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Discrete analysis of non-symmetric buildings

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Abstract

In this paper the discrete concept in mathematical modeling of non-symmetric multistory buildings under static and dynamic lateral load is presented. Buildings with orthogonal configuration of vertical structural elements are particularly considered. The attempt of consistent approach to discrete analysis of buildings which includes the possibility of arbitrary configuration in plan of the frame and shear wall elements is given. This work is more concerned with formulation of the corresponding mathematical model then with the possibilities to solve the obtained equations.

1 Introduction and basic assumptions

Buildings are one of the basic human needs. Therefore, buildings are designed and built in a great variety of forms, so they are definitely the most numerous civil engineering structures. Due to great possibilities of their configurations and shapes, buildings are also among the most complex structural systems with respect to their mechanical behaviour. Consequently, development of the appropriate mathematical models related to mechanical behaviour of buildings is still a challenge to researches, even though the existing literature is quite numerous. Some of it is given in [1] - [13].

The basic structural components of any building are the horizontal slabs and vertical supporting elements. The slabs are dividing the space within the building into separate stories and they are also connecting the vertical elements into the unique structural entity the building represents. One of the basic assumptions in mathematical modeling of buildings is the overall linearly elastic behaviour which allows the principle of superposition, ie. the separation of gravitational and horizontal load analysis. Consequently, the actual design of buildings consists of two independent parts: gravitational (vertical) and lateral (horizontal) load analysis. The former part represents the "routine" part of engineering design, while the later analysis is the "advanced" one, since the lateral load analysis is a more complex one and usually is dominant in the overall stability of buildings.

This work is related to horizontal load analysis of non-symmetric buildings and at this stage it is more concerned with formulation of corresponding mathematical model then with the possibilities to solve obtained equations. Also, the present analysis represents the discrete concept in mathematical modeling, as opposed to continuous approaches, see [12], [13]. Therefore, due to discrete approach, the analysis is naturally oriented towards numerical solutions and use of computers.

The main assumptions in the present discrete horizontal load analysis of buildings, besides the usual assumptions of the linear theory of structures (statical, geometrical and material linearity), are the following:

- All horizontal slabs are considered as infinitely stiff in their planes;
- Mass of the building is concentrated and contained in each slab only;
- Axial deformations of vertical supporting elements are neglected;
- Vertical elements are planar structural elements, ie. they have finite stiffness in their plane and negligible out-of-plane stiffness.

The last assumption is not necessary and sometimes is not adopted. However, the usual structural concept of vertical elements is generally consistent with such an assumption.

The consequence of the adopted assumptions is that the building is visualized as a finite set of rigid laminae which are free to move in parallel horizontal planes, while the vertical elements are treated as (elastic) constrains for such a motion. Therefore, the building with N stories, ie. with N slabs, has $3N$ degrees of freedom. The generalized coordinates of the system correspond to description of planar motion of each slab, ie. they are the two independent coordinates of displacement vector of the reference point of each slab and one coordinate of the rotation vector of each slab. In static lateral load analysis the choice of reference points is not so crucial, while in dynamic analysis it is rather convenient to choose the centers of mass of each slab as a reference points.

The literature related to the analysis of buildings is very substantial, so the review of it would be rather consuming. However, one must point out the famous TABS programme, see [2], as one of the best and the most widely used commercial computer program devoted especially to buildings. The basic approach in TABS is that a building is idealized by a system of independent planar frame and shear wall elements interconnected by floor diaphragms which are rigid in their planes. The TABS programme can simultaneously analyze three cases of gravitational and two cases of lateral static loading conditions. Dynamic analysis in TABS is devoted primarily to earthquake analysis and consists of the time history due to the given ground acceleration or the response spectrum analysis.

Nonsymmetrical nonrectangular buildings which have frame and shear walls located arbitrarily in plan can be also considered by TABS. However, the use of TABS "for structures in which frames are not arranged in a reasonably rectangular fashion in plan is questionable", see [2], p.7. This paper represents the analysis which is close to the approach used in TABS and also in some other work, see for example, [3] - [7]. However, the main contribution of the present paper is the attempt to give the consistent approach to discrete analysis of buildings which includes the possibility of arbitrary configuration in plan of the frame and shear wall elements.

2 Static lateral load analysis

2.1 Generalized coordinates and displacement of a point

The representative slab which is free to move in the horizontal Oxy plane, but whose motion is consistent with the small displacement assumption, is presented in Fig. 1.

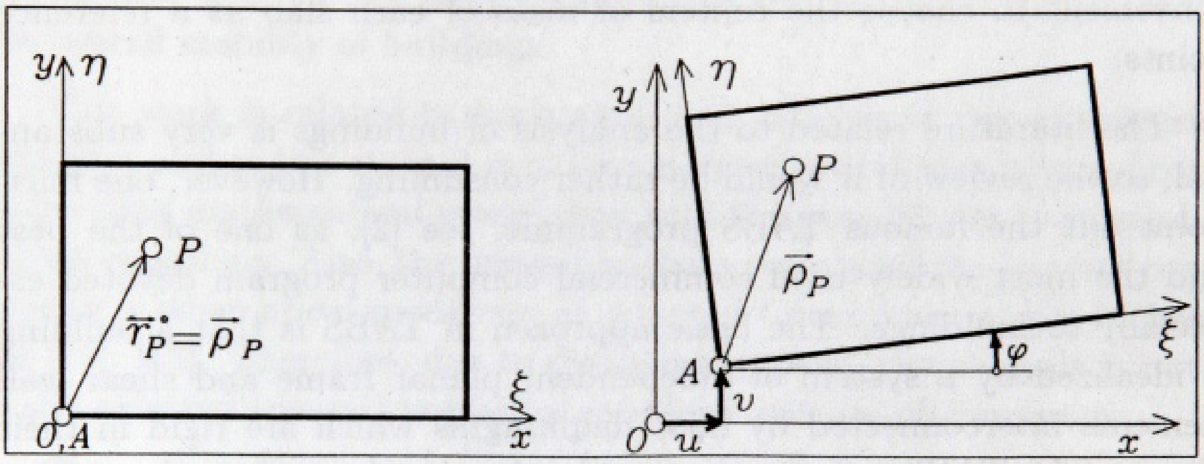

Fig. 1. - Small displacements of a slab from its equilibrium position.

