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Gauss's principle in a parametric formulation of mechanics

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Abstract

In this paper it is demonstrated that the Gauss's principle also can be formulated in a parametric formulation of the mechanics of rheonomic systems, which is based on a family of varied paths, and on the transition to a new parameter, retaining the time as the independent variable. In this manner, one obtains the Gauss's principle, both in a differential form and in the form of a principle of least constraint, including as well their presentation in generalized coordinates. From so formulated principle follows an extended system of corresponding Lagrange's and Gibbs-Appell's equations, and in all so obtained relations appears one additional term or equation, arising from the nonstationarity of constraints, which is characteristic for this parametric formulation of mechanics.

1 Introduction

As it is known [1,2], the Gauss's general principle of mechanics expresses a characteristic of the motion of a mechanical system with arbitrary constraints in a infinitesimal time interval in comparison with its imagined motion with the same values of all position and velocity vectors, bur without the presence of constraints. Then the real motion of this system is determined by the condition

$$(\mathbf{F}_\nu - m_\nu \mathbf{a}) \cdot \delta \ddot{\mathbf{r}}_\nu = 0, \qquad (\nu = 1, 2, ..., N)\,, \tag{1}$$

where the summation over the repeated indices always would be understood, regardless of the position of this index, and this is the differential form of Gauss's principle. Its other, equivalent form can be obtained by introducing so-called "constraint", and this principle then asserts that at the real motion of a system this constraint has the minimal value

$$\delta Z = 0, \qquad Z = \frac{1}{m_\nu}\left(\mathbf{a}_\nu - \frac{\mathbf{F}_\nu}{m_\nu}\right)^2. \tag{2}$$

On the other hand, the mechanics of rheonomic systems is formulated by the author himself in a parametric manner [3], which is based on a family of varied paths $\mathbf{r}_\nu = \mathbf{r}_\nu(t, \lambda)$, drawn from their initial position, and on the transition to a new parameter depending on the chosen path, keeping the time as the independent variable. In such formulation the equations of constraints can be expressed by introducing this parameter $\tau = \tau(t, \lambda)$ instead the time along any of these paths ($\lambda = const$) in the form

$$f_\mu(\mathbf{r}_\nu, \tau) = 0, \qquad (\mu = 1, 2, ..., k)\,, \tag{3}$$

which yields the conditions for the virtual displacements

$$\frac{\partial f_\mu}{\partial \mathbf{r}_\nu} \cdot \delta \mathbf{r}_\nu = -\frac{\partial f_\mu}{\partial \tau}\delta\tau. \tag{4}$$

If this function for the real motion is taken as a additional generalized coordinate $q^o = \tau_o(t)$, the relations between the position vectors and these coordinates receive the form

$$\mathbf{r}_\nu = \mathbf{r}_\nu\left(q^o, q^1, ..., q^n\right) = \mathbf{r}_\nu(q^\alpha)\,, \qquad (\alpha = 0, 1, ..., n) \tag{5}$$

The results following from such formulation are in accordance with those obtained by V. Vujičić [4] in his modification of the mechanics of rheonomic systems. Here, *inter alia*, the Gauss's principle is formulated also in the second of these forms, where appear the specific additional terms or equations, corresponding to this additional, so-called rheonomic coordinate q^o, but without any connection with the first form of this principle.

In this paper we shall demonstrate that the Gauss's principle can be formulated in this parametric formulation of mechanics also, both in the form of a differential principle and in the form of a principle of least constraint, and that from so formulated principle in generalized coordinates follows the corresponding systems of Lagrange's and Gibbs-Appell's equations.

2 Gauss's conditions and ideal reaction forces

Let us consider the motion of a mechanical system with k holonomic nonstationary constraints $f_\mu(\mathbf{r}_\nu, \tau) = 0$ in the infinitesimal parameter interval $(\tau, \tau + d\tau)$ in this parametric formulation of mechanics. Then for any particle we have

$$\mathbf{r}_\nu(\tau + d\tau) = \mathbf{r}_\nu(\tau) + \mathbf{v}_\nu d\tau + \mathbf{a}_\nu \frac{d\tau^2}{2} + \ldots, \tag{6}$$

and for an imagined motion of this system with the same values of all position and velocity vectors, but free i.e. without the constraints

$$\mathbf{r}'_\nu(\tau + d\tau) = \mathbf{r}_\nu(\tau) + \mathbf{v}_\nu d\tau + \frac{\mathbf{F}_\nu}{m_\nu}\frac{d\tau^2}{2} + \ldots\,. \tag{7}$$

