TEORIJSKA I PRIMENJENA MEHANIKA 20, pp 245–252, 1994. UDK 532

DAMAGE OF THE UNIAXIALLY REINFORCED COMPOSITES BY INTERFACIAL ARC MICROCRACK

D. Šumarac

(Received 19.09.1994.)

1. Introduction

Composite materials are ideal for structural applications where high strength-to-weight and stiffness-to-weight ratios are required (Jones, 1975 [3]). Aircraft, spacecraft and recently bridges in Civil Engineering, are typical weight sensitive structures in which composite materials are cost-effective. There are three commonly accepted types of composite materials: Fibrous composites which consist of fibers in a matrix; Laminate composites which consist of layers of various materials; Particulate composites which are composed of particles in a matrix. In this paper fibrous composites will be considered. Despite they are high performance, those materials are very sensitive to initial defects, which have decisive influence on they are strength and durability. In the fibrous composites initial defects are interfacial arc microcracks, which are usually consequence of the curing process or temperature mismatch. In this paper only stationary damage model, with the constant crack size, would be analyzed. Finally the Young's modulus, Poisson's ratio, and complete compliance matrix in the plane perpendicular to the fiber direction would be given, for uniform distribution of cracks.

2. Compliance of the Undamaged Material

Matrix with the longitudinal fibers randomly distributed is transversely

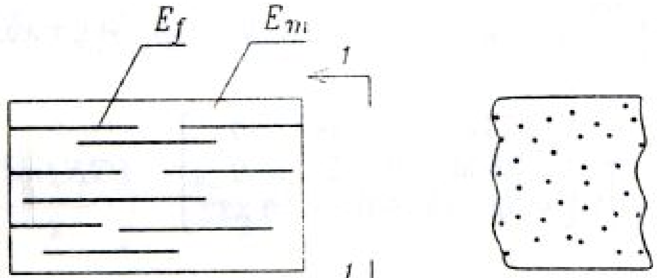

Fig.1. Uniaxially reinforced composite

isotropic material. In this paper the same composite, as in the article written by Ju (1991) [4], will be chosen.

This material consists of epoxy matrix, Fig.1, with the characteristics:

$$\mu_m = 2.39\,GN/m^2; \quad \nu_m = 0.35; \quad E_m = 6.47\,GN/m^2, \tag{2.1}$$

and glass fibers with the constants:

$$\mu_f = 44.26\,GN/m^2; \quad \nu_f = 0.22; \quad E_f = 107.85\,GN/m^2, \tag{2.2}$$

In the above expressions μ, ν and E stand for shear modulus, Poisson's ratio and Young's modulus respectively. Let the volume fractions of those two materials are:

$$V_m = 0.8; \quad V_f = 0.2. \tag{2.3}$$

For such composed material there is no way to find exact value of the elastic constants. The approximate, averaged, values will be presented in this paragraph. The Young's modulus in the direction of the fibers is:

$$E_L = V_m E_m + V_f E_f = 0.8 \times 6.47 + 0.2 \times 107.85 = 26.75\,GN/m^2, \tag{2.4}$$

which is very well known as the rule of mixtures. For more explanation about the rule of mixtures see Jones 1975 [3]. Young's modulus perpendicular to the direction of fibers is calculated on the assumption that the stress in the matrix and in the fiber in this direction is the same. This leads to:

$$E_T = E_0 = \frac{E_f E_m}{V_m E_f + V_f E_m} = 7.97\,GN/m^2 \tag{2.5}$$

There are other approximations to calculate E_T, which can be found also in reference [3]. Overall Poisson's ratio in the plane perpendicular to fibers can be calculated using rule of mixtures from (2.1) to (2.3) as:

$$\nu_0 = V_m \nu_m + V_f \nu_f = 0.8 \times 0.35 + 0.2 \times 0.22 = 0.324. \tag{2.6}$$

In the plane perpendicular to the direction of fibers material behaves isotropically, with the overall constants E_0 and ν_0 given by formulas (2.5) and (2.6). In this way the overall compliance matrix that represents the response of undamaged material in the plane perpendicular to fibers reads:

$$[S^\bullet] = \frac{1}{E_0}\begin{bmatrix} (1-\nu_0^2) & -\nu_0(1+\nu_0) & 0 \\ -\nu_0(1+\nu_0) & (1-\nu_0^2) & 0 \\ 0 & 0 & 2(1+\nu_0) \end{bmatrix} = $$

$$= \begin{bmatrix} 0.112 & -0.054 & 0 \\ -0.054 & 0.112 & 0 \\ 0 & 0 & 0.332 \end{bmatrix} \quad (GN/m^2)^{-1} \tag{2.7}$$

