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AN APPROXIMATE METHOD FOR OBTAINING SELF-EXCITED VIBRATION OF THE ROTOR

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1. Introduction

Some of the forces which act on the rotor cause self-excited vibrations. Usually these are the internal friction forces or the forces in squeeze bearing etc. Due to the fact that the self-excited vibrations are harmful for the rotor it is necessary to analyze them. The model of the rotor is a symmetric shaft disc system supported in squeeze bearings. The differential equation which describes the vibrations of the rotor is non-linear and its general form is

$$\ddot{z} + c_1 z + f(z,\, \dot{z},\, cc) = 0, \tag{1}$$

where $z = x + iy$ is a complex deflection function of rotor's center, x, y are the coordinates of rotor's center, $i = \sqrt{-1}$ is the imaginary unit, c_1 is the coefficient of linear rigidity, cc are the complex conjugated functions, $\dot{()} \equiv \frac{\mathrm{d}}{\mathrm{d}t}$ and t is time. To make the analysis more proper it is useful to find the analytical solution of the equation (1). As it is known the general analytical solution for (1) is not known. Because of that the approximate analytical solution is obtained for some special cases of the equation (1). For the case when the nonlinearity is small and the differential equation is

$$\ddot{z} + c_1 z + \varepsilon f(z,\, \dot{z},\, cc) = 0, \tag{2}$$

in the papers [1] and [2] the well known Bogoliubov–Mitropolski method and the Krylov–Bogolubov method of slow amplitude and phase variation is adopted for solving coupled differential equations with complex function. In the papers [3] and [4] the same methods are applied for obtaining the solutions of the equation describing the vibrations of the strong nonlinear cubic rotor system

$$\ddot{z} + c_1 z + c_3 z^3 = \varepsilon f(z,\, \dot{z},\, cc), \tag{3}$$

where c_3 is a coefficient of nonlinear rigidity, ε is a small parameter and εf is a small nonlinear function. In this paper the vibrations of quasi cubic rotor system

[5] which model is

$$\ddot{z} + c_1 z + c_3 \frac{z^3}{z\bar{z}} = \varepsilon f(z, \dot{z}, cc), \tag{4}$$

are analyzed. The modified Krylov–Bogolubov method is developed for solving such a differential equation. The method is based on the procedure developed for solving strong nonlinear Duffing equation [6], [7]. For the generating function ($\varepsilon = 0$) the exact analytical solution for strong nonlinear differential equation is obtained. A perturbation procedure is introduced. The trial solution for (4) is assumed in the form of generating solution. The time variable amplitude and phase are obtained. Two examples are considered: first, when a force which is a linear function of velocity acts and then when the force is non-linear. The solutions are compared with numerical one obtained by Runge–Kutta method.

2. Analytical solving method

Let us consider the generating equation when $\varepsilon = 0$ i.e.,

$$\ddot{z} + c_1 z + c_3 \frac{z^3}{z\bar{z}} = 0. \tag{5}$$

The generating solution is

$$z = A[\operatorname{cn}(\omega t + \Theta, k^2) + i \operatorname{sn}(\omega t + \Theta, k^2)], \tag{6}$$

where sn and cn are Jacobian elliptic functions [8], $\omega^2 = c_1 + c_3$, $k^2 = 2c_3/(c_1+c_3)$ is the parameter of elliptic function, A and Θ are constants obtained according to the initial values.

Let us assume the trial solution of (5) in the form (6), as

$$z = A(t)[\operatorname{cn}(\psi(t), 1) + \operatorname{sn}(\psi(t), 1)] \tag{7}$$

where $\psi(t) = \omega t + \Theta(t)$ and A and Θ are time dependent functions. The parameter of the elliptic function k and the frequency ω are constant values obtained for generating solution. Let us write (7) in simplified form

$$z = A(t)(\operatorname{cn} + i \operatorname{sn}). \tag{7'}$$

The first and second time derivatives are

$$\begin{aligned} \dot{z} &= A(t)\, i\omega\, \operatorname{dn}[\operatorname{cn} + i \operatorname{sn}], \\ \ddot{z} &= -A(t)\,\omega\,[\omega + \dot{\Theta}(t)](\operatorname{dn}^2 + i\, k^2 \operatorname{sn}\operatorname{cn})(\operatorname{cn} + i \operatorname{sn}) + \dot{A}(t)\, i\omega\, \operatorname{dn}(\operatorname{cn} + i \operatorname{sn}), \end{aligned} \tag{8}$$

where $(\dot{\cdot}) \equiv \dfrac{\mathrm{d}}{\mathrm{d}t}$.