The inertial frame of reference $Oxyz$ and material frame $A\xi\eta\zeta$ are chosen to coincide when the slab is at rest (in equilibrium) in the absence of any horizontal forces. For such a case the vertical elements, not shown in Fig. 1, are in unstressed configuration. The chosen generalized coordinates describing small planar motion of the slab are components of displacement vector of the reference point A : $\mathbf{d}_A = \{u, v, 0\}$ and rotation vector of the slab $\boldsymbol{\phi} = \{0, 0, \varphi\}$, see Fig.1.

The position vector of some point P after small motion of the slab is given by $\mathbf{r}_P = \mathbf{d}_A + \boldsymbol{\rho}_P$ where $\boldsymbol{\rho}_P = \overrightarrow{AP} = \{\xi, \eta, 0\}$ is expressed relative to material frame $A\xi\eta\zeta$ and is a constant vector for a given point. Therefore, displacement vector of a point P is given by $\mathbf{d}_P = \mathbf{r}_P - \mathbf{r}_{P_0}$ where $\mathbf{r}_{P_0}$ is the initial position vector of P. Since displacement of the slab is generally small, so $\cos\varphi \approx 1$ and $\sin\varphi \approx \varphi$, displacement vector of the point P, when expressed relative to inertial frame $Oxyz$ is obtained as

$$\begin{aligned}\mathbf{d}_P &= \mathbf{r}_P - \mathbf{r}_{P_0} \\ &= (u + \xi\cos\varphi - \eta\sin\varphi)\,\mathbf{i} + (v + \xi\sin\varphi + \eta\cos\varphi)\mathbf{j} \qquad (1) \\ &= (u - \eta\varphi)\mathbf{i} + (v + \xi\varphi)\mathbf{j}\end{aligned}$$

In the initial position reference frames are coinciding, so $\xi = x$, $\eta = y$, $\mathbf{i} = \boldsymbol{\lambda}$, $\mathbf{j} = \boldsymbol{\mu}$, where $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are unit vectors of axes $\xi\eta$.

2.2 Buildings with orthogonal configuration of vertical elements

The usual structural configuration of a building is consistent with the assumption that vertical elements are considered as planar structural systems, see Fig. 2. The exception to this are the so-called "central cores" of a building which represent the complex reinforced concrete walls forming vertical communication space (elevator shafts) within a building. If such complex walls do not have rather substantial dimensions, eg. as in a case of several elevators in a row, then their horizontal (ie. lateral) stiffness is usually smaller then the stiffness of other planar vertical elements, Fig. 2.

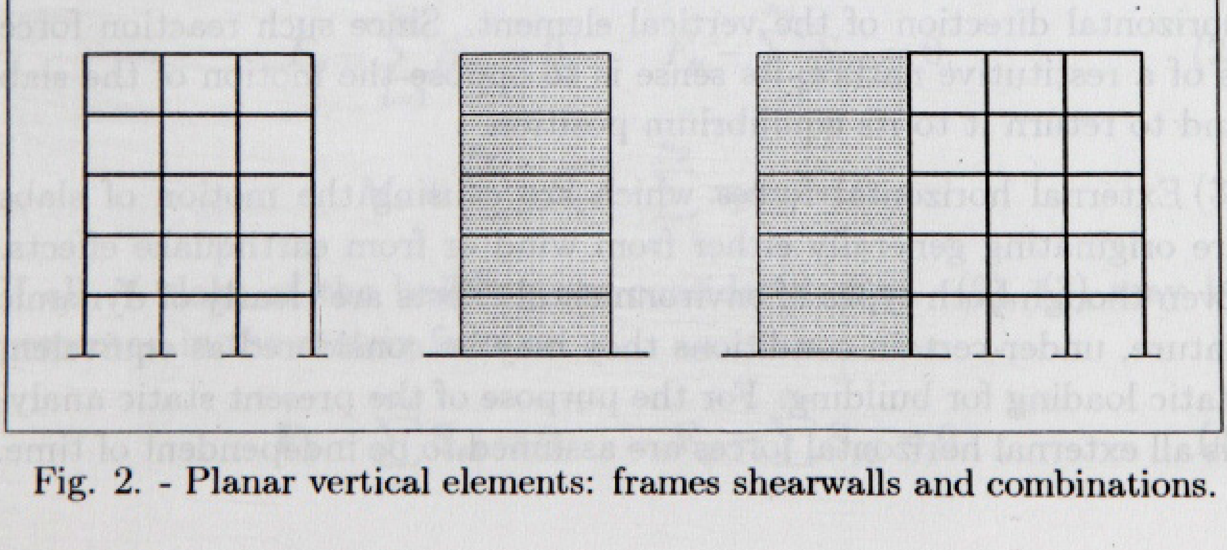

Fig. 2. - Planar vertical elements: frames shearwalls and combinations.

Also, buildings are usually regular in plan which means they are approximately symmetrical with respect to two orthogonal horizontal directions. Typical situation is the rectangular plan of a building as shown in Fig. 1. However, the plan of a building is often of a more complex shape, usually some combination of rectangular regions, see Fig. 3. In all such cases the normal situation is that vertical structural elements are mutually orthogonal, ie. vertical elements are parallel to x and y directions.

A building is considered to be symmetrical in the structural sense if the center of mass and shear center of each slab are coinciding, or if they are relatively close together. Without further elaboration, the shear center is defined as the unique point of the slab with the following property: if the resultant external horizontal force is acting through such a point, then the corresponding motion of the slab is translation without any rotation. Even though a building may seem as geometrically symmetrical with respect to two vertical planes, the actual structural properties, mass distribution and even configuration of vertical elements are usually not completely symmetrical, so most of the buildings are more or less non-symmetric. Of course, if a building is non-symmetric in geometrical sense, then it is usually, but not necessarily, non-symmetric in a structural sense, too.

