \tag{6}$$

where z is the coordinate in the airfoil plane, z_o is the coordinate of the leading edge singular point, chosen to be halfway from the leading edge to the center of curvature, z_1 is the coordinate of the trailing edge singular point, ζ_1 is the coordinate of the airfoil shape in the near circle plane, and β is defined by

$$\beta = \frac{1}{2 - \frac{\tau}{\pi}},$$

where τ is the trailing edge angle of the airfoil. One can obtain from Eq. (6) the following

$$\zeta_1 = \beta \frac{z_o (z - z_1)^{\beta} - z_1 (z - z_o)^{\beta}}{(z - z_1)^{\beta} - (z - z_o)^{\beta}}, \tag{7}$$

$$z = \frac{z_o (\zeta_1 - \beta z_1)^{\frac{1}{\beta}} - z_1 (\zeta_1 - \beta z_o)^{\frac{1}{\beta}}}{(\zeta_1 - \beta z_1)^{\frac{1}{\beta}} - (\zeta_1 - \beta z_o)^{\frac{1}{\beta}}}, \tag{8}$$

$$\frac{d\zeta_1}{dz} = \beta \zeta_1 \left[\frac{z_o (z - z_1)^{\beta-1} - z_1 (z - z_o)^{\beta-1}}{z_o (z - z_1)^{\beta} - z_1 (z - z_o)^{\beta}} - \frac{(z - z_1)^{\beta-1} - (z - z_o)^{\beta-1}}{(z - z_1)^{\beta} - (z - z_o)^{\beta}}\right] \tag{9}$$

When the formulae (7) is applied to NACA 4412 airfoil it is obtained the shape shown on the Fig. 1.

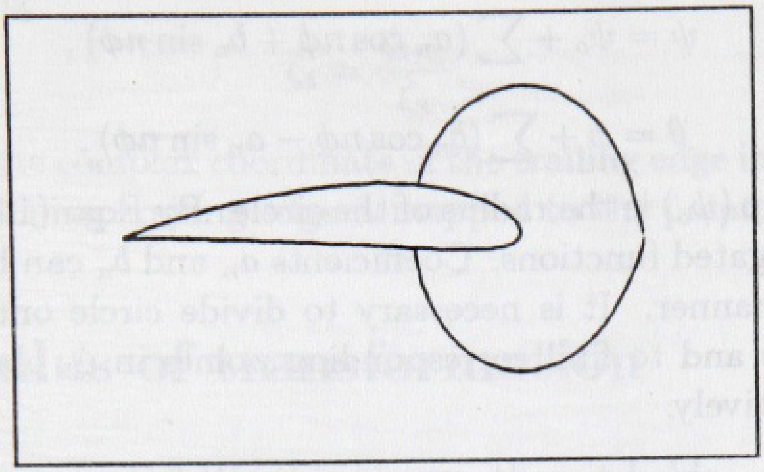

Fig. 1. - Transformation of the airfoil shape to a near circle plane.

Next step is to translate center of gravity of the near circle to the coordinate origin. center of gravity of the near circle in the ζ_1 plane can be found by the application of the following expression

$$\zeta_1^T = \frac{\sum_{i=1}^{n-1}\left(\zeta_{1_{i+1}} + \zeta_{1_i}\right)\left|\zeta_{1_{i+1}} - \zeta_{1_i}\right|}{2\sum_{i=1}^{n-1}\left|\zeta_{1_{i+1}} - \zeta_{1_i}\right|}. \tag{10}$$