Substituting (7) and (8) into (4) and separating the real and imaginary terms it is

$$\begin{aligned} A\,\dot{\Theta}\,\omega\,\operatorname{dn}^2 &= -\varepsilon \Re[f(\operatorname{cn} - i\operatorname{sn})], \\ \dot{A}\,\omega\,\operatorname{dn} - A\,\dot{\Theta}\,\omega \operatorname{sn}\operatorname{cn} &= \varepsilon \Im[f(\operatorname{cn} - i\operatorname{sn})], \end{aligned} \tag{9}$$

where $\Re$ and $\Im$ represent the real and imaginary part of the functions.

The period of the Jacobian elliptic functions is $4K(k^2)$, where $K(k^2) \equiv K$ is the complete elliptic integral of the first kind. Due to the fact that the elliptic functions are periodic functions the averaging procedure is applied. According to (9) the amplitude and phase variation is obtained

$$\dot{\Theta} = -\frac{\varepsilon}{A\omega} < \Re[f(\mathrm{cn} - i\mathrm{sn})] >,$$
$$\dot{A} = \frac{\varepsilon}{\omega} < \mathrm{dn}\Im[f(\mathrm{cn} - i\mathrm{sn})] >, \tag{10}$$

where $< \cdots >= \int_0^{4K} \ldots \mathrm{d}\psi$.

3. Numerical solving method

Let us transform the differential eq. (4) in the form

$$\ddot{x} + c_1\, x - c_3 \frac{x^3 - xy^2}{x^2 + y^2} = \varepsilon\, \Re(f), \tag{11}$$
$$\ddot{y} + c_1\, y + c_3 \frac{y^3 - x^2 y}{x^2 + y^2} = \varepsilon\, \Im(f).$$

Using the Runge–Kutta method the numerical solutions are denoted. The initial conditions are $t = 0$, $x_0 = A_0\, \mathrm{cn}(\Theta_0,\, k^2)$, $\dot{x}_0 = -A_0\, \omega\, \mathrm{sn}(\Theta_0,\, k^2)\, \mathrm{dn}(\Theta_0,\, k^2)$, $y_0 = A_0\, \mathrm{sn}(\Theta_0,\, k^2)$, $\dot{y}_0 = A_0\, \omega\, \mathrm{cn}(\Theta_0,\, k^2)\, \mathrm{dn}(\Theta_0,\, k^2)$.

4. Comparison of the solutions

Two examples are considered: first, the damping force is linear and the second, the damping force is nonlinear.

Let us consider an example of rotor vibrations when a force which is a linear function of velocity acts. The differential equation of rotor motion is

$$\ddot{z} + 3z + \frac{z^3}{z\bar{z}} = \varepsilon\, \dot{z}, \tag{12}$$

where ε is a small damping coefficient. The value of small parameter is $\varepsilon = 0.1$. For the initial conditions $A_0 = 0.1$, $\Theta_0 = 0$ the analytical and numerical solutions are obtained. The amplitudes of vibration $A(t)$ obtained analytically and numerically are compared since the amplitude gives the rotor's energy and is fundamental to self-excited vibrations $z(t)$. The analytically obtained amplitude is after expressions (11)

$$A(t) = A_0\, e^{\varepsilon\, t}. \tag{13}$$

In Fig. 1 the results are plotted.