In this way, all the quantities $\mathbf{r}_\nu$ and $\dot{\mathbf{r}}_\nu$, as well τ and $\dot{\tau}$ are fixed, so that the corresponding Gauss's conditions here are given by

$$\begin{aligned} \delta\mathbf{r}_\nu &= \delta\dot{\mathbf{r}}_\nu = 0, \\ \delta\tau &= \delta\dot{\tau} = 0. \end{aligned} \tag{8}$$

The conditions for the variations $\delta\ddot{\mathbf{r}}_\nu$ can be founded by differentiating relations (4) two times, by analogy with the habitual proof (see [5], p.

377)

$$= \frac{d^2}{dt^2}\left(\frac{\partial f_\mu}{\partial \mathbf{r}_\nu}\right) \cdot \delta \mathbf{r}_\nu + 2\frac{d}{dt}\left(\frac{\partial f_\mu}{\partial \mathbf{r}_\nu}\right) \delta \dot{\mathbf{r}}_\nu + \frac{\partial f_\mu}{\partial \mathbf{r}_\nu} \delta \ddot{\mathbf{r}}_\nu =$$

$$- \frac{d^2}{dt^2}\left(\frac{\partial f_\mu}{\partial \tau}\right) \cdot \delta \tau - 2\frac{d}{dt}\left(\frac{\partial f_\mu}{\partial \tau}\right) \delta \dot{\tau} - \frac{\partial f_\mu}{\partial \tau} \delta \ddot{\tau},$$

which, because of Gauss's conditions (8), is reduced to

$$\frac{\partial f_\mu}{\partial \mathbf{r}_\nu} \delta \ddot{\mathbf{r}}_\nu = - \frac{\partial f_\mu}{\partial \tau} \delta \ddot{\tau}, \qquad (\mu = 1, 2, ..., k). \tag{9}$$

Then , the total work of ideal reaction forces on the "displacements" $\delta \ddot{\mathbf{r}}_\nu$ is determined by

$$\mathbf{R}^{id}_\nu \cdot \delta \ddot{\mathbf{r}}_\nu = \lambda_\mu \frac{\partial f_\mu}{\partial \mathbf{r}_\nu} \cdot \delta \ddot{\mathbf{r}}_\nu = -\lambda_\mu \frac{\partial f_\mu}{\partial \tau} \delta \ddot{\tau},$$

therefore

$$\mathbf{R}^{id}_\nu \cdot \delta \ddot{\mathbf{r}}_\nu = R_o \delta \ddot{\tau}, \tag{10}$$

where R_o is given by

$$R_o = -\lambda_\mu \frac{\partial f_\mu}{\partial \tau} = \mathbf{R}^{id}_o \cdot \frac{\partial \mathbf{r}_\nu}{\partial q^o}. \tag{11}$$

Here R_o designates the generalized ideal reaction force corresponding to the additional coordinate $q^o = \tau_o(t)$, arising from the nonstationarity of constraints and introduced by Vujičić [4]. Consequently, this total work of ideal reaction forces $\mathbf{R}^{id}_\nu \cdot \delta \ddot{\mathbf{r}}_\nu$ in the case of nonstationary constraints in this formalism is different from zero, which represents a characteristic difference in comparison with the usual formulation.

3 Two equivalent forms of Gauss's principle

In order to formulate here the Gauss's principle, let us start from the fundamental equation of dynamics

$$m_\nu \mathbf{a}_\nu = \mathbf{F}_\nu + \mathbf{R}^{id}_\nu + \mathbf{R}^*_\nu, \qquad (\nu = 1, 2, ..., N), \tag{12}$$

where the reaction forces are decomposed into the ideal $\mathbf{R}_\nu^{id}$ and non-ideal ones $\mathbf{R}_\nu^*$. From here, multiplying by $\delta\ddot{\mathbf{r}}_\nu$ and summing over the index ν, we find

$$(\mathbf{F}_\nu + \mathbf{R}_\nu^* - m_\nu \mathbf{a}_\nu) \cdot \delta\ddot{\mathbf{r}}_\nu = -\mathbf{R}_\nu^{id} \cdot \delta\ddot{\mathbf{r}}_\nu,$$

and if one substitutes the expression on the right-hand side by (10) one obtains

$$(\mathbf{F}_\nu + \mathbf{R}_\nu^* - m_\nu \mathbf{a}_\nu) \cdot \delta\ddot{\mathbf{r}}_\nu = -R_o \delta\ddot{\tau}. \tag{13}$$

It is the Gauss's differential principle in this parametric formulation of mechanics, where the term $R_o\delta\ddot{\tau}$ expresses the influence of the nonstationarity of constraints on the motion of the system.