Coordinates of the near circle after translation can be found

$$\zeta_{2_i} = \zeta_{1_i} - \zeta_1^T. \tag{11}$$

Now remains to map near circle to circle in ζ_3 plane. We have applied the following series

$$\zeta_2 = \zeta_3 \exp\left(\sum \frac{c_n'}{\zeta_3^n}\right), \tag{12}$$

by which we actually map the circle to near circle. With ζ_3 are given coordinates in the circle plane, and $c_n' = a_n' + b_n'$. If the coordinates $\zeta_2 = \exp(\psi + i\theta)$ and $\zeta_3 = \exp(\psi + i\theta)$ are written into polar form we get

$$\begin{aligned} \exp(\psi + i\theta) = \; & \exp(\psi + i\theta) \times \\ & \exp\left(\sum (a_n \cos n\phi + b_n \sin n\phi)\right. \\ & \left. + i\,(b_n \cos n\phi - a_n \sin n\phi)\right), \end{aligned} \tag{13}$$

where $a_n = a'_n / R^n$ and $b_n = b'_n / R^n$, or

$$\psi = \psi_o + \sum \left(a_n \cos n\phi + b_n \sin n\phi\right), \tag{14}$$

$$\theta = \phi + \sum \left(b_n \cos n\phi - a_n \sin n\phi\right), \tag{15}$$

where $R = \exp(\psi_o)$ is the radius of the circle. By Eqs. (14) and (15) are defined conjugated functions. Coefficients a_n and b_n can be determined in iterative manner. It is necessary to divide circle onto 2^M equally spaced points and to find corresponding points in ζ_2 plane. This can be done iteratively.

1. Suppose $a_n = b_n = 0$, then $\theta_i = \phi_i$ from Eq. (15).
2. Left side of Eq. (14) is now known, $\psi(\theta_i)$. Coefficients a_n and b_n can be determined by the application of the fast Fourier transformation analysis.
3. Fourier synthesis step should be performed in Eq. (15) to obtain new estimate of angle $\theta(\phi)$. Steps 2. and 3. should be repeated until convergence is reached.

We have now defined mapping functions, but it is necessary to know on which point trailing edge of the airfoil is mapped into circle plane. Angle θ_{TE} of the trailing edge in the near circle plane is known. The closest θ_i to θ_{TE} should be found. Taylor series of angle $\theta(\phi)$ near angle ϕ_i is given by

$$\theta_{TE} = \theta(\phi_i) + \frac{d\theta}{d\phi}(\beta - \phi_i) + ..., \tag{16}$$

from which $\beta = \phi_{TE}$ can be found

$$\beta = \phi_i + \frac{\theta_{TE} - \theta(\phi_i)}{\dfrac{d\theta}{d\phi}}. \tag{17}$$

Derivative $d\theta/d\phi$ can be calculated from Eq. (15)

$$\frac{d\theta}{d\phi} = 1 - \sum n\left(a_n \cos n\phi + b_n \sin n\phi\right). \tag{18}$$

Exterior of the circle can be mapped onto interior of the unit circle by the mapping

$$\zeta_4 = \frac{\zeta_{3_{TE}}}{\zeta_3}, \tag{19}$$

where $\zeta_{3_{TE}}$ is the complex coordinate of the trailing edge in the ζ_3 plane. With such mapping trailing edge is mapped to the point $\zeta_4 = +1$.

4 Modulus of transformation

If the complex potential $W(z)$ is known, than it holds

$$W(z) = W(z(\zeta_1)) = \ldots = W(z(\zeta_1(\zeta_2(\zeta_3)))) \cdot \tag{20}$$

Conjugate complex velocity in the $z-$plane can be found by differentiating former equation with respect to z

$$\bar{V} = \frac{dW}{dz} = \frac{dW}{d\zeta_3}\frac{d\zeta_3}{d\zeta_2}\frac{d\zeta_2}{d\zeta_1}\frac{d\zeta_1}{dz}. \tag{21}$$

With $dW/d\zeta_3$ is defined conjugate complex velocity in the ζ_3-plane, while rest of expression on the right side of Eq. (21) is modulus of transformation $d\zeta_3/dz$. Let us find all components of that modulus.

