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PROPOSAL FOR AN ALGORITHM FOR INCREASING THE ACCURACY OF AN OPTIMAL ESTIMATION OF THE GENERALIZED KALMAN DISCRETE FILTER FOR NON–LINEAR SYSTEMS

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The starting point is a non–linear discrete object model

$$\underline{x}_{k+1} = \underline{f}\,[\underline{x}_k,\, k] + \underline{G}\,[\underline{x}_k,\, k]\,\underline{w}_k \tag{1}$$

$$\underline{z}_k = \underline{h}\,[\underline{x}_k,\, k] + \underline{\nu}_k \tag{2}$$

The discrete algorithm of the estimation which is optimal according to the criterion of maximal a posterior probability has the following form [4], [9]:

$$\hat{\underline{x}}_{k+1} = \hat{\underline{x}}_{k+1/k} + \underline{K}_{k+1}\left[\underline{z}_{k+1} - \underline{h}\left(\hat{\underline{x}}_{k+1/k},\, k+1\right)\right] \tag{3}$$

$$\hat{\underline{x}}_{k+1/k} = \underline{f}\,(\hat{\underline{x}}_k,\, k) \tag{4}$$

where a priori covariant matrix of an error estimation is covariant matrix of an error estimation

$$\underline{\Sigma}_{k+1/k} = \underline{f}_{\underline{x}}\,(\hat{\underline{x}}_k,\, k)\,\underline{\Sigma}_k \underline{f}_{\underline{x}}^{T}\,(\hat{\underline{x}}_k,\, k) + \underline{G}\,(\hat{\underline{x}}_k,\, k)\,\underline{Q}_k \underline{G}^{T}\,(\hat{\underline{x}}_k,\, k) \tag{5}$$

$$\underline{\Sigma}_{k+1} = \underline{\Sigma}_{k+1/k} - \underline{\Sigma}_{k+1/k}\underline{h}_{\underline{x}}^{T}\left(\hat{\underline{x}}_{k+1/k},\, k+1\right)\left[\underline{\Sigma}_{k+1/k} - \underline{h}_{\underline{x}}^{T}\left(\hat{\underline{x}}_{k+1/k},\, k+1\right) + \underline{R}_{k+1}\right]^{-1} \underline{h}_{\underline{x}}\left(\hat{\underline{x}}_{k+1/k},\, k+1\right)\underline{\Sigma}_{k+1/k} \tag{6}$$

The coefficient of the generalized Kalman filter intensification matrix can be calculated from the relation

$$\underline{K}_{k+1} = \underline{\Sigma}_{k+1}\underline{h}_{\underline{x}}^{T}\left(\hat{\underline{x}}_{k+1/k},\, k+1\right)\underline{\Sigma}_{k+1}^{-1} \tag{7}$$

The inital conditions are:

$$\hat{\underline{x}}_{k_0} = M[\underline{x}_0]; \quad \underline{\Sigma}_{k_0} = \underline{\Sigma}_{x_0} \tag{8}$$

If the equations [1] and [2] do not describe the dynamic system with sufficient accuracy and if can covarient matrices $\underline{Q}$ and $\underline{R}$ do not describe random disturbances which are present in the system with sufficient accuracy then it is possible that the optimal estimation diverges (behaves in an unstable way).

The equations which determine the matrices $\underline{\Sigma}_k$, $\underline{\Sigma}_{k+1/k}$ and $\underline{K}_k$, for the case of a generalized Kalman filter involve only the given a priori information which cannot be further corrected. That's why every deviation in choosing a priori parameters leads to the modification of the intensification matrix $\underline{K}_k$ and can be a reason for the filter divergence. Errors in computation can also cause divergence of the optimal estimation algorithm. If $\|\underline{\Sigma}_k\|$ is decreasing, the moment will come when the matrix elements $\underline{\Sigma}_k$ become co-measurable with computation errors and this causes the optimal estimation algorithm divergence. For the generalized optimal Kalman filter the observations which have been made at any previous moment carry the same information just like those made at any given moment. In that sense the general Kalman filter gives a greater estimation accuracy since it processes the results of previous operations which have arrived once again.

In real conditions and due to deviations in determining a priori parameters as well as to errors in computation a greater impact of ongoing measurements is needed. However, if the object dynamics is not taken accurately and its mathematical modelling this can lead to great errors in the part where the measurements have not been fully taken. In that case it is possible to cause the estimation divergence as a consequence of the mathematical model deviation from the real object behavior.

As a way of preventing the divergence it is possible to rely on an adjustment of the covariant matrix $\underline{\Sigma}_{k+1/k}$ based on experience so that a greater attention is paid to the ongoing measurements (since they are supposed to mean "more" in forming an optimal estimation).

