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EXPERIMENTAL STUDY OF A NON–NEWTONIAN FLOW

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1. Introduction

Hyperconcentrated water-clay mixtures exhibit non-Newtonian behaviour, and in order to describe the flow characteristics of such fluids, rheological relationships need to be established. In the present paper such a relationship is presented for water-clay mixtures in terms of volumetric concentration of the solid phase. The well-known Bingham model can be used to describe these fluids, and the two parameters of the model – the yield stress and the plastic viscosity, are to be determined by laboratory measurements. These parameters can be subsequently used to calculate the friction coefficients for the pressurized and free-surface one-dimensional flows. The method of calculation will be illustrated by numerical examples, using the measured rheological characteristics of the analysed water-clay mixtures.

2. Rheological characteristics of investigated fluids

The considered fluids are water-kaolinite clay mixtures with different concentration of the solid particles with the mean diameter of 0.006 mm. The specific density of the clay material is 2.65, and its chemical composition: S_iO_2 (≈ 50 %), Al_2O_3 (37 %), CaO (< 5 %) and Fe_2O_3(< 3 %). Nine liquid-solid mixtures have been investigated, whose density (ρ_m), and concentrations (by volume C_v, and by weight C_w) are presented in Table 1.

C_v (%)	2.0	4.0	6.2	8.6	11.2	13.9	16.9	20.1	23.6
C_w (%)	5	10	15	20	25	30	35	40	45
ρ_m (kg/m^3)	1032	1066	1103	1142	1184	1230	1279	1332	1389

Table 1. Basic physical characteristics of investigated mixtures

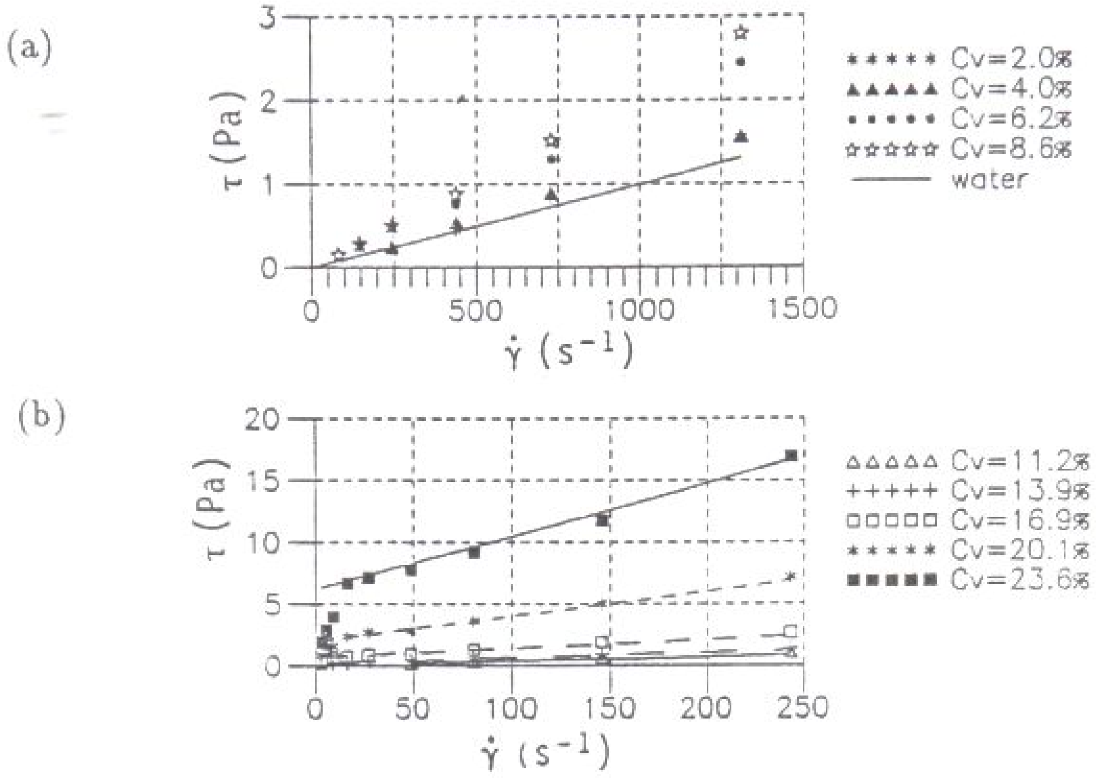

Fig. 1 Rheological relationships for water-clay mixtures:
(a) volumetric concentrations up to 8.6%
(b) volumetric concentrations above 11.2%

The rheological properties have been determined by the coaxial cylinder rotary viscometer, a Couette flow type of device, used to measure torque which resists rotation at a known angular speed [3], [13]. The rheological diagrams depicted in Fig. 1 show the relationship between the applied shear rate, specified by the angular speed of the viscometer, and the shear stress, determined from the recorded torque[1]. The shear stress (τ) and the shear rate ($\dot{\gamma}$) are obtained as [13]:

$$\tau = \frac{M}{2r_1^2\pi h} \tag{1}$$

$$\dot{\gamma} = \frac{2\Omega}{1-(r_1/r_2)^2} \tag{2}$$

where M and Ω are the moment at the rotary cylinder and its angular velocity, while r_1, r_2, and h, are the radii of the inner (rotary) and the outer (stationary) cylinder, and the height of the inner cylinder, respectively. The given relationships are of the "integral type", which means that are valid for a small control volume – annular gap of the viscometer, and are for this reason, called "pseudo-shear" diagrams [13].