Modulus of transformation between planes z and ζ_1, $d\zeta_1/dz$ is given by Eq. (9). Modulus $d\zeta_2/d\zeta_1$ is equal to 1. Modulus $d\zeta_3/d\zeta_2$ can be given in the following form

$$\frac{d\zeta_3}{d\zeta_2} = \frac{\dfrac{d\zeta_3}{d\phi}}{\dfrac{d\zeta_2}{d\phi}} = \frac{\dfrac{d}{d\phi}\left(e^{\psi_o + i\phi}\right)}{\dfrac{d}{d\phi}\left(e^{\psi + i\theta}\right)}, \tag{22}$$

or

$$\frac{d\zeta_3}{d\zeta_2} = i\frac{\zeta_3}{\zeta_2}\frac{1}{\dfrac{d\psi}{d\phi} + i\dfrac{d\theta}{d\phi}}. \tag{23}$$

Let us consider now transformation of points outside circle line. Point in the circle plane is defined by $\zeta_3 = r\exp(i\phi)$ or $\zeta_3 = \exp\ln(r + i\phi)$.

Eq. (12), after substitution of expression for ζ_3, now looks

$$\zeta_2 = r \exp(i\phi) \exp\left[\sum c'_n / \left(r^n e^{i\phi}\right)\right], \tag{24}$$

or

$$\psi = \ln r + \sum \left(\frac{R}{r}\right)^n (a_n \cos n\phi + b_n \sin n\phi), \tag{25}$$

$$\theta = \phi + \sum \left(\frac{R}{r}\right)^n (b_n \cos n\phi - a_n \sin n\phi). \tag{26}$$

Differentiating Eq. (24) with respect to ζ_3 one obtains

$$\frac{d\zeta_3}{d\zeta_2} = \frac{\zeta_2}{\zeta_3}\left(1 - \sum \frac{nc'_n}{\zeta_3}\right), \tag{27}$$

or

$$\frac{d\zeta_3}{d\zeta_2} = \frac{\exp\left(-\sum \frac{c'_n}{\zeta_3^n}\right)}{1 - \sum \frac{nc'_n}{\zeta_3}}. \tag{28}$$

Further transformation is eased by the fact that transformation between ζ_2 and z-plane is analytic. Correspondent lines in ζ_2 and z-plane are shown in Fig. 2.

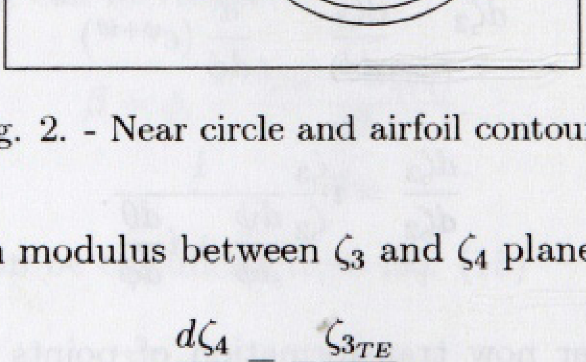

Fig. 2. - Near circle and airfoil contours.

Transformation modulus between ζ_3 and ζ_4 plane is easily found as

$$\frac{d\zeta_4}{d\zeta_3} = -\frac{\zeta_{3TE}}{\zeta_3^2}. \tag{29}$$

5 Thomson's theorem

Let here be no rigid boundaries, and let the complex potential of the flow is given by $f(z)$, where the singularities of the $f(z)$ are at the distance greater then a from the origin. Circular cylinder $|z| = R$ can be introduced in the flow by the following complex potential

$$W = f(z) + \bar{f}\left(\frac{R^2}{z}\right). \tag{30}$$

Eq. (30) is called Thomson's theorem, [8]. Complex potential of the stream inclined at angle α to x-axis is $f(z) = V_\infty \exp(-i\alpha) z$. Circular cylinder, with origin at the z, is introduced in the flow field by the application of Eq. (30)

$$W(z) = V_\infty e^{-i\alpha}(z - z_o) + \frac{R^2 V_\infty e^{i\alpha}}{z - z_o}. \tag{31}$$