In [1], [2] and [3] there are several ways presented dealing with the problem of the estimation divergence elimination. The basic idea of all these approaches is ultimately connected with the limitation of the coefficients of the intensification matrix $\underline{K}_k$.

Another approach [5] is more complex and it involves forming of adaptive algorithms which can be used to perform an optimal estimation not only of the state vectors but also of the object parameters or statistics of input and measuring disturbances.

These authors suggest an algorithm for increasing the optimal estimation accuracy starting from the criterion referred to in the latter approach [2], [5] and [9]. This criterion which serves as an indicator of the generalized Kalman filter divergence is determined with the inequality

$$\underline{\bar{\nu}}_k^T \underline{\bar{\nu}}_k \leq \gamma S_p \left[M \left(\underline{\nu}_k \underline{\nu}_k^T \right) \right] \tag{9}$$

where S_p denotes a matrix trace whereas γ reserve coefficient ($\gamma \geq 1$).

$$\underline{\bar{\nu}}_k = \underline{z}_k - \underline{h}_{\underline{x}} \left(\underline{\hat{x}}_{k/k-1}, k \right)$$

$$M \left[\underline{\bar{\nu}}_k \underline{\bar{\nu}}_k^T \right] = \underline{h}_{\underline{x}} \left(\underline{\hat{x}}_{k/k-1}, k \right) \underline{\Sigma}_{k/k-1} \underline{h}_{\underline{x}}^T \left(\underline{\hat{x}}_{k/k-1}, k \right) + \underline{R}_k$$

If the filter divergence is consequence of increasing the real estimation error beyond the allowed one, then the condition (9) can be used as a criterion in deciding upon divergence. If the matrix $M \left[\underline{\bar{\nu}}_k \underline{\bar{\nu}}_k^T \right]$ is replaced by its theoretical value, the criterion (9) obtains the form

$$\underline{\bar{\nu}}_k^T \underline{\bar{\nu}}_k \leq \gamma S_p \left[\underline{h}_{\underline{x}} \left(\underline{\hat{x}}_{k/k-1}, k \right) \underline{\Sigma}_{k/k-1} \underline{h}_{\underline{x}}^T \left(\underline{\hat{x}}_{k/k-1}, k \right) + \underline{R}_k \right] \tag{10}$$

If the condition (9) is not fulfilled then it means that the estimation real error overcomes more than γ times its theoretical value, that is the filter gives an estimation which considerably diverges from the real value of the state vector. The method of "freezing" the intensification matrix $\underline{K}_k$ consists of the following: if there is a divergence (as shown by the criterion (9)) then the intensification matrix of the filter $\underline{K}_k$ gets "frozen" (the procedure of computing the matrix $\underline{K}_k$ is not carried out) whereas its last value (when the condition (9) was fulfilled) is preserved. $\underline{K}_k$ remains constant until the criterion (9) is fulfilled. The method of "freezing" the intensification matrix $\underline{K}_k$ enables further reduction of the norm $\|\underline{K}_k\|$ which as a consequence leads to an increase of the importance of the ongoing measurements in comparison with the case when the filter is operationg in its usual optimal regime. It is obvious that the effect of "freezing" $\underline{K}_k$ upon divergence of the optimal estimations is the same as that of the empirical increase of the importance of the matrix $\underline{\Sigma}_{k+1/k}$.

Unlike an increase of the importance of the covariant matrix based on experience this can also be done in another way [9]. The measuring disturbance covariant matrix which has previously been denoted as $\underline{R}_k$ can be also represented in the following form:

$$M \left[\underline{\nu}_j \underline{\nu}_j^T \right]_{t=t_k} = \underline{R}_{j/k} = b^{k-j} \underline{R}_j; \quad k \geq j \tag{11}$$

where b– scalar moving within the range $b = 1 \div 1.4$. $\underline{R}_{j/k}$–meaning that the covariant matrix $\underline{R}_j$ is at the moment j used for obtaining an estimation of the state vector at the moment k. It is not equal to $\underline{R}_j$ as in the case of the generalized Kalman filter but it is b^{k-j} times greater. From the expression for $\underline{K}_{k+1}$ it is obvious that the state vector $\underline{\hat{x}}_{k+1}$ optimal estimation involves all the covariant matrices of the measuring disturbance from $\underline{R}_1$ to $\underline{R}_{k+1}$. In other words, it has been proposed that the ongoing estimation should involve the importance of the covariant matrix of the measuring disturbance of all the previous measurements. The more any moment j is away from the ongoing moment k, the more importance should be allotted to the covariant matrix $\underline{R}$ which corresponds to the moment j.