Let us assume that there are n_x vertical structural elements parallel to x axis and n_y vertical elements parallel to y axis. Such planar vertical elements are assumed to have only in-plane lateral stiffness. Vertical elements are representing constraints for horizontal motion of slabs, so it means that the influence of a vertical element on a motion of a slab is represented by internal concentrated constraint (restitutive) force in horizontal direction of the vertical element. Since such reaction force is of a restitutive nature, its sense is to oppose the motion of the slab and to return it to its equilibrium position.

External horizontal forces which are causing the motion of slabs are originating generally either from wind or from earthquake effects. Even though both types of environmental effects are clearly of dynamic nature, under certain conditions they may be considered as equivalent static loading for building. For the purpose of the present static analysis all external horizontal forces are assumed to be independent of time.

Force components F_x, F_y and the couple M_z shown in Fig. 3 are the result of reduction upon the reference point of all external horizontal forces acting on a slab. Therefore, the considered slab is under the action of external forces F_x, F_y and M_z, which represent any given external loading acting on a slab, and internal constraint forces S_{xi} and $S_{yj}, (i = 1, 2, ..., n_x, \ j = 1, 2, ..., n_y)$, which are acting in direction of each vertical element. Having in mind the small displacement assumption, the equilibrium conditions of forces are established on undeformed configuration of a slab, as presented in Fig. 3.

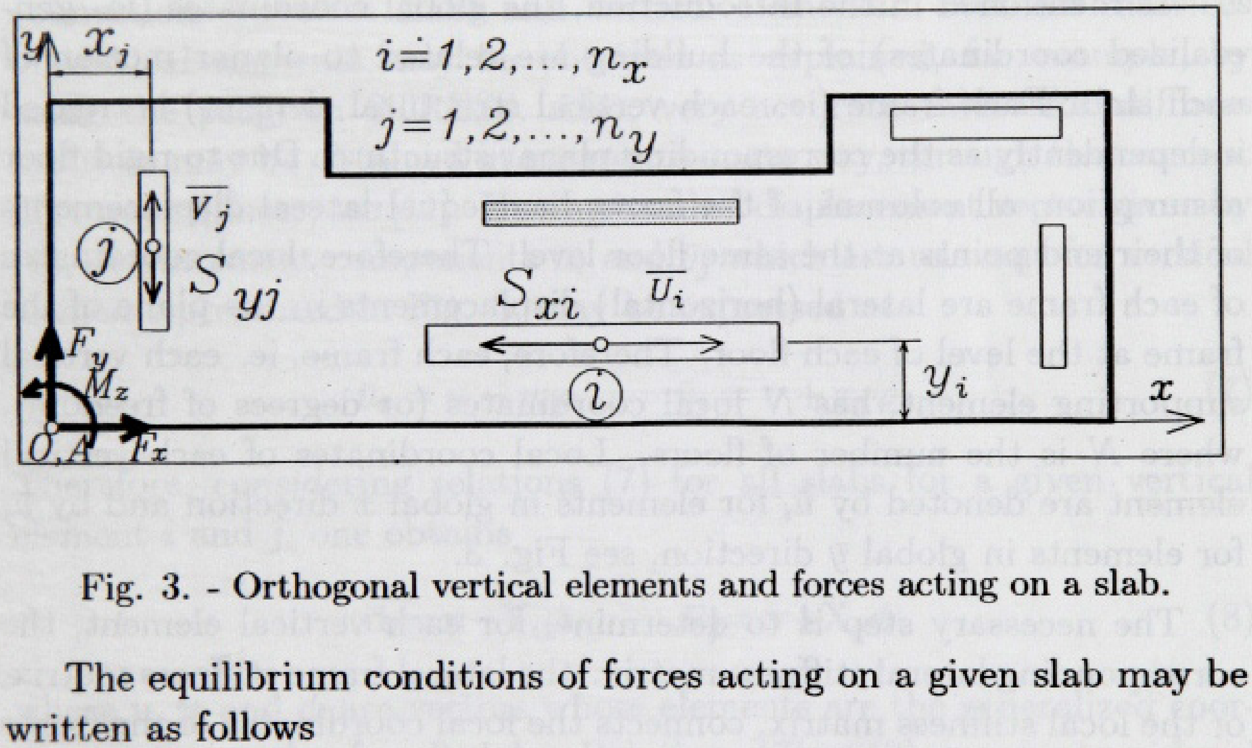

Fig. 3. - Orthogonal vertical elements and forces acting on a slab.

The equilibrium conditions of forces acting on a given slab may be written as follows

$$F_x - \sum_{i=1}^{n_x} S_{xi} = 0, \quad F_y - \sum_{j=1}^{n_y} S_{yj} = 0, \tag{2}$$

$$M_z + \sum_{i=1}^{n_x} y_i S_{xi} - \sum_{j=1}^{n_y} x_j S_{yj} = 0. \tag{3}$$

If all N slabs of the building are considered, Eqs. (2), (3) may be presented in the matrix form as

$$\mathbf{F}_x - \sum_{i=1}^{n_x} \mathbf{S}_{xi} = 0, \qquad \mathbf{F}_y - \sum_{j=1}^{n_y} \mathbf{S}_{yj} = 0, \tag{4}$$

$$\mathbf{M}_z + \sum_{i=1}^{n_x} \mathrm{Y}_i \mathbf{S}_{xi} - \sum_{j=1}^{n_y} \mathrm{X}_j \mathbf{S}_{yj} = 0. \tag{5}$$

In Eqs. (4), (5) $\mathbf{F}_x$, $\mathbf{F}_y$ and $\mathbf{M}_z$ are vectors whose elements are resultant external forces acting on each slab, $\mathbf{S}_{xi}$ and $\mathbf{S}_{yj}$ are vectors with internal restitutive forces between slabs and vertical structural elements in x and y directions, while X_j and Y_i are diagonal matrices whose elements are x_j and y_i coordinates of vertical elements with respect to Oxy system. since vertical elements belong to a single vertical plane, all coordinates x_j or y_i for a given vertical element are constant, so matrices X_j and Y_i should normally be of a form $x_j\mathrm{I}$ and $y_i\mathrm{I}$, where I is the unit matrix of order N.