Complex potential of the vortex is given by the following equation

$$f(z) = i\frac{\Gamma}{2\pi}\ln(z - z_v), \tag{32}$$

where z_v is the coordinate of the center of vortex. Introduction of solid circle into flow field is done again by the application of Thomson's theorem

$$W(z) = \frac{i\Gamma}{2\pi}\ln(z - z_o - z_v) + \frac{i\Gamma}{2\pi}\ln\left(\frac{R^2}{z - z_o} - \bar{z}_v\right). \tag{33}$$

Former equation can be rearranged into following form

$$W(z) = \frac{i\Gamma}{2\pi}\ln[z - (z_o + z_v)] - \frac{i\Gamma}{2\pi}\ln(z - z_o) + \frac{i\Gamma}{2\pi}\ln\left[z - \left(z_o + \frac{R^2}{\bar{z}_v}\right)\right], \tag{34}$$

where constant $\ln(-\bar{z}_v)$ is dropped from the expression for $W(z)$. Two last terms in the expression for complex potential do not contribute to circulation inside cylinder because they cancel each other. Complex potential for the circular cylinder of the radius R, with center at z_o for complex potential $f(z)$ is introduced together with vortex with center at z_o by the formula

$$W(z) = f(z - z_o) + \bar{f}\left(\frac{R^2}{z - z_o}\right) + \frac{i\Gamma}{2\pi} \ln(z - z_o). \tag{35}$$

We can determine magnitude of the vortex Γ for which point $z = z_1$ is stagnation point on the circle. If we differentiate (35) with respect to z, and equalize obtained expression with zero, we get

$$f'(z_1 - z_o) - \frac{R^2}{(z_1 - z_o)^2}\bar{f}'\left(\frac{R^2}{z - z_o}\right) + \frac{i\Gamma}{2\pi(z_1 - z_o)} = 0.$$

Former equation can be solved for Γ

$$\frac{\Gamma}{2\pi i} = (z_1 - z_o) f'(z_1 - z_o) - \frac{R^2}{(z_1 - z_o)^2}\bar{f}'\left(\frac{R^2}{z - z_o}\right). \tag{36}$$

If $f(z) = V_\infty \exp(-i\alpha)z$ and $z_1 = R\exp(i\beta)$, expression for Γ becomes

$$\Gamma = 4\pi R V_\infty \sin(\alpha - \beta). \tag{37}$$

Lift coefficient c_y can be found from the equation

$$\rho V_\infty \Gamma = \rho V_\infty^2 c_y S,$$

where S is the wing surface, in our case $S = l \cdot 1$,namely

$$c_y = \frac{2\Gamma}{V_\infty l} = 8\pi \frac{R}{l} \sin(\alpha - \beta). \tag{38}$$

Conjugated complex velocity $\bar{v}$ for the flow around cylinder is given by the equation

$$\frac{\bar{v}}{V_\infty} = \frac{1}{V_\infty}\frac{dW}{dz} = 2ie^{-i\phi}\left[\sin(\phi - \alpha) + \sin(\alpha - \beta)\right]. \tag{39}$$

Velocity obtained by Eq. (39) can be easily transformed to airfoil plane according to Eq. (21). Pressure coefficient C_p is defined by

$$C_p = 1 - \left|\frac{\bar{v}}{V_\infty}\right|^2. \tag{40}$$

Fig. 3 show distribution of pressure coefficient for the airfoil NACA 4412 and $\alpha = 4$.

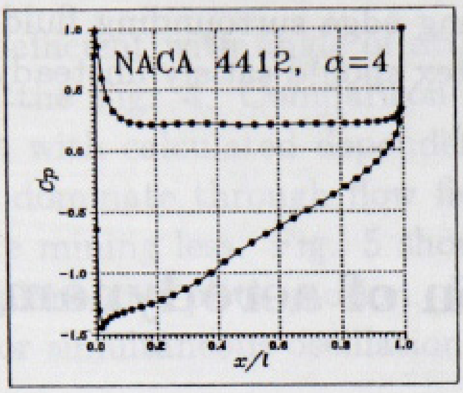

Fig. 3. - Airfoil NACA 4412, $\alpha = 4^o$.