The other, equivalent form of this principle can be obtained by introducing the notion of "constraint" in the extended sense of Gauss

$$Z \cong m_\nu \left(\mathbf{a}_\nu - \frac{\mathbf{F}_\nu + \mathbf{R}_\nu^*}{m_\nu} \right)^2 = \frac{1}{m_\nu} (\mathbf{F}_\nu + \mathbf{R}_\nu^* - m_\nu \mathbf{a}_\nu)^2 . \tag{14}$$

If one varies this expression, bearing in mind the conditions (8), one yields

$$\delta Z = -2 (\mathbf{F}_\nu + \mathbf{R}_\nu^* - m_\nu \mathbf{a}_\nu) \cdot \delta\ddot{\mathbf{r}}_\nu,$$

and on the basis of relation (13) it passes into

$$\delta Z = 2R_o \delta\ddot{\tau}. \tag{15}$$

This relation can be transformed to a concise form, analogous to the usual one, putting

$$\tilde{P} = -\int R_o \delta\ddot{\tau}, \quad \Leftrightarrow \quad \delta\tilde{P} = -R_o \delta\ddot{\tau}, \tag{16}$$

where by it passes into

$$\delta\tilde{Z} = 0, \qquad \tilde{Z} = Z + 2\tilde{P}, \tag{17}$$

and so introduced quantity $\tilde{P}$ can be named, by analogy with the rheonomic potential $P = -\int R_o \delta q^o$, [4],the Gaussian rheonomic potential. It represents the other form of Gauss's principle in this formulation of mechanics, the principle of least constraint and differs from the habitual one by an additional term $2\tilde{P}$.

4 Gauss's principle in generalized coordinates

The concept of constraint and the Gauss's principle of least constraint in the usual formulation of mechanics are also formulated in generalized coordinates. In this way, J. Synge ([6], p. 283) introduced this notion under the name of dynamical curvature and postulated the principle itself, but without any connection with its definition (14). Vujičić [7] elaborated this problem in detail, demonstrating how the constraint (14) can be transformed to generalized coordinates and from that obtained the corresponding equations of motion. Later, the same author formulated this principle in his modification of the mechanics of rheonomic systems, where appear the additional terms and equations, corresponding to the supplementary coordinate q^o.

At this point, let us depart from the first form of Gauss's principle (13) and transform it to the generalized coordinates. So, on the basis of the relations (5) we have

$$\dot{\mathbf{r}}_\nu = \frac{\partial \mathbf{r}_\nu}{\partial q^\alpha}\dot{q}^\alpha, \qquad \ddot{\mathbf{r}}_\nu = \frac{\partial^2 \mathbf{r}_\nu}{\partial q^\alpha \partial q^\beta}\dot{q}^\alpha \dot{q}^\beta + \frac{\partial \mathbf{r}_\nu}{\partial q^\alpha}\ddot{q}^\alpha, \tag{18}$$

and from here, keeping in mind the Gauss's conditions (8), we obtain

$$\delta\ddot{\mathbf{r}}_\nu = \frac{\partial \mathbf{r}_\nu}{\partial q^\alpha}\delta\ddot{q}^\alpha, \qquad (\alpha = 1, 2, ..., n). \tag{19}$$

Then, the first part of (13) passes into

$$(\mathbf{F}_\nu + \mathbf{R}^*_\nu)\cdot\delta\ddot{\mathbf{r}}_\nu = (\mathbf{F}_\nu + \mathbf{R}^*_\nu)\cdot\frac{\partial \mathbf{r}_\nu}{\partial q^\alpha}\delta\ddot{q}^\alpha = (Q_\alpha + R^*_\alpha)\,\delta\ddot{q}^\alpha,$$

or

$$(\mathbf{F}_\nu + \mathbf{R}^*_\nu)\cdot\delta\ddot{\mathbf{r}}_\nu = \tilde{Q}_\alpha\delta\ddot{q}^\alpha, \qquad \tilde{Q}_\alpha = Q_\alpha + R^*_\alpha, \tag{20}$$

where Q_α and R^*_α are respectively the generalized forces and nonideal reaction ones. Its second part can be transformed in an analogous way as in the habitual analytical mechanics