3. Compliance du to Presence of a Single Arc Microcrack

Consider the interface arc microcrack shown in Fig.2 under remotely applied stresses. In the subsequent sections it will be assumed that the crack size

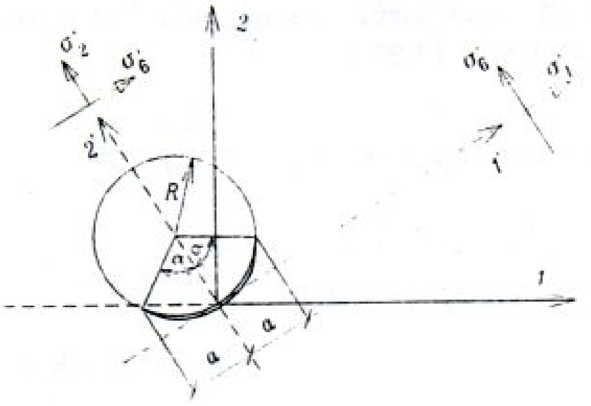

Fig. 2 Single arc microcrack

is small compared with the radius of the fiber ($a \ll R$, or $\alpha \to 0$, where R represents averaged value of the fiber radius). This assumption is in agreement with the goal of the paper, which is to consider the initial damage, obviously supposed to be small. Second assumption is, that the crack is embedded in the material which is with the unknown material constants that should be determined. This assumption is in accordance with the well known Self Consistent Method which will be applied in this paper. There are papers that are considering the interface cracks embedded between two dissimilar media (Toya 1974 [6]) which was used in the paper written by Ju (1991) [4]. This approach even seems better at the beginning, leads to cumbersome numerical procedure even for small damage (crack size). Also assumption of self-consistency will wipe out the strong accuracy which was used at the beginning.

From the paper written by Cotterell and Rice (1980) [1], for arbitrary size of the crack, the stress intensity factors are:

$$K_I = \sqrt{\pi a}\left\{\left[\left(\frac{\sigma_2' - \sigma_1'}{2}\right) - \frac{\sigma_2' - \sigma_1'}{2}\sin^2(\alpha/2)\cos^2(\alpha/2)\right]\frac{\cos(\alpha/2)}{(1+\sin^2(\alpha/2))} + \frac{\sigma_2' - \sigma_1'}{2}\cos(3\alpha/2) - \sigma_6'\left[\sin(3\alpha/2) + \sin^3(\alpha/2)\right]\right\} \tag{3.1}$$

$$K_{II} = \sqrt{\pi a}\left\{\left[\left(\frac{\sigma_2' - \sigma_1'}{2}\right) - \frac{\sigma_2' - \sigma_1'}{2}\sin^2(\alpha/2)\cos^2(\alpha/2)\right]\frac{\sin(\alpha/2)}{(1+\sin^2(\alpha/2))} + \frac{\sigma_2' - \sigma_1'}{2}\sin(3\alpha/2) + \sigma_6'\left[\cos(3\alpha/2) + \cos(\alpha/2)\sin^2(\alpha/2)\right]\right\} \tag{3.2}$$

where Voigt's notation was used:

$$\sigma_2' = \sigma_{yy}'; \quad \sigma_1' = \sigma_{xx}'; \quad \sigma_6' = \sigma_{xy}'. \tag{3.3}$$

For small α ($\alpha \to 0$, small crack size), taking only linear term with respect to α, it is obtained from (3.1) and (3.2):

$$K_I = \sqrt{\pi a}\left(\sigma_2' - \frac{3}{2}\alpha\sigma_6'\right) = \sqrt{\pi a}\left(\sigma_2' - \frac{3}{2}\frac{a}{R}\sigma_6'\right) \tag{3.4}$$

$$K_{II} = \sqrt{\pi a}\left[\sigma_6' + \left(\sigma_2' - \frac{\sigma_1'}{2}\right)\alpha\right] = \sqrt{\pi a}\left[\sigma_6' + \left(\sigma_2' - \frac{\sigma_1'}{2}\right)\frac{a}{R}\right]. \tag{3.5}$$