As mentioned in the Introduction, the global coordinates (ie. generalized coordinates) of the building are related to planar motion of each slab. Each frame (ie. each vertical structural element) is treated independently as the corresponding planar structure. Due to rigid floor assumption, all columns of the frame have equal lateral displacements of their end points at the same floor level. Therefore, local coordinates of each frame are lateral (horizontal) displacements in the plane of the frame at the level of each floor. Therefore, each frame, ie. each vertical supporting element, has N local coordinates (or degrees of freedom), where N is the number of floors. Local coordinates of each vertical element are denoted by $\bar{u}_i$ for elements in global x direction and by $\bar{v}_j$ for elements in global y direction, see Fig. 3.

The necessary step is to determine, for each vertical element, the corresponding lateral stiffness matrix. the lateral frame stiffness matrix, or the local stiffness matrix, connects the local coordinates of the frame $\bar{u}_i$ (or $\bar{v}_j$) and the corresponding lateral (constraint) S_{xi} (or S_{yj}). Lateral stiffness matrix of each frame may be obtained in different ways, one of them is by using the well known program STRESS or the equivalent. Namely, for each frame with different properties and therefore with different lateral stiffness, the corresponding STRESS model should be prepared. In order to simulate the rigid floor assumption, artificially high value for axial stiffness of each horizontal beam is adopted. Also, at all floor levels at one side of the frame, the additional supports that prevent only horizontal displacements of floors are adopted. Such a model is then solved successively for N loading cases where each load case is defined as a unit horizontal displacement of a support at each floor level. The elements of the corresponding frame lateral stiffness matrix are obtained as reaction forces of artificial supports at floor levels.

If the lateral stiffness matrices of vertical elements in x and y directions are denoted by K_x and K_y then the vectors of internal restitutive forces $\mathbf{S}_{xi}$ and $\mathbf{S}_{yj}$ may be expressed as

$$\mathbf{S}_{xi} = \mathrm{K}_{xi}\bar{\mathbf{u}}_i, \qquad \mathbf{S}_{yj} = \mathrm{K}_{yj}\bar{\mathbf{v}}_j. \tag{6}$$

In relations (6) vectors $\bar{\mathbf{u}}_i$ and $\bar{\mathbf{v}}_j$ represent vectors whose elements are displacement components in x and y directions of the points of connections between slabs and vertical elements. Stiffness matrices of vertical elements may be obtained as explained, for example, by using the program STRESS. Also, very accurate values for stiffness matrices may be obtained in an approximate way, through the concept of storey stiffness, see [11]. Having in mind displacement vector given by (1), displacement components $\bar{u}_i$ and $\bar{v}_j$ which are relevant for vertical elements presented in Fig. 3, may be expressed as

$$\bar{u}_i = u - y_i\varphi, \qquad \bar{v}_j = v + x_j\varphi. \tag{7}$$

Therefore, considering relations (7) for all slabs for a given vertical element i and j, one obtains

$$\bar{\mathbf{u}}_i = \mathbf{u} - \mathrm{Y}_i\boldsymbol{\phi}, \qquad \bar{\mathbf{v}}_j = \mathbf{v} + \mathrm{X}_j\boldsymbol{\phi}, \tag{8}$$

where $\mathbf{u}$, $\mathbf{v}$ and $\boldsymbol{\phi}$ are vectors whose elements are the generalized coordinates u, v and φ for all slabs. Relations (7) or (8) may be considered as transformation relations between the local and global coordinates. When substituting relations (6) and (8) into equilibrium equations (4), (5) one obtains

$$\mathbf{F}_x = \sum_{i=1}^{n_x}\mathrm{K}_{xi}\mathbf{u} - \sum_{i=1}^{n_x}\mathrm{K}_{xi}\mathrm{Y}_i\boldsymbol{\phi}, \qquad \mathbf{F}_y = \sum_{j=1}^{n_y}\mathrm{K}_{yj}\mathbf{v} + \sum_{j=1}^{n_y}\mathrm{K}_{yj}\mathrm{X}_j\boldsymbol{\phi}, \tag{9}$$

$$\mathbf{M}_z = -\sum_{i=1}^{n_x}\mathrm{Y}_i\mathrm{K}_{xi}\mathbf{u} + \sum_{i=1}^{n_x}\mathrm{Y}_i\mathrm{K}_{xi}\mathrm{Y}_i\boldsymbol{\phi} + \sum_{j=1}^{n_y}\mathrm{X}_j\mathrm{K}_{yj}\mathbf{v} + \sum_{j=1}^{n_y}\mathrm{X}_j\mathrm{K}_{yj}\mathrm{X}_j\boldsymbol{\phi}. \tag{10}$$

Obtained three sets of N equations (9), (10) may be also written as a single matrix equation

$$\begin{bmatrix} \mathrm{K}_{xx} & 0 & \mathrm{K}_{x\varphi} \\ 0 & \mathrm{K}_{yy} & \mathrm{K}_{y\varphi} \\ \mathrm{K}_{x\varphi} & \mathrm{K}_{y\varphi} & \mathrm{K}_{\varphi\varphi} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{v} \\ \boldsymbol{\phi} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_x \\ \mathbf{F}_y \\ \mathbf{M}_z \end{Bmatrix}, \tag{11}$$

or in compact form with obvious notation

$$\mathrm{K}\boldsymbol{\delta} = \mathbf{f}. \tag{12}$$

The matrix K given by (11) is the global stiffness matrix of a building. Submatrices of the global stiffness matrix K are given by