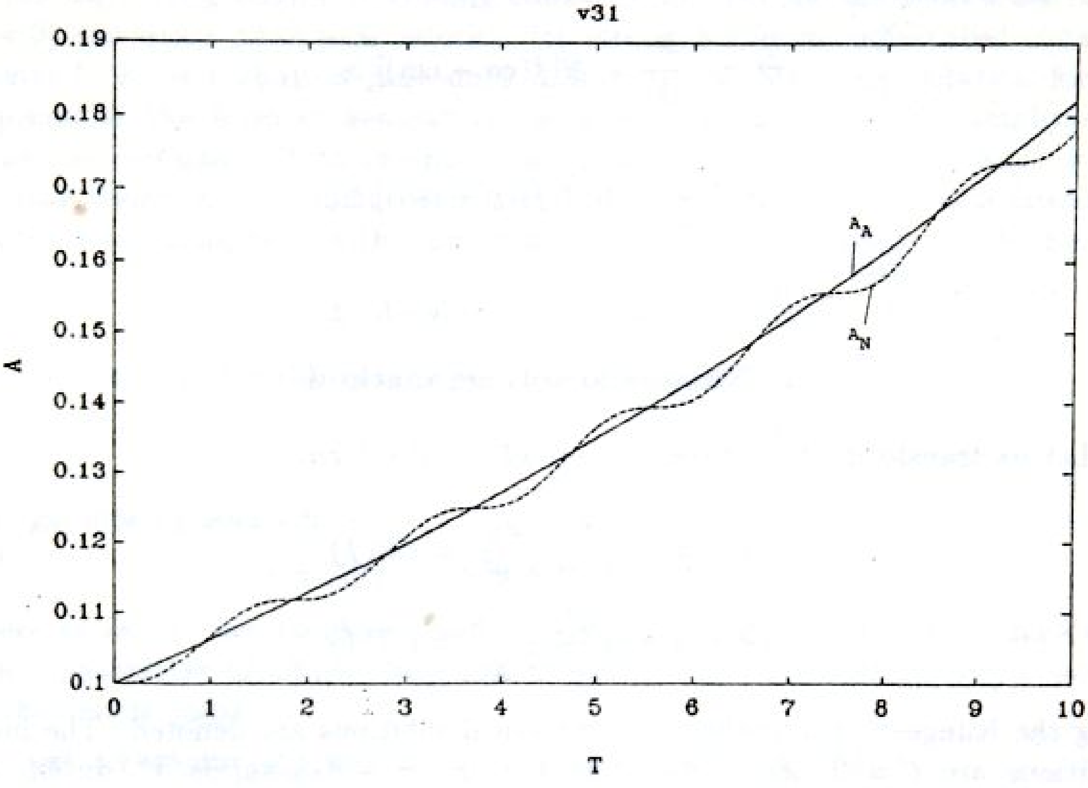

Figure 1

For the case when the damping force is nonlinear the differential equation is

$$\ddot{z} + 3\,z + \frac{z^3}{z\bar{z}} = \varepsilon\,\dot{z}(z,\,\bar{z}). \tag{14}$$

For the initial conditions $A_0 = 0.1$ and $\Theta_0 = 0$, the approximate analytical solution for amplitude is

$$A(t) = \frac{A_0}{(1 - 2\,A_0^2\,\varepsilon\,t)^{1/2}}. \tag{15}$$

In Fig. 2 the amplitudes obtained analytically and numerically are plotted. Comparing the analytical and numerical solutions it can be seen that the difference between analytical and numerical solutions is negligible. It means that the suggested analytical method gives us good results.