$$\begin{aligned} m_\nu \mathbf{a}_\nu \cdot \delta\ddot{\mathbf{r}}_\nu &= m_\nu \dot{\mathbf{v}}_\nu \cdot \frac{\partial \mathbf{r}_\nu}{\partial q^\alpha}\delta\ddot{q}^\alpha \\ &= \left[\frac{d}{dt}\left(m_\nu \mathbf{v}_\nu \cdot \frac{\partial \mathbf{r}_\nu}{\partial q^\alpha}\right) - m_\nu \mathbf{v}_\nu \frac{d}{dt}\left(\frac{\partial \mathbf{r}_\nu}{\partial q^\alpha}\right)\right]\delta\ddot{q}^\alpha \\ &= \left(\frac{d}{dt}\frac{\partial T}{\partial \dot{q}^\alpha} - \frac{\partial T}{\partial q^\alpha}\right)\delta\ddot{q}^\alpha, \end{aligned}$$

where T is the kinetic energy of the system, so that the relation (13) can be transformed to

$$\left(\tilde{Q}_\alpha - \frac{d}{dt}\frac{\partial T}{\partial \dot{q}^\alpha} + \frac{\partial T}{\partial q^\alpha}\right)\delta\ddot{q}^\alpha = -R_o \delta\ddot{q}^o. \tag{21}$$

That is the Gauss's differential principle in generalized coordinates, with characteristic term $R_o\delta\ddot{q}^o$ of this formalism of mechanics. To obtain the corresponding differential equations of motion, let us write it in a concise form

$$\left(\tilde{Q}_\alpha^{ex} - \frac{d}{dt}\frac{\partial T}{\partial \dot{q}^\alpha} + \frac{\partial T}{\partial q^\alpha}\right)\delta\ddot{q}^\alpha = 0, \tag{22}$$

where

$$\tilde{Q}_\alpha^{ex} = \begin{cases} Q_i + R_i^* & \text{for} \quad \alpha = i = 1, 2, ..., n \\ Q_o + R_o^* + R_o & \text{for} \quad \alpha = 0. \end{cases} \tag{23}$$

Because of the independence of the variations $\delta\ddot{q}^\alpha$ from here follows an extended system of Lagrange's equations

$$\tilde{Q}_\alpha^{ex} - \frac{d}{dt}\frac{\partial T}{\partial \dot{q}^\alpha} + \frac{\partial T}{\partial q^\alpha} = 0, \qquad (\alpha = 0, 1, 2, ..., n), \tag{24}$$

or explicitly

$$\begin{aligned} \frac{d}{dt}\frac{\partial T}{\partial \dot{q}^\alpha} - \frac{\partial T}{\partial q^\alpha} &= Q_i + R_i^*, \qquad (i = 1, 2, ..., n), \\ \frac{d}{dt}\frac{\partial T}{\partial \dot{q}^\alpha} - \frac{\partial T}{\partial q^\alpha} &= Q_o + R_o^* + R_o, \end{aligned} \tag{25}$$

in accordance with the result of Vujičić [4], obtained from Hamilton's principle.

These Lagrange's equations can be written also in the covariant form, as in the habitual case, but with the difference in that here exist $n+1$ generalized coordinates, therefore all the summations over the index α must be effectuated from 0 to n. So, if one writes explicitly the expression on the left-hand side of (24), beginning from the corresponding kinetic energy, and on the other hand from the covariant components of generalized acceleration as the absolute derivative $a^\alpha = \delta v^\alpha/\delta t$, repeating the corresponding calculations (see [6], p.280), one obtains

$$a_\alpha = g_{\alpha\beta}\frac{\delta \dot{q}^\beta}{\delta t} = \frac{d}{dt}\frac{\partial T}{\partial \dot{q}^\alpha} - \frac{\partial T}{\partial q^\alpha}. \tag{26}$$

Then the Gauss's differential principle (22) receive the covariant, tensor form

$$\left(a_\alpha - \tilde{Q}_\alpha^{ex}\right)\delta\ddot{q}^\alpha = 0, \tag{27}$$

and the Lagrange's equations (24) can be written concisely as

$$a_\alpha = \tilde{Q}_\alpha^{ex} \qquad \Leftrightarrow \qquad a^\beta = \tilde{Q}^\beta_{ex}, \quad (\alpha,\beta = 0,1,2,...,n), \tag{28}$$

thus in the usual way, but with the extended number of equations.