$$\mathrm{K}_{xx} = \sum_{i=1}^{n_x} \mathrm{K}_{xi}, \qquad \mathrm{K}_{yy} = \sum_{j=1}^{n_y} \mathrm{K}_{yj}, \tag{13}$$

$$\mathrm{K}_{x\varphi} = \mathrm{K}_{\varphi x}^T = -\sum_{i=1}^{n_x} \mathrm{Y}_i \mathrm{K}_{xi} \quad \left(= -\sum_{i=1}^{n_x} \mathrm{K}_{xi} y_i \right), \tag{14}$$

$$\mathrm{K}_{y\varphi} = \mathrm{K}_{\varphi y}^T = \sum_{j=1}^{n_y} \mathrm{X}_j \mathrm{K}_{yj} \quad \left(= \sum_{j=1}^{n_y} \mathrm{K}_{yj} x_j \right), \tag{15}$$

$$\mathrm{K}_{\varphi\varphi} = \sum_{i=1}^{n_x} \mathrm{Y}_i \mathrm{K}_{xi} \mathrm{Y}_i + \sum_{j=1}^{n_y} \mathrm{X}_j \mathrm{K}_{yj} \mathrm{X}_j \quad \left(= \sum_{i=1}^{n_x} \mathrm{K}_{xi} y_i^2 + \sum_{j=1}^{n_y} \mathrm{K}_{yj} x_j^2 \right). \tag{16}$$

The expressions given in brackets correspond to the normal case when matrices X_j and Y_i are given as $x_j\mathrm{I}$ and $y_i\mathrm{I}$. Therefore, the static problem of a non-symmetric building with orthogonal vertical elements is given by the system (12) of $3N$ linear algebraic equations.

The solution of the system (12) is easily obtained as

$$\boldsymbol{\delta} = \mathrm{K}^{-1}\mathbf{f}. \tag{17}$$

With generalized coordinates determined by (17), displacement vectors for each vertical element are given by (8), so the internal constraint forces between the slabs and vertical elements are given by (6), or by

$$\mathbf{S}_{xi} = \mathrm{K}_{xi}\mathbf{u} - \mathrm{K}_{xi}\mathrm{Y}_i\boldsymbol{\phi}, \qquad (i = 1, 2, ..., n_x), \tag{18}$$

$$\mathbf{S}_{yj} = \mathrm{K}_{yj}\mathbf{v} - \mathrm{K}_{yj}\mathrm{X}_j\boldsymbol{\phi}, \qquad (j = 1, 2, ..., n_y). \tag{19}$$

Obtained internal constraint forces $\mathbf{S}_{xi}$ and $\mathbf{S}_{yj}$ are now considered as the given external forces in the independent analysis of each vertical element, which should naturally follow. The analysis of each frame of shear wall subjected to forces $\mathbf{S}_{xi}$ or $\mathbf{S}_{yj}$ is of course the necessary step in order to establish if each vertical element is capable to resist the loading which is being distributed to it through overall behaviour of the building.

It is possible to introduce the notion of the mentioned shear centers and also similarly defined torsion centers of the building. The purpose of that would be to apply then the appropriate transformation of generalized coordinates in order to obtain three independent sets of N equations each, instead of previous set of $3N$ coupled equations (12), see for example [5]. However, it will not be pursued here, since, generally speaking, the solution of the coupled set of equations (12) is not hard to obtain, because the number of stories N is usually relatively low $(N < 30)$. Therefore, the total number of equations is normally less then 100.

2.3 Buildings with arbitrary configuration of vertical elements

Let us consider a building with non-symmetric (or even symmetric) geometry in plan and with total number of n_v vertical elements whose configuration is arbitrary. The term arbitrary configuration of vertical elements means that vertical elements are not all parallel to inertial x or y axes, as presented in Fig. 4. As in the case of orthogonal vertical elements, only planar vertical elements are considered, as shown in Fig. 2.

Let us define the location the of each vertical element by the position of the centroid of the element $C_i\,(x_i, y_i)$ and by oriented direction of the element given by the angle $\alpha_i = \angle\,(x, \bar{x}_i)$, where $\bar{x}_i$ is the local horizontal axis in direction of the element, see Fig. 4.

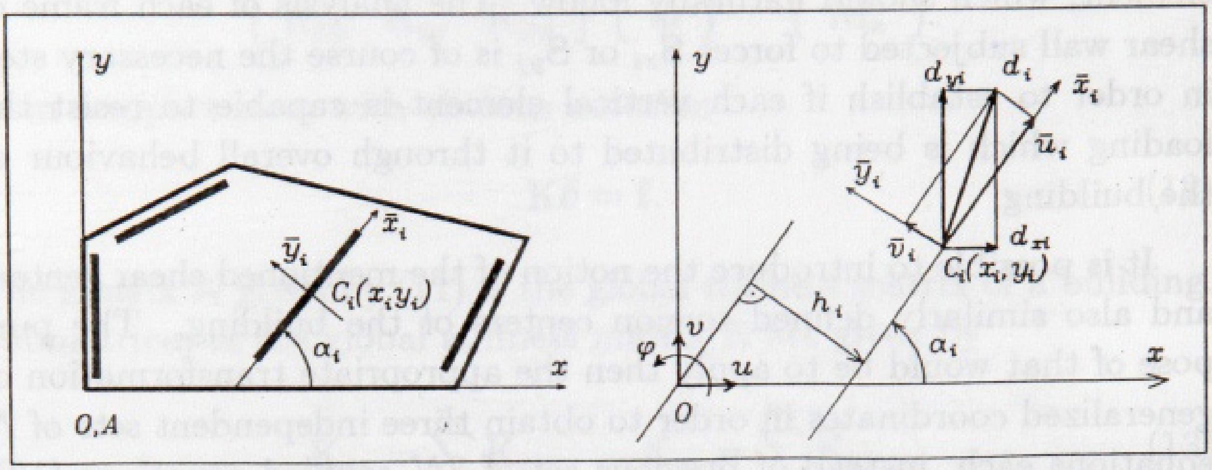

Fig. 4. - Arbitrary configuration of vertical elements and displacement of a point.