Instead of the constant parameter b the authors suggest an algorithm whose scalar multiplier is not constant but instead, it changes automatically at each step of an optimal estimation calculation. The criterion used here as an indicator of the optimal estimation divergence also has the form (9). The value of the parameter γ should also be determined where the scalar product $\underline{\bar{\nu}}_{k+1}^T \cdot \underline{\bar{\nu}}_{k+1}$ has a minimal value allowed for by the inequality (9). In other words, a value for γ should be determined at which the condition for the optimal estimation convergence (9) is the most intense. If it is $\gamma \geq 1$, the value is $\gamma = 1$ since at that moment the strongest convergence condition is

$$\underline{\bar{\nu}}_{k+1}^T \underline{\bar{\nu}}_{k+1} \leq S_p \left[M \left(\underline{\nu}_{k+1} \underline{\nu}_{k+1}^T \right) \right] \tag{12}$$

From the equation (12) its more general form is obtained

$$\begin{aligned} \underline{\bar{\nu}}_{k+1}^T \underline{\bar{\nu}}_{k+1} &= M \left[\underline{\nu}_{k+1} \underline{\nu}_{k+1}^T \right] = \\ &= b_{k+1} \underline{h}_{\underline{x}} \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \underline{f}_{\underline{x}} \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \cdot \\ &\underline{\Sigma}_k \underline{f}_{\underline{x}}^T \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \underline{h}_{\underline{x}}^T \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) + \\ &+ \underline{h}_{\underline{x}} \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \cdot \\ &\underline{G} \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \underline{Q}_k \underline{G}^T \left(\underline{\hat{x}}_{k+1}, k+1 \right) \underline{h}_{\underline{x}}^T \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) + \underline{R}_{k+1} \end{aligned} \tag{13}$$

As it is

$$M \left[\underline{\nu}_{k+1} \underline{\nu}_{k+1}^T \right] = \underline{h}_{\underline{x}} \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \underline{\Sigma}_{k+1/k} \underline{h}_{\underline{x}}^T \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) + \underline{R}_{k+1}$$

it follows that

$$\begin{aligned} b_{k+1} \underline{I} = &\\ &\left[\underline{\nu}_{k+1} \underline{\nu}_{k+1}^T - \underline{h}_{\underline{x}} \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \underline{G} \left(\underline{\hat{x}}_{k+1}, k+1 \right) \right. \\ &\left. \underline{Q}_k \underline{G}^T \left(\underline{\hat{x}}_{k+1}, k+1 \right) \underline{h}_{\underline{x}}^T \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) + \underline{R}_{k+1} \right] \\ &\left[\underline{h}_{\underline{x}} \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \underline{f}_{\underline{x}} \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \underline{\Sigma}_k \underline{f}_{\underline{x}}^T \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \underline{h}_{\underline{x}}^T \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \right]^{-1} \end{aligned} \tag{14}$$

$$\begin{aligned} b_{k+1} = &\\ &\frac{1}{m} S_p \left\{ \left[\underline{\nu}_{k+1} \underline{\nu}_{k+1}^T - \underline{h}_{\underline{x}} \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \underline{G} \left(\underline{\hat{x}}_{k+1}, k+1 \right) \cdot \right. \right. \\ &\left. \underline{Q}_k \underline{G}^T \left(\underline{\hat{x}}_{k+1}, k+1 \right) \underline{h}_{\underline{x}}^T \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) + \underline{R}_{k+1} \right] \\ &\left. \left[\underline{h}_{\underline{x}} \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \underline{f}_{\underline{x}} \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \underline{\Sigma}_k \underline{f}_{\underline{x}}^T \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \underline{h}_{\underline{x}}^T \left(\underline{\hat{x}}_{k+1/k}, k+1 \right) \right]^{-1} \right\} \end{aligned} \tag{15}$$

where m is the number of the measured changeable states.