5 The constraint in generalized coordinates

The constraint itself also can be formulated in generalized coordinates, departing from its definition (14) and extending to this case the corresponding proof of Vujičić [7] from the usual analytical mechanics. In this manner, passing from the forces $\mathbf{F}_\nu$ to generalized ones Q_α by

$$Q_\alpha = \mathbf{F}_\nu \cdot \frac{\partial \mathbf{r}_\nu}{\partial q^\alpha}, \tag{29}$$

and decomposing the acceleration vector $\mathbf{a}_\nu$ along the basic vectors $\partial\mathbf{r}_\nu/\partial q^\alpha$ as

$$\mathbf{a}_\nu = a^\alpha \frac{\partial \mathbf{r}_\nu}{\partial q^\alpha}, \quad \left(a^\alpha = \frac{\delta \dot{q}^\alpha}{\delta t}\right), \tag{30}$$

the expression (14) for the constraint will be transformed to the following form

$$Z = g_{\alpha\beta}\left(a^{\alpha} - \tilde{Q}^{\alpha}_{ex}\right)\left(a^{\beta} - \tilde{Q}^{\beta}_{ex}\right), \tag{31}$$

where $g_{\alpha\beta}$ are components of the metric tensor.

To obtain the corresponding Gauss's principle, let us start from the variation of this expression

$$\delta Z = g_{\alpha\beta}\left(a^{\alpha} - \tilde{Q}^{\alpha}_{ex}\right)\delta a^{\beta} + g_{\alpha\beta}\left(a^{\beta} - \tilde{Q}^{\beta}_{ex}\right)\delta a^{\alpha},$$

or, passing to the covariant components and keeping in mind that $\delta a^{\alpha} = \delta\ddot{q}^{\alpha}$,

$$\delta Z = 2\left(a_{\alpha} - \tilde{Q}^{ex}_{\alpha}\right)\delta\ddot{q}^{\alpha}. \tag{32}$$

From here, on the basis of the first form (27) of this principle, one obtains immediately

$$\delta Z = \frac{\partial Z}{\partial \ddot{q}^{\alpha}}\delta\ddot{q}^{\alpha} = 0, \tag{33}$$

and this is the Gauss's principle of least constraint in generalized coordinates, which is equivalent to $\partial Z/\partial\ddot{q}^{\alpha} = 0$,i.e. to the equations of motion (28).

So defined constraint for the real motion of the system is minimal in comparison with its values along the other neighboring paths satisfying the Gauss's conditions (8). Namely, for the motion with $a^{\alpha} + \delta a^{\alpha}$ we have

$$\begin{aligned}\bar{Z} &= g_{\alpha\beta}\left(a^{\alpha} + \delta a^{\alpha} - \tilde{Q}^{\alpha}_{ex}\right)\left(a^{\beta} + \delta a^{\beta} - \tilde{Q}^{\beta}_{ex}\right) \\ &= g_{\alpha\beta}\left(a^{\alpha} - \tilde{Q}^{\alpha}_{ex}\right)\left(a^{\beta} - \tilde{Q}^{\beta}_{ex}\right) + 2g_{\alpha\beta}\left(a^{\alpha} - \tilde{Q}^{\alpha}_{ex}\right)\delta a^{\alpha} + g_{\alpha\beta}\delta a^{\alpha}\delta a^{\beta},\end{aligned}$$

and since the second term according to (32) vanishes, this relation is reduced to

$$\bar{Z} = Z + g_{\alpha\beta}\delta a^{\alpha}\delta a^{\beta}. \tag{34}$$

Here the last term as the positive definite form is always positive, so that $Z < \bar{Z}$ for any value of δa^{α}.

These results are in accordance with the ones obtained by Vujičić [4], but in another way, without the proof of this principle and its connection with the first, differential form of it.

6 Gauss's principle and Gibbs-Appell's equations

Let us demonstrate also that the Gauss's principle in this parametric formulation of mechanics can be transformed to an Appelian form, from which instead of the usual Gibbs-Appell's equations [8] follows an extended system of these ones. To this aim, we can begin from the Gauss's differential principle (13) and transform its second part as

$$m_\nu \mathbf{a}_\nu \cdot \delta\ddot{\mathbf{r}}_\nu = m_\nu \ddot{\mathbf{r}}_\nu \cdot \delta\ddot{\mathbf{r}}_\nu = \delta\left(\frac{1}{2} m_\nu \ddot{\mathbf{r}}_\nu \cdot \ddot{\mathbf{r}}_\nu\right).$$