It is easy to verify, see Fig. 4, that displacement component of the point C_i of the slab in direction of the local axis $\bar{x}_i$ is given by

$$\bar{u}_i = dx_i \cos\alpha_i + dy_i \sin\alpha_i, \tag{20}$$

where dx_i and dy_i are components of displacement vector $\mathbf{d}_i$ of the point C_i in direction of x and y axes and are given by

$$dx_i = u - y_i\varphi, \qquad dy_i = v + x_i\varphi. \tag{21}$$

Therefore, displacement component $\bar{u}_i$ in direction of the vertical element i may be expressed as

$$\bar{u}_i = u\cos\alpha_i + v\sin\alpha_i + h_i\varphi, \tag{22}$$

where

$$h_i = x_i \sin\alpha_i - y_i \cos\alpha_i, \tag{23}$$

and represents the normal distance of the local axis $\bar{x}_i$ from the origin O.

Instead of notation K_{xi} and K_{yj} for the lateral stiffness matrices of mutually orthogonal vertical elements, it is more convenient to denote now the lateral stiffness matrix of any vertical element by $K_{\bar{x}_i}$. Therefore, instead of relations (6), the corresponding force - displacement relations are now written as

$$\mathbf{S}_{\bar{x}_i} = \mathrm{K}_{\bar{x}_i}\, \bar{\mathbf{u}}_i, \qquad (i = 1, 2, ..., n_v)\,. \tag{24}$$

The vector $\bar{\mathbf{u}}_i$ represents displacements $\bar{u}_i$ in direction of the local axis of a given vertical element for all slabs. This vector, considering relations (22), may be expressed as

$$\bar{\mathbf{u}}_i = \mathbf{u}\cos\alpha_i + \mathbf{v}\sin\alpha_i + \boldsymbol{\phi} h_i, \tag{25}$$

where $\mathbf{u}$, $\mathbf{v}$ and $\boldsymbol{\phi}$ are the vectors whose elements are the generalized coordinates of the problem. For a given vertical element, coordinates of its centroid C_i and direction angle α_i are normally constant for all slabs.

Like mentioned before, due to small displacement assumption, equilibrium conditions of all forces acting on each slab are considered on undeformed configuration. Fig. 5 represents an isolated slab under action of an arbitrary set of external horizontal forces, and the corresponding set of internal reaction forces.

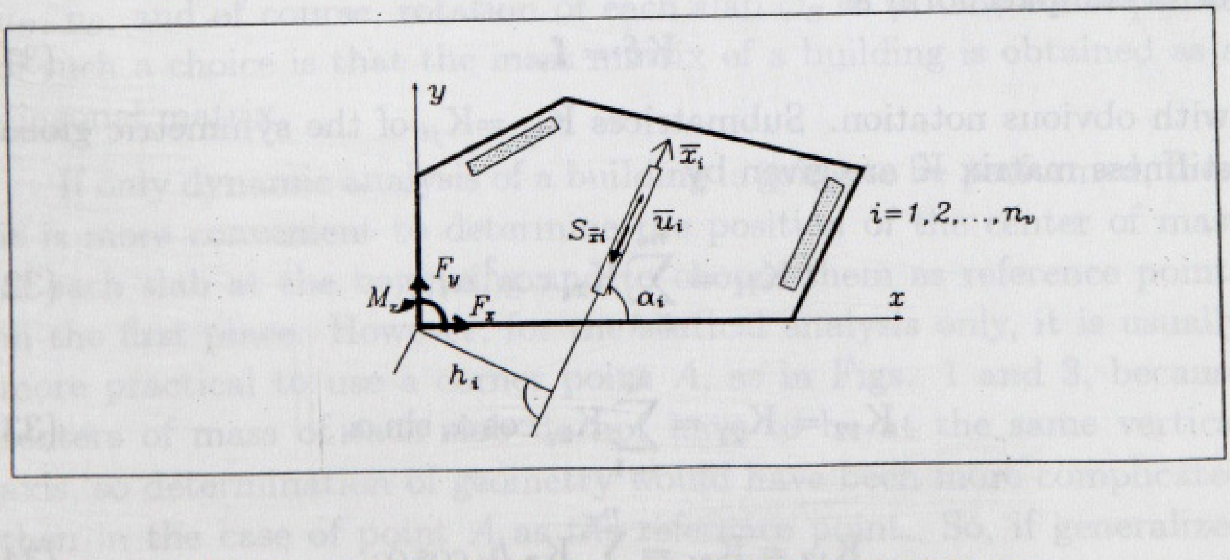

Fig. 5. - Equilibrium of external and internal forces.

Arbitrary external forces are represented by F_x, F_y and M_z, while corresponding set of reactive forces are given by $S_{\bar{x}_i}$ and represent the influence of vertical elements upon considered slab. The equilibrium equations of forces acting upon the isolated slab are given by

$$F_x - \sum_{i=1}^{n_v} S_{\bar{x}_i} \cos\alpha_i = 0, \qquad F_y - \sum_{i=1}^{n_v} S_{\bar{y}_i} \sin\alpha_i = 0, \tag{26}$$

$$M_z - \sum_{i=1}^{n_v} S_{\bar{x}_i} h_i = 0. \tag{27}$$

When considering equilibrium equations for all slabs, Eqs. (26), (27) become matrix equations

$$\mathbf{F}_x - \sum_{i=1}^{n_v} \mathbf{S}_{\bar{x}_i} \cos\alpha_i = 0, \qquad \mathbf{F}_y - \sum_{i=1}^{n_v} \mathbf{S}_{\bar{y}_i} \sin\alpha_i = 0, \tag{28}$$

$$\mathbf{M}_z - \sum_{i=1}^{n_v} \mathbf{S}_{\bar{x}_i} h_i = 0. \tag{29}$$

If relations (24) and (25) are introduced into equations (28), (29) global equations of equilibrium may be written as