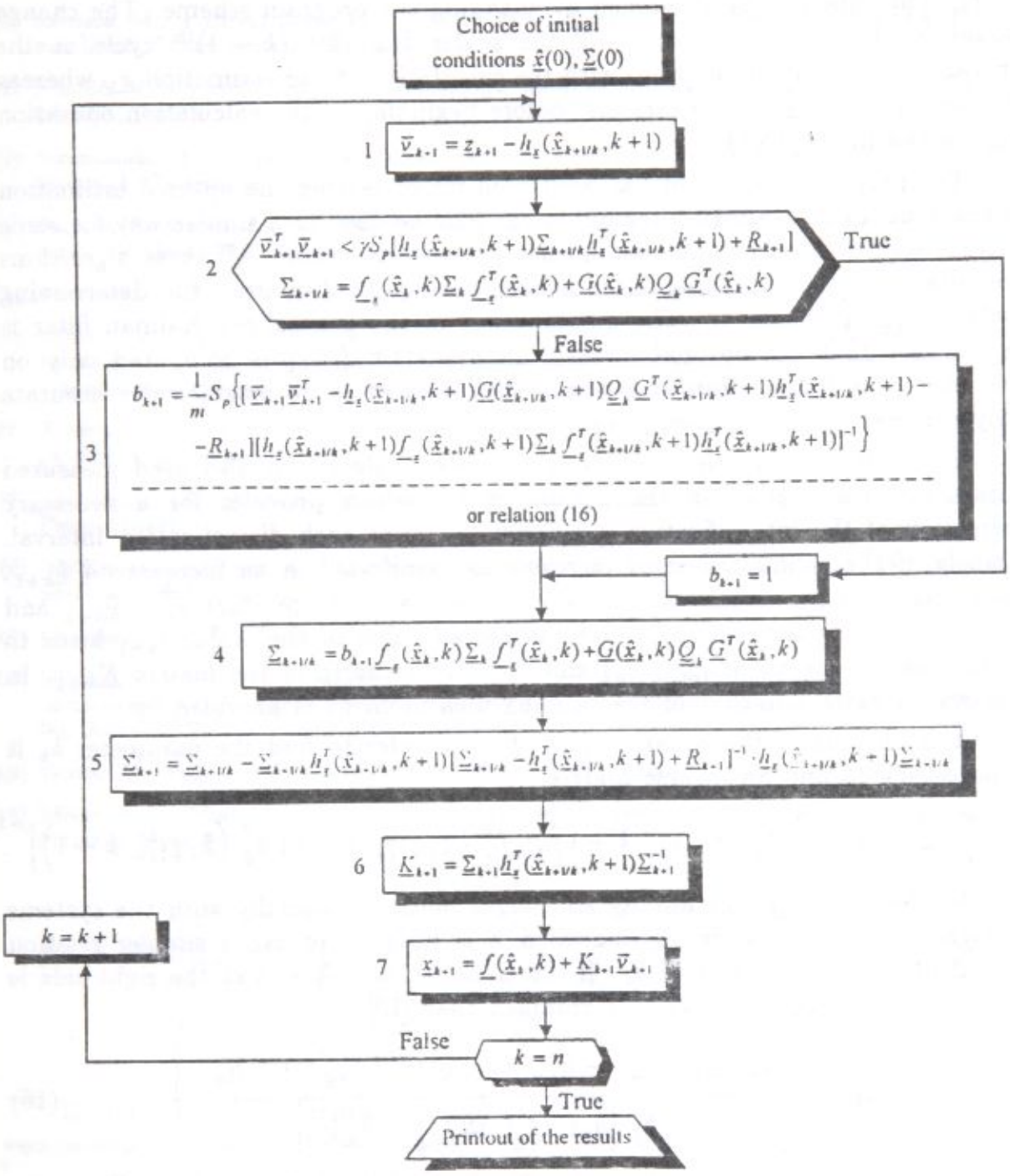

Figure 1. Proposed algorithm for increasing the estimation accuracy

The inverse matrix in the expression (15), that is (14) exists since $\underline{\Sigma}_k$ is a positively determined matrix by definition [2]. The algorithmic scheme for the proposed algorithm for increasing the estimation accuracy is shown in Fig. 1.

It can be seen in the scheme that the estimation $\underline{\hat{x}}_{k+1}$ lags behind the measurement $\underline{z}_{k+1}$ for the time needed to compute the expressions in the blocks (1–7). The time can be shortened by changing the program scheme. The change would involve the computation of the scalar b_{k+1} for $(k+1)^{\text{th}}$ cycle at the previous step k, simultaneously with the calculation of the estimation $\underline{x}_k$ whereas the measurement $\underline{z}_{k+1}$ appears just before beginning of the calculation operation $\underline{\hat{x}}_{k+1}$ given in the block 7.

The proposed algorithm has been used for correcting the optimal estimation of the generalized Kalman filter but it can also be used in a similar way for some other discrete algorithm for non–linear optimal estimation. All these algorithms are adaptive since the calculated scalar can be further used for determining matrix $\underline{\Sigma}_{k+1/k}$ (block 4). The disadvantage of the generalized Kalman filter is in the fact that the optimal intensification matrix $\underline{K}_{k+1}$ is computed only on the basis of a priori parameters and if they happen to be insufficiently accurate they can be a cause of divergence.