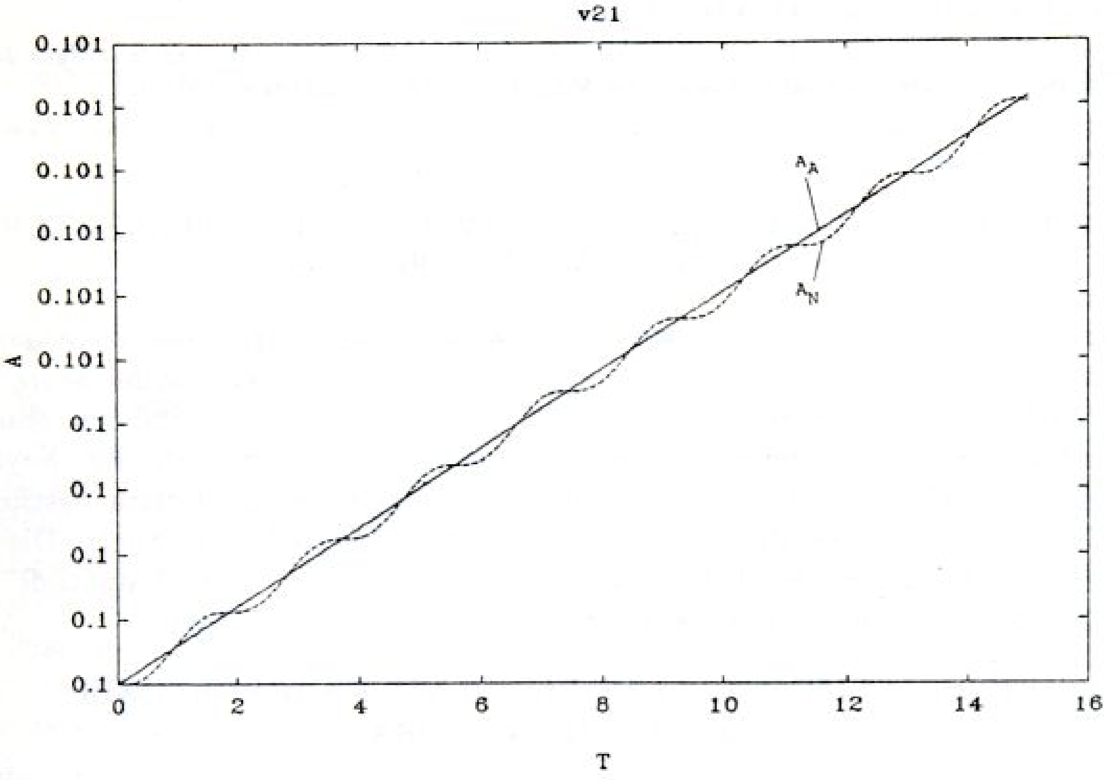

Figure 2

5. Conclusion

In the nonlinear rotor system on which the linear or nonlinear small squeeze friction force acts the self-excited vibrations occur. The amplitude of vibrations has a tendency of increasing which is a property of self-excited vibrations. The amplitude of vibrations increases faster for the linear force, than for the non linear. The increase of the amplitude causes an instability in rotor motion.

REFERENCES

[1] Cvetićanin, L., *An Asymptotic Solution for Weak Nonlinear Vibrations of the Rotor*, Mechanism and Machine Theory, **28**, No. 4 (1993), 495–505.

[2] Cvetićanin, L., *Approximate Analytical Solutions to a Class of Non-Linear Equations with Complex Functions*, Journal of Sound and Vibration, **157**, No.2 (1992), 289–302.

[3] Cvetićanin, L., *An Approximate Solution for a System of Two Coupled Differential Equations*, Journal of Sound and Vibration, **152**, No.2 (1992), 375–380.

[4] Cvetićanin, L., Glavardanov, V., *Analytical Method for Denoting of the Vibrations of the Rotot with Cubic Nonlinearity*, Facta Universitatis, Ser. Mechanics, Automatic Control and Robotics, 1, No. 2 (1992), 177–186.

[5] Tondl, A., *Avtokolebanije mahanicheskih sistem*, Mir, Moscow, (1979).

[6] Coppola, V.T., Rand, R.,H., *Averaging Using Elliptic Functions: Approximation of Limit Cycles*, Acta Mechanica, 81 (1990), 125–142.

[7] Yuste, S.B., Bejarano, J.D. *Improvement of a Krylov- Bogoliubov Method that Uses Jacobi Elliptic Functions*, Journal of Sound and Vibration, 139, No.1 (1990), 151–163.

[8] Byrd, P.F., Friedman, M.D. *Handbook of Elliptic Integrals for Engineers and Physicists*, Springer Verlag, Berlin (1954).

EINE VERFAHRUNG FUR DIE BESTIMMUNG SELBSTERREGTER SCHWINGUNGEN DES ROTORS

In dieser Arbeit werden die Selbsterregte Schwingungen des nichtlineares Rotors bestimmt. Das mathematsche Modell des Rotors ist eine nichtlineare Differenzialgleichung der zweite Ordnung mit komplexer Funktion und mit starken und schwachen nichtlinearen Gliedern. Die Schwingungen sind mit Krylov–Bogoliubov Verfahrung und Jacobien elliptischen Funktien analytisch bestimmt. Die Losung der Differenzialgleichungen ist auch numerisch berechnet. Die analytische und numerische Ergebnisse sind vergleichbar. Die Amplitude der Schwingungen nimmt zu und die Rotation des Rotors ist unstabil.

PRIBLIŽNI METOD ZA ODREĐIVANJE SAMOPOBUDNIH OSCILACIJA ROTORA

U ovom radu određene su samopobudne oscilacije kvazi-kubnog nelinearnog rotora. Na rotor pored navedene nelinearnosti deluju i male nelinearne sile. Matematički model rotora je nelinearna diferencijalna jednačina drugog reda sa kompleksnom funkcijom pomeranja. Oscilacije su približno određene analitički primenom metode Krilov–Bogoljubova. Rešenje strogo nelinearne diferencijalne jednačine je opisano Jakobijevim eliptičkim funkcijama. Tačno rešenje se perturbuje i odredi se vremenska promenljiva amplituda i faza. Diferencijalna jednačina se reši i numerički primenom Runge–Kuta metode. Navedene metode su primenjene na dva primera: prvi, kada na rotor deluje sila koja je linearna funkcija brzine pomeranja i koja je mala, i drugi, kada je ta sila mala ali nelinearna. Poređenjem rešenja dobivenih analitički i numerički može se zaključiti da su razlike zanemarljivo male. Amplituda samopobudnih oscilacija ima tendenciju porasta i ukazuje na nestabilnost obrtanja rotora.

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