$$\begin{bmatrix} \mathrm{K}_{11} & \mathrm{K}_{12} & \mathrm{K}_{13} \\ \mathrm{K}_{21} & \mathrm{K}_{22} & \mathrm{K}_{23} \\ \mathrm{K}_{31} & \mathrm{K}_{32} & \mathrm{K}_{33} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{v} \\ \boldsymbol{\phi} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_x \\ \mathbf{F}_y \\ \mathbf{M}_z \end{Bmatrix}, \tag{30}$$

or in compact form as

$$\mathrm{K}\boldsymbol{\delta} = \mathbf{f}, \tag{31}$$

with obvious notation. Submatrices $\mathrm{K}_{ij} = \mathrm{K}_{ji}$ of the symmetric global stiffness matrix K are given by

$$\mathrm{K}_{11} = \sum_{i=1}^{n_v} \mathrm{K}_{\bar{x}_i} \cos^2\alpha_i, \tag{32}$$

$$\mathrm{K}_{12} = \mathrm{K}_{21} = \sum_{i=1}^{n_v} \mathrm{K}_{\bar{x}_i} \cos\alpha_i \sin\alpha_i, \tag{33}$$

$$\mathrm{K}_{13} = \mathrm{K}_{31} = \sum_{i=1}^{n_v} \mathrm{K}_{\bar{x}_i} h_i \cos\alpha_i, \tag{34}$$

$$K_{22} = \sum_{i=1}^{n_v} K_{\bar{x}_i} \sin^2 \alpha_i, \tag{35}$$

$$K_{23} = \sum_{i=1}^{n_v} K_{\bar{x}_i} h_i \sin \alpha_i, \tag{36}$$

$$K_{33} = \sum_{i=1}^{n_v} K_{\bar{x}_i} h_i^2. \tag{37}$$

The solution of equations of equilibrium (31) is easily obtained as $\boldsymbol{\delta} = K^{-1}\mathbf{f}$. With obtained generalized coordinates $\boldsymbol{\delta}^T = \{\mathbf{u}, \mathbf{v}, \boldsymbol{\phi}\}$, local displacement vector $\bar{\mathbf{u}}_i$ of the particular vertical element is obtained by (25) and the corresponding constraint forces $\mathbf{S}_{\bar{x}_i}$ are then given by (24). After that, the classical analysis of each vertical element (frame or shear wall) is performed with $\mathbf{S}_{\bar{x}_i}$ as the given external forces acting on each floor of the frame.

3 Dynamic lateral load analysis

3.1 Translation of generalized coordinates

In dynamic analysis it is more convenient to choose the center of mass S of each slab as a reference point, instead of some other point A, so generalized coordinates are displacement components of centers of mass: u_S, v_S, and of course, rotation of each slab $\varphi_S = \varphi$. The consequence of such a choice is that the mass matrix of a building is obtained as a diagonal matrix.

If only dynamic analysis of a building is going to be performed, then it is more convenient to determine the position of the center of mass of each slab at the beginning and to choose them as reference points in the first place. However, for the statical analysis only, it is usually more practical to use a corner point A, as in Figs. 1 and 3, because centers of mass of each slab do not have to be at the same vertical axis, so determination of geometry would have been more complicated then in the case of point A as the reference point. So, if generalized coordinates u, v and φ related to point A are already being used, it is easy to perform simple transformation of coordinates introducing translated coordinate systems located in centers of mass $S(x_S, y_S)$ of each slab. Representative slab is presented in Fig. 6.

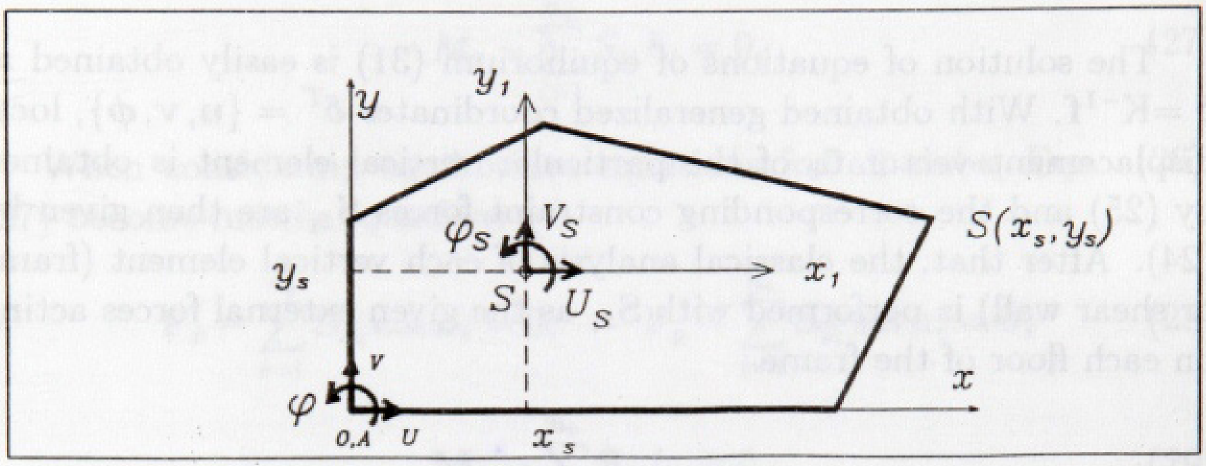

Fig. 6. - Translation of generalized coordinates.