If we introduce the quantity

$$S = \frac{1}{2} m_\nu \ddot{\mathbf{r}}_\nu^2, \tag{35}$$

which is well-known acceleration energy or Appell's function, the Gauss's principle (13) obtains the form

$$(\mathbf{F}_\nu + \mathbf{R}_\nu^*) \cdot \delta\ddot{\mathbf{r}}_\nu - \delta S = -R_o \delta\ddot{\tau}. \tag{36}$$

In order to transform this principle to generalized coordinates, we must reformulate the Gauss's conditions (8) in these coordinates

$$\delta q^\alpha = 0, \qquad \delta\dot{q}^\alpha = 0, \qquad (\alpha = 0, 1, ..., n). \tag{37}$$

Then the first two terms, according to (20) and (37), can be transformed to

$$(\mathbf{F}_\nu + \mathbf{R}_\nu^*) \cdot \delta\ddot{\mathbf{r}}_\nu = \tilde{Q}_\alpha \delta\ddot{q}^\alpha, \qquad \delta S = \frac{\partial S}{\partial \ddot{q}^\alpha} \delta\ddot{q}^\alpha,$$

so that relation (36) passes into

$$\left(\tilde{Q}_\alpha - \frac{\partial S}{\partial \ddot{q}^\alpha}\right) \delta\ddot{q}^\alpha = -R_o \delta\ddot{q}^o, \tag{38}$$

and it is the corresponding Gauss's differential principle in Appelian form. The acceleration energy (35) also can be expressed in generalized coordinates, with the aid of (30)

$$S = \frac{1}{2} m_\nu \frac{\partial \mathbf{r}_\nu}{\partial q^\alpha} \frac{\partial \mathbf{r}_\nu}{\partial q^\beta} a^\alpha a^\beta = \frac{1}{2} g_{\alpha\beta} a^\alpha a^\beta, \tag{39}$$

where $g_{\alpha\beta}$ represents the metric tensor in this formalism

$$g_{\alpha\beta} = m_\nu \frac{\partial \mathbf{r}_\nu}{\partial q^\alpha} \frac{\partial \mathbf{r}_\nu}{\partial q^\beta}. \tag{40}$$

To obtain the corresponding differential equations of motion, let us write the relation (38) concisely in the form

$$\left(\tilde{Q}_\alpha^{ex} - \frac{\partial S}{\partial \ddot{q}^\alpha} \right) \delta \ddot{q}^\alpha = 0, \tag{41}$$

where $\tilde{Q}_\alpha^{ex}$ is given by (23) and $q^o = \tau_o(t)$. Because of the independence of the variations $\delta \ddot{q}^\alpha$ from here follows

$$\tilde{Q}_\alpha^{ex} - \frac{\partial S}{\partial \ddot{q}^\alpha} = 0, \quad (\alpha = 0, 1, ..., n), \tag{42}$$

or explicitly

$$\frac{\partial S}{\partial \ddot{q}^i} = Q_i + R_i^* \qquad \text{for} \qquad (i = 1, 2, ..., n)$$

$$\frac{\partial S}{\partial \ddot{q}^o} = Q_o + R_o^* + R_o. \tag{43}$$

This is an extended system of Gibbbs-Appell's equations, where the last equation determines the generalized reaction force R_o, corresponding to the additional generalized coordinate q^o, in a similar manner as in the system of Lagrange's equations (25).

Finally, let us remark that these Gibbs-Appell's equations can also be obtained from equivalent Gauss's principle of least constraint, departing from (31) in the developed form and applying the corresponding Gauss's principle $\delta Z = 0$.

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Gauss-ov princip u parametarskoj formulaciji mehanike

U ovom radu pokazano je da se i Gauss-ov princip može formulisati u parametarskoj formulaciji mehanike reonomnih sistema, koja se zasniva na familiji variranih putanja i na prelazu na novi parametar, zadržavajući vreme kao nezavisno promenljivu. Na taj način dobija se odgovarajući Gauss-ov princip, kako u diferencijalnom obliku, tako i u vidu principa najmanje prinude, uključujući i njihov prikaz u generalisanim koordinatama. Iz tako formulisanog Gauss-ovog principa proizilazi proširen sistem odgovarajućih Lagrange-evih kao i Gibbs-Appell-ovih jednačina. U svim dobijenim relacijama pojavljuju se dodatni član ili jednačina, koji potiču od nestacionarnosti jednačina veza, što je karakteristično za ovu parametrsku formulaciju mehanike.