6 Generation of free vortices

Transformation modulus between ζ_3 and z plane is singular at the trailing edge of the airfoil. That means that velocity in the circle plane should be divided by zero. To avoid infinite velocities at the trailing edge Kutta-Jukowsky condition should be applied, which states thus velocity in the circle plane, at the corresponding point of the trailing edge, should be also zero in order to obtain finite velocities at the trailing edge in the airfoil, z plane,. Change of the incidence angle of the airfoil cause change of the circulation of the airfoil which must be compensated by the free vortex to satisfy Kelvin's condition

$$\frac{d\Gamma}{dt} = 0. \tag{41}$$

Let us consider usual situation when there are number of already generated vortices. Complex potential in the circle plane is given by the following equation

$$W(\zeta_3) = V_\infty e^{-i\alpha}\zeta_3 + V_\infty e^{i\alpha}\frac{R^2}{\zeta_3} + \frac{i\Gamma_o}{2\pi}\ln\zeta_3$$

$$+\sum\left(\frac{i\Gamma_k}{2\pi}\right)\left[\ln\left(\zeta_3 - \zeta_{3_k}\right) - \ln\zeta_3 + \ln\left(\zeta_3 - \frac{R^2}{\bar{\zeta}_3}\right)\right], \tag{42}$$

where ζ_{3_k} is the position of the k-th free vortex in the circle plane. Position of the newly generated free vortex is specified in advance, according to the trailing edge surrounding fluid velocity. Strength of the generated free vortex should satisfy unsteady Kutta condition [9] - [11].

7 Calculation of aerodynamic force

Let us suppose that the mapping from the interior of the unit circle to the exterior of the airfoil is done by the following series

$$z = f(z) = \frac{k}{\zeta} + k_o + k_1\zeta + k_2\zeta^2 + \dots, \tag{43}$$

where coefficients k_i are determined as it is previously explained. According to [7] complex potential of the flow can be expressed as a three component function

$$w_o(z) = U_o w_1(z) + V_o w_2(z) + \Omega w_3(z), \tag{44}$$

where index $_o$ means regular part of the complex potential. In [7] it is shown that

$$U_o w_1(\zeta) + V_o w_2(\zeta) = \bar{q}_o f(\zeta) - \frac{\bar{q}_o k}{\zeta} - q_o \bar{k}\zeta,$$

$$w_3(\zeta) = -\frac{1}{4\pi}\oint_C f(\zeta')\,\bar{f}\left(\frac{1}{\zeta'}\right)\frac{\zeta' + \zeta d\zeta'}{\zeta' - \zeta d\zeta'}. \tag{45}$$

While the force on arbitrary moving contour can be calculated from

$$X + iY = -i\rho z_M \frac{d\Gamma}{dt} + \frac{i\rho}{2}\oint_C \overline{\left(\frac{dw}{dz}\right)^2} + \frac{d}{dt}\left[\rho\frac{d(Sz^*)}{dt} + i\rho\oint_C z\frac{dw}{dz}dz\right], \tag{46}$$

where z_M is the coordinate of the arbitrary point M.

8 Calculated examples

Variation of the lift coefficient with angle of attack for the $\alpha = 2.5 + 4\sin(0.3)$ is shown on the Fig. 4. Comparison with experiments [12] shows good agreement with calculated dependence for the case when viscous effects do not dominate through flow field. When separation occurs comparisons are mining less. Fig. 5 shows lift coefficient variation with combined plane and pitch motion. Fig. 6 shows variation of the lift coefficient for simultaneous oscillation of the airfoil in the x and y direction.