Potential energy increase due to presence of a single crack is obtained via Fracture mechanics as:

$$\psi^{*(k)} = (1-\nu^2)\int_{-a}^{a}\frac{K_I^2 + K_{II}^2}{E}\,da = \frac{\pi a^2}{E}(1-\nu^2)\left[\left(\sigma_2'\right)^2\left(1+\frac{a^2}{2R^2}\right) + \right.$$
$$\left. + \left(\sigma_1'\right)^2\frac{a^2}{8R^2} - \sigma_1'\sigma_2'\frac{a^2}{2R^2} - \sigma_1'\sigma_6'\frac{2a}{3R} - \sigma_2'\sigma_6'\frac{2a}{3R} + \left(\sigma_6'\right)^2\left(1+\frac{9a^2}{8R^2}\right)\right]. \tag{3.6}$$

In deriving the above expression, equations (3.4) and (3.5) were used. The increase of the compliance due to presence of one crack are obtained by differentiating expression (3.6) with the governing stresses:

$$S_{ij}^{*'(k)} = \frac{\partial^2\psi^{*(k)}}{\partial\sigma_i'\partial\sigma_j'}. \tag{3.7}$$

Substitution of (3.6) into (3.7) leads to:

$$S_{ij}^{*(k)'} = \frac{2\pi a^2}{E}(1-\nu^2)\left[\frac{a^2}{8R^2}\delta_{1i}\delta_{1j} - \frac{a^2}{4R^2}(\delta_{1i}\delta_{2j} + \delta_{2i}\delta_{1j}) + \right.$$
$$+ \left(1+\frac{a^2}{2R^2}\right)\delta_{2i}\delta_{2j} - \frac{a}{3R}(\delta_{1i}\delta_{6j} + \delta_{6i}\delta_{1j}) - \frac{a}{3R}(\delta_{2i}\delta_{6j} + \delta_{6i}\delta_{2j}) +$$
$$\left. + \left(1+\frac{9a^2}{8R^2}\right)\delta_{6i}\delta_{6j}\right]. \tag{3.8}$$

In the case of straight crack ($R \to \infty$, or $a/R \to 0$), from (3.8) it follows:

$$S_{ij}^{*(k)'} = \frac{2\pi a^2}{E}(1-\nu^2)(\delta_{2i}\delta_{2j} + \delta_{6i}\delta_{6j}), \tag{3.9}$$

which is very well known expression (see Šumarac and Krajčinović 1989 [9]). Compliance (3.8) are given in the local coordinate system. Using transformation rule:

$$S_{ij}^{*(k)} = S_{mn}^{*(k)'}g_{mi}g_{nj}, \tag{3.10}$$

the compliances in the global coordinate system are obtained, where transformation matrix g_{ij} is given by (Horii and Nemat Nasser, 1983 [2]):

$$g_{ij} = \begin{bmatrix} \cos^2\Theta & \sin^2\Theta & \sin 2\Theta \\ \sin^2\Theta & \cos^2\Theta & -\sin 2\Theta \\ -\frac{1}{2}\sin 2\Theta & \frac{1}{2}\sin 2\Theta & \cos 2\Theta \end{bmatrix}. \tag{3.11}$$

Substituting (3.11) into (3.10) finally it is obtained:

$$S_{ij}^{*(k)} = \frac{2\pi a^2}{E}(1-\nu^2)\Bigg[\frac{a^2}{8R^2}g_{1i}g_{1j} - \frac{a^2}{4R^2}(g_{1i}g_{2j} + g_{2i}g_{1j}) + \left(1+\frac{a^2}{2R^2}\right)g_{2i}g_{2j} - \frac{a}{3R}(g_{1i}g_{6j} + g_{6i}g_{1j}) - \frac{a}{3R}(g_{2i}g_{6j} + g_{6i}g_{2j}) + \left(1+\frac{9a^2}{8R^2}\right)g_{6i}g_{6j}\Bigg]. \tag{3.12}$$

It is very easy to calculate particular values of the compliance from the expression (3.12).