Transformation relations between generalized coordinates located in reference points A, as the original one, and S, as the new one, are given by

$$\left\{\begin{array}{c} u_S \\ v_S \\ \varphi_S \end{array}\right\} = \left[\begin{array}{ccc} 1 & 0 & -y_S \\ 0 & 1 & x_S \\ 0 & 0 & 1 \end{array}\right] \left\{\begin{array}{c} u \\ v \\ \varphi \end{array}\right\} \Leftrightarrow \left\{\begin{array}{c} u \\ v \\ \varphi \end{array}\right\} = \left[\begin{array}{ccc} 1 & 0 & y_S \\ 0 & 1 & -x_S \\ 0 & 0 & 1 \end{array}\right] \left\{\begin{array}{c} u_S \\ v_S \\ \varphi_S \end{array}\right\} \tag{38}$$

where x_S and y_S are coordinates of center of mass S with respect to inertial reference frame Oxy. Introducing diagonal matrices whose elements are locations of centers of mass of each slab as

$$X_S = diag\left(x_S^1, x_S^2, ..., x_S^N\right), \quad \text{and} \quad Y_S = diag\left(y_S^1, y_S^2, ..., y_S^N\right),$$

transformation relations (38) are written for all slabs as

$$\left\{\begin{matrix} \mathbf{u} \\ \mathbf{v} \\ \boldsymbol{\phi} \end{matrix}\right\} = \begin{bmatrix} \mathrm{I} & 0 & \mathrm{Y}_S \\ 0 & \mathrm{I} & -\mathrm{X}_S \\ 0 & 0 & \mathrm{I} \end{bmatrix} \left\{\begin{matrix} \mathbf{u}_S \\ \mathbf{v}_S \\ \boldsymbol{\phi}_S \end{matrix}\right\}, \quad \text{or} \quad \boldsymbol{\delta} = \mathrm{T}\boldsymbol{\delta}_S, \tag{39}$$

where T is the transformation matrix of the order $3N$. It is easy to verify that relation (31) may be transformed into

$$\mathrm{K}_S \boldsymbol{\delta}_S = \mathbf{f}_S, \tag{40}$$

where

$$\mathrm{K}_S = \mathrm{T}^T \mathrm{K} \mathrm{T}, \tag{41}$$

$$\mathbf{f}_S = \mathrm{T}^T \mathbf{f}. \tag{42}$$

The matrix K_S is the global stiffness matrix of a building which corresponds to generalized coordinates $\boldsymbol{\delta}_S$ related to centers of mass, while $\mathbf{f}_S$ is the global force vector obtained after reduction of all external forces upon centers of mass.

3.2 Differential equations of motion

Differential equations of motion of the building may be derived in various mutually equivalent ways. One way is to consider isolated slabs and to apply for each slab the laws of momentum and moment of momentum

$$m\mathbf{a}_S = \mathbf{F}_R, \qquad \frac{d\mathbf{D}^{(S)}}{dt} = \mathbf{M}_R^{(S)}. \tag{43}$$

In scalar form Eqs. (43) are obtained as

$$m\ddot{u}_S = F_{Rx}, \quad m\ddot{v}_S = F_{Ry}, \quad J_S\ddot{\varphi}_S = M_{Rz}^{(S)}, \tag{44}$$

where m and J_S are total mass and mass moment of inertia of a slab with respect to central axis perpendicular to the slab. The right hand side of Eqs. (44) are the components of all external and internal forces after their reduction upon the center of mass. External forces are the given forces which are some functions of time, and are arising, for example, from the wind loading, while the internal forces are restoring forces which are representing influences of vertical elements upon a slab.

If Eqs. (44) are considered for all slabs and if the following diagonal matrices, which contain masses and mass moments of inertia of all slabs, are introduced: $\mathrm{m} = diag\,(m_1, m_2, ..., m_N)$ and $\mathrm{J} = diag\,(J_{S1}, J_{S2}, ..., J_{SN})$, then Eqs. (44), written for all slabs, become

$$M\ddot{\boldsymbol{\delta}}_S = \mathbf{f}_S - K_S\boldsymbol{\delta}_S. \tag{45}$$

The usual form of obtained equations is

$$M\ddot{\boldsymbol{\delta}}_S + K_S\boldsymbol{\delta}_S = \mathbf{f}_S\,(t)\,, \tag{46}$$

where M is the global mass matrix of the building given by

$$\mathrm{M} = \begin{bmatrix} \mathrm{m} & & \\ & \mathrm{m} & \\ & & \mathrm{J}_S \end{bmatrix}. \tag{47}$$

Of course, it is possible to include the viscous damping forces in the usual way, by adopting the global viscous matrix in the proportional form as $\mathrm{C}_S = \alpha\mathrm{M} + \beta\mathrm{K}_S$. In that case, equations of motion are given as

$$M\ddot{\boldsymbol{\delta}}_S + C_S\dot{\boldsymbol{\delta}} + K_S\boldsymbol{\delta}_S = \mathbf{f}_S\,(t)\,, \tag{48}$$

The solution of differential equations (46) or (48) is obtained by the usual methods of structural dynamics. Generally, there are two basic approaches: either the modal analysis, or direct numerical integration. In both cases it is very convenient that the mass matrix is a diagonal one. Therefore, the additional effort to transform initial stiffness matrix K into the new one K_S according to (41) and also to transform the initially defined loading $\mathbf{f}$ into $\mathbf{f}_S$ is definitely the worth-while effort. When the generalized displacements as the function of time are obtained, $\boldsymbol{\delta}_S = \boldsymbol{\delta}_S\,(t)$, the in-plane displacements of each vertical element are determined according to relations (25) and (39), so reaction forces transmitted to each vertical element are then given by (24). The final step is the separate analysis of each vertical frame or shear wall in order to determine its local behaviour and consequently to determine the overall behaviour of the whole building.

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Diskretna analiza nesimetričnih zgrada

U radu se prikazuje diskretni matematički model za opis mehaničkog ponašanja nesimetričnih višespratnih zgrada pod dejstvom statičkih i dinamičkih horizontalnih sila. Posebno je razmatran verovatno najčešći slučaj konfiguracije zgrada kada su vertikalni noseći elenmenti rasporedjeni u medjusobno ortogonalnim ravnima. Takodje je razmatrana i proizvoljna konfiguracija vertikalnih elemenata i oblika tavanica. Težište rada je u matematičkoj formulaciji statičkog i dinamičkog ponašanja nesimetričnih zgrada, a ne u načinu rešavanja dobijenih jednačina ravnoteže ili kretanja.