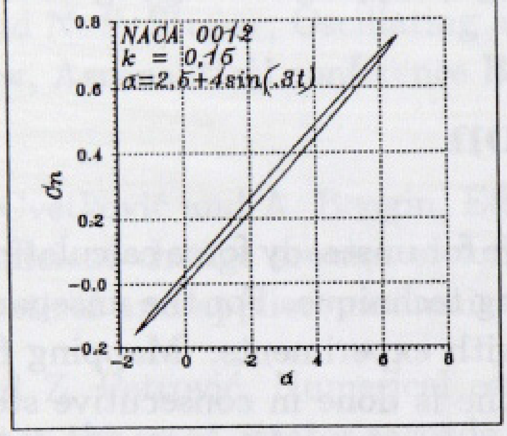

Fig. 4. - Variation of C_n with $\alpha = 2.5 + 4\sin(.3t)$.

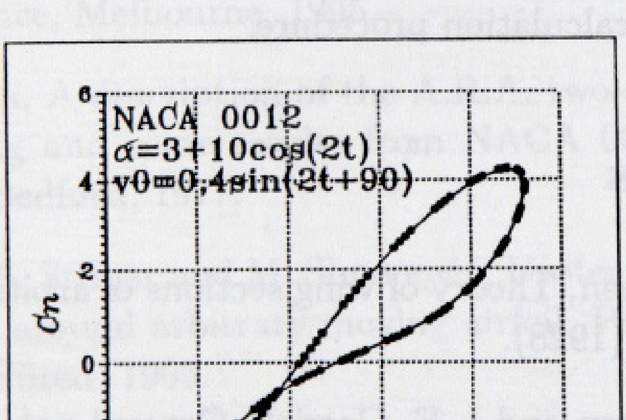

Fig. 5. - Normal coefficient variation for combined pitch and plane motion.

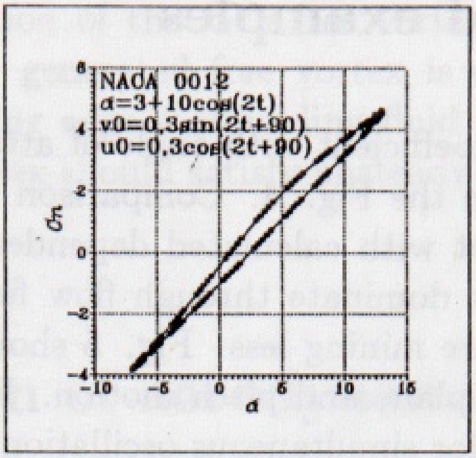

Fig. 6. - Variation of C_n for simultaneous variation of pitching angle, flapping and lagging motion.

9 Conclusion

Calculation procedure for unsteady force calculations is developed based on conformal mapping technique. For the unseparated flow results reasonably well agree with experiments. Mapping from the airfoil plane to the unit circle plane is done in consecutive steps. Calculations are accelerated by the application of fast Fourier transformation for determination of the mapping coefficients. Few examples are presented which illustrate calculation procedure.

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Varijacija koeficijenta uzgona proizvoljnog oscilirajućeg aerodinamičkog profila

Kompleksni potencijal strujanja oko aeroprofila je podeljen na deo koji odgovara stacionarnom kružnom strujanju i deo koji odgovara nestacionarnom strujanju. Stacionarno potencijalno strujanje je dalje podeljeno na tri komponente. Nestacionarni potencijal je upotrebljen za simulaciju slobodnog stvaranja vrtloga i njihovo prenošenje. Račun je sproveden pomoću konformnog preslikavanja spoljašnjs konture aeroprofila u krug, dok je trag aeroprofila preslikan u presek tog kruga i realne ose u transformisanoj ravni. Nestacionarno strujanje oko kruga u prisustvu singulariteta je rešeno u ravni kruga i preslikano nazad u ravan aeroprofila. Uporebljen je Thomson-ov princip odbijanja da bi se očuvao kružni oblik u transformisanoj ravni. Novo proizvedeni slobodni vrtlozi su postavljeni u blizinu traga aeroprofila, dok njihov intenzitet mora da zadovolji teoremu Kelvin-a i nestacionarni uslov Kutta-Jukovsky. Nestacionarne sile su odredjene integracijom nestacionarne jednačine Bernoulli-ja. Dobijeni rezultati su uporedjeni sa rezultatima ispitivanja u vazdušnom tunelu.