4. Compliance due to Presence of an Ensemble of Cracks

Once the increase of the compliance (3.12) due to presence of one crack is known, the total contribution of all cracks would not be difficult to be obtained. In the case of many cracks, the total compliance would be:

$$S_{ij} = S_{ij}^0 + S_{ij}^*, \tag{4.1}$$

where S_{ij}^0 are the compliances of the undamaged material, determined by Young's modulus and Poisson's ratio given by (2.5) and (2.6). S_{ij}^* stands for the increase of the compliance due to presence of all cracks. Instead of summing contributions of all particular cracks, the averaged values of the compliance increase would be multiplied by the number of cracks, i.e.:

$$S_{ij}^* = N\int_{\Theta_{\min}}^{\Theta_{\max}} S_{ij}^{(k)*}(\Theta)\,p(\Theta)\,\mathrm{d}\Theta, \tag{4.2}$$

where N is the number of cracks per unit area (unit cell) (see Horii and Nemat-Nasser, 1983 [2], and Šumarac and Krajčinović 1987 [7]). Taking for simplicity that the distribution of orientation of cracks is uniform, i.e. $\Theta_{\min} = 0$ and $\Theta_{\max} = \pi$, then:

$$p(\Theta) = \frac{1}{\pi}, \tag{4.3}$$

where $p(\Theta)$ is uniform distribution density function of Θ. Introducing (4.3) into (4.2), and taking governing coefficients of matrix g_{ij}, after lengthy integration and algebra it is obtained:

$$S_{11}^* = S_{22}^* = \frac{\varpi}{E}(1-\nu^2); \quad S_{66}^* = \frac{2\varpi}{E}(1-\nu^2); \quad (S_{ij}^* = 0 \quad \text{otherwise}), \tag{4.4}$$

where:

$$\overline{\omega} = N\pi a^2\left(1+\frac{a^2}{R^2}\right) = \omega\left(1+\frac{a^2}{R^2}\right), \tag{4.5}$$

is the measure of the damage for the arc microcracks, and $\omega = N\pi a^2$ is the measure of the damage in the case of the straight cracks (see Krajčinović 1989 [5] and Šumarac and Krajčinović 1987 [7]). For the straight crack, $(a^2/R^2) \to 0$, from (4.4), using (4.5), it follows:

$$S^*_{11} = S^*_{22} = \frac{\omega}{E}(1-\nu^2); \quad S^*_{66} = \frac{2\omega}{E}(1-\nu^2); \quad (S^*_{ij} = 0 \quad \text{otherwise}), \tag{4.6}$$

which is the same as obtained in the above mentioned papers.

Composite material with the randomly distributed longitudinal fibers, weakened by the uniform distribution of arc microcracks, is isotropic in the plane perpendicular to the fibers. Then, the overall material constants (E and ν) are representing its response. Equations (4.1) accompanied with the equations (4.4) are the system of three equations that are not identically equal to zero. Two of them are sufficient to find solution for E and ν, and it could be easily checked that the third is identically satisfied. Taking for $S_{11} = (1-\nu^2)/E$ and for $S_{12} = -\nu(1+\nu)/E$, from (4.1) and (4.4), the solution of the system of two equations is:

$$\frac{E}{E_0} = \frac{1-\overline{\omega}}{1+\nu_0}\,\frac{(1+\nu_0-2\nu_0\overline{\omega})}{(1-\nu_0\overline{\omega})^2} \tag{4.7}$$

$$\frac{\nu}{\nu_0} = \frac{1-\overline{\omega}}{1-\nu_0\overline{\omega}}. \tag{4.8}$$

As was mentioned above, the equation for S_{66} is identically satisfied. From the equations (4.7) and (4.8) it could be seen that for $\overline{\omega} = 0$ (no damage) $E/E_0 = \nu/\nu_0 = 1$, which is to be expected. Also for $\overline{\omega} = 1$ (material is completely damaged), $E = \nu = 0$, which is also characteristic of the Self-consistent model. In the case of the straight cracks ($a/R \to 0$), from (4.7) and (4.8), the result of Šumarac (1987) [8] is recovered. Once the expressions (4.7) and (4.8) are known, the total compliance matrix is:

$$[S] = \frac{1}{E_0}\begin{bmatrix} \dfrac{(1-\nu_0^2)}{1-\overline{\omega}} & -\nu_0(1+\nu_0) & 0 \\ -\nu_0(1+\nu_0) & \dfrac{(1-\nu_0^2)}{1-\overline{\omega}} & 0 \\ 0 & 0 & \dfrac{2(1+\nu_0)(1-\nu_0\overline{\omega}}{1-\overline{\omega})} \end{bmatrix}. \tag{4.9}$$

As a numerical example, for already chosen material in the second paragraph, for $\overline{\omega} = 0.1$ it is obtained $E/E_0 = 0.91$ ($E = 7.29\,GN/m^2$) and $\nu/\nu_0 = 0.93$ ($\nu = 0.3$). From this calculation it could be seen that for small amount of damage of 10%, Young modulus is decreased for 9% if it is compared with the undamaged material. The compliance matrix for this amount of damage is:

$$[S] = \begin{bmatrix} 0.125 & -0.054 & 0 \\ -0.054 & 0.125 & 0 \\ 0 & 0 & 0.357 \end{bmatrix} (GN/m^2)^{-1}. \tag{4.10}$$

Obviously, as is expected, the elements of the compliance matrix of damaged material are increased which could be easily checked comparing expressions (4.10) with (2.7).