In addition to a priori values the proposed algorithm also used measured parameters (indirectly via the parameter b_k) which provides for a necessary correction of the intensification coefficient $\underline{K}_{k+1}$ at each discretization interval. Namely, if the estimation error increases as manifested in an increase od $\bar{\underline{\nu}}_{k+1}$, then this leads to an increase of the value of the scalar product $\bar{\underline{\nu}}^T_{k+1} \cdot \bar{\underline{\nu}}_{k+1}$ and thus to that of the scalar b_{k+1}. The increased value of the scalar b_{k+1} leads to an increase of the norm $|\underline{\Sigma}_{k+1/k}|$ and that of the norm of the matrix $\underline{K}_{k+1}$. In this way a greater impact of the ongoing measurement is provided for.

It is obvious in the equation (15) that in order to find the parameter b_k it is necessary to find an inverse matrix

$$\left[\underline{h}_{\underline{x}}\left(\underline{\hat{x}}_{k+1/k}, k+1\right) \underline{f}_{\underline{x}}\left(\underline{\hat{x}}_{k+1/k}, k+1\right) \underline{\Sigma}_k \underline{f}^T_{\underline{x}}\left(\underline{\hat{x}}_{k+1/k}, k+1\right) \underline{h}^T_{\underline{x}}\left(\underline{\hat{x}}_{k+1/k}, k+1\right)\right]^{-1}$$

As the procedure for finding an inverse matrix, especially with the systems of high order is not a simple operation it is possible to use a simpler relation for calculating the parameter b_k. If the trace of the left and of the right side is found in the expression (14) it is obtained that [10]

$$b_{k+1} = \frac{S_p\left[\underline{\nu}_{k+1}\underline{\nu}^T_{k+1} - \underline{h}_{\underline{x}}(\cdot)\underline{G}(:)\underline{Q}_k\underline{G}^T(:)\underline{h}^T_{\underline{x}}(\cdot) + \underline{R}_{k+1}\right]}{S_p\left[\underline{h}_{\underline{x}}(\cdot)\underline{f}_{\underline{x}}(\cdot)\underline{\Sigma}_k\underline{f}^T_{\underline{x}}(\cdot)\underline{h}^T_{\underline{x}}(\cdot)\right]} \tag{16}$$

where $(\cdot)$ and $(:)$ means $\left(\underline{\hat{x}}_{k+1/k}, k+1\right)$ and $\left(\underline{\hat{x}}_{k+1}, k+1\right)$, respectively.

This approximate expression can be used as an alternative to the expresion (15) and the authors suggest its exclusive use since it is much simpler than the expression (15) whereas at the same time it gives almost the same results.

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ПРЕДЛОЖЕНИЕ ОДНОГО АЛГОРИФМА ДЛЯ УВЕЛИЧЕНИЯ ТОЧНОСТИ ОПТИМАЛЪНОЙ ОЦЕНКИ РАШИРЕННОГО ДИСКРЕТНОГО ФИЛЪТРА КАЛМАНА ДЛЯ НЕЛИНЕЙНИЕ СИСТЕМИ

В работе на основании знакомых критериев и результатов для увеличения точности оптималной оценки вектора состояния раширенного дискретного фильтра Калмана для нелинейных систем [1], [2], [3] и [5] предложение одного адаптивный алгорифм. Этот алгорифм подходит от предложения что ковариационая матрица шума наблюдения адаптивно уточнаеця в процесе реализации вычислительной процедури. Таким образом предложений алгорифм для увеличения точности оптимальной или псевдо–оптимальной оценки можно пользоваться и для остальных дискретных алгорифмов оценки вектора состояния нелинейних систем [2] и [4].

PREDLOG JEDNOG ALGORITMA ZA POVEĆANJE TAČNOSTI OPTIMALNE OCENE UOPŠTENOG DISKRETNOG FILTRA KALMANA ZA NELINEARNE SISTEME

U radu je na osnovu poznatih kriterijuma i rezultata za povećanje tačnosti optimalne ocene veličina stanja uopštenog diskretnog filtra Kalmana za nelinearne sisteme [1], [2], [3] i [5], predložen jedan adaptivni algoritam. Ovaj algoritam polazi od pretpostavke da se kovarijaciona matrica mernog poremećaja adaptivno podešava u toku realizacije računske procedure. Ovako predloženi algoritam za povećanje tačnosti optimalne ili pseudo optimalne ocene može se koristiti i kod drugih algoritama za diskretnu ocenu veličina stanja kod nelinearnih sistema [2] i [4].

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