5. Conclusions

Current development of industry needs new type of materials such are composites. Those types of materials are with the very high strength and stiffness and low weight, which is ideal for application, but they are very sensitive to the crack-like defects. In this paper, the influence of distributed damage (arc microcracks) on the stiffness of uniaxially reinforced fibrous composites was analyzed. It is shown that for small damage of let say 10% the decrease of Young's modulus is 9% if it is compared with the undamaged material.

Acknowledgement. The author gratefully acknowledges the financial support provided by the Scientific Fund of Serbia, to the Department of Civil Engineering, University of Belgrade, through the grant number 1701, and partially by SMNTR thorough the grant number OSI262, which made this work possible.

REFERENCES

[1] Cotterell, B., and Rice, J.R., *Slightly curved or kinked cracks*, I.J. of Fracture, **16** (1980), 155-169.

[2] Horii, H., and Nemat-Nasser, S., *Overall Moduli of Solids with Microcracks: Load Induced Anisotropy*, J. Mech. Phys. Solids, **31** (1983), 155-171.

[3] Jones, M.R., *Mechanics of Composite Materials*, Scripta Book Company, (1975).

[4] Ju,J. W.,*A Micromechanical Damage Model for Uniaxially Reinforced Composites Weakened by Interfacial Arc Microcracks*, J. of App. Mechanics, **58** (1991), 923-930.

[5] Krajčinivić, D., *Damage Mechanics*, Mech. of Materials, **8** (1989), 117-197.

[6] Toya, M., *A Crack Along the Interface of a Circular Inclusion Embedded in an Infinite Solid*, J. Mech. Phys. Solids, **22** (1974), 325-348.

[7] Šumarac, D., and Krajčinović, D., *A Self-consistent Model for Microcrack Weakened Solids*, Mech. of Materials, **6** (1987), 39-52.

[8] Šumarac, D., *Self-consistent Model for the Brittle Response of Solids*, Ph.D. Thesis, UIC, Chicago, (1987).

[9] Šumarac, D., and Krajčinović, D., *A Mesomechanical Model for Brittle Deformation Processes*, Part II, J. Appl. Mech., **56** (1989), 57-62.

[10] Šumarac, D., *Elastic Parameters of the Body Weakened by the Elliptical Voids*, Theor. and Appl. Mechanics, **18** (1992), 119-130.

ДЕФЕКТ КОЛЬЦЕВЫМИ ТРЕЩИНАМЫ ОДНОНАПРАВЛЕННЫХ ВОЛОКНИСТЫХ КОМПОЗИТОВ

В настоящей работе, пользуясь приёмом механики разрушений и механики дефектов, предложен вполне аналитический метод определения матрицы податливости, или модуля упругости и коэффициента Пуассона, для волокнистых композиционных материалов, при наличии кольцевых трещин. В отличие то подходов встречающихся в литератури, имеющих за основание точние выражения, а закончивающихся

численным методом решения, в этой работе то самого начала введено предположение о малом размере дефекта. Это предположение, вместе с применением метода самосогласования, результировало простыми аналитическими выражениями для искоммых характеристик материала, удобных для применений в практике.

OŠTEĆENJE KRUŽNIM PRSLINAMA KOMPOZITA ARMIRANOG JEDNOAKSIJALNIM VLAKNIMA

U radu se, polazeći od mehanike loma i mehanike oštećenja, daje analitički postupak odredjivanja matrice deformabilnosti, odnosno modula elastičnosti i Poasonovog broja, kompozitnog materijala oslabljenog kružnim prslinama na spoju matrice i vlakana. Za razliku od rezultata u literaturi, koji polaze od strožijih predpostavki u samom početku a što vodi u numeričko dobijanje rešenja, u ovom radu se pošlo od pretpostavke malog oštećenja. To je uz korišćenje samokonsistentnog modela dovelo do jednostavnih analitičkih izraza praktičnih za svakodnevnu primenu u praksi.

Dragoslav Šumarac
Civil Engineering, University of Belgrade,
Bul. Revolucije 73
11000 Belgrade, Yugoslavia