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[1] The measurements have been obtained by a commercial "Rheotest 2" rotary viscometer in the Laboratory for Physical Chemistry and Electro-Chemistry, Faculty for Technology and Mining, University of Belgrade.

3. Rheological model of investigated fluids

Considering the theoretical principles of rheological modelling and the results of laboratory experiments, it has been decided to describe the investigated water-clay mixtures by the simple Bingham model [2], [10], which gives a linear shear stress – shear rate relationship:

$$\tau = \tau_c + \eta\dot{\gamma} \tag{3}$$

In the above equation, the two parameters – the yield stress τ_c, and the plastic viscosity η, are to be determined experimentally. These parameters are dependent on the concentration of the solid particles in water.

The empirical relationships in Fig. 1-(a) are linear, without an apparent yield stress. This means that the water-clay mixtures with volumetric concentrations of up to 8.6% can be considered as Newtonian fluids. A closer inspection of laboratory results pertaining to mixtures with concentrations above 11.2%, Fig. 1-(b), reveals a nonlinear behaviour in the zone of small shear rates. However, if this narrow zone of nonlinearity is neglected, the given relationships can be considered linear, and the Bingham model can be assumed to be valid. The slope of each linear relationship represents the plastic viscosity of the given mixture (η), while the intercept on the ordinate is its yield stress (τ_c). Both values are concentration dependent (Table 2):

Concentration C_v (%)	11.2	13.9	16.9	20.1	23.6
Yield stress τ_c (Pa)	0.10	0.20	0.69	1.96	6.19
Plastic viscosity η (mPas)	3.30	6.90	12.10	20.10	43.00

Table 2. Parameters of the Bingham model for the investigated mixtures

These results, analytically represented by exponential curves:

$$\eta = 0.621\exp(0.173C_v) \quad \tau_c = 0.002\exp(0.342C_v) \tag{4}$$

are shown in Fig. 2.

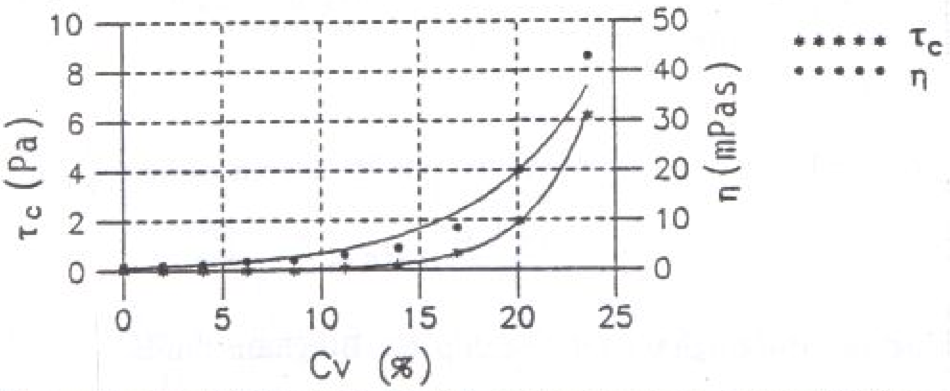

Fig. 2 Dependence of Bingham model parameters on concentration of solid particles

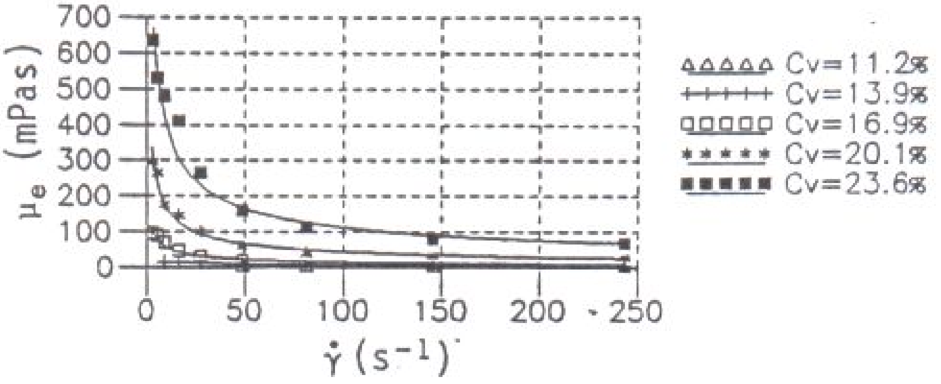

Fig. 3 Dependence of effective viscosity of the mixtures on the rate of shear [6]

The coefficient of effective viscosity ($\mu_e = \tau/\dot{\gamma}$) can be calculated from the experimental data. Its dependence on the rate of shear, i.e. concentration of solid particles, is shown in Fig. 3.

4. Pressurized flow analysis

The flow of hyperconcentrated water-clay mixtures is laminar, and the estimation of the friction coefficient

$$f_1 = \frac{\tau}{\frac{1}{2}\rho_m V^2} \tag{5}$$

is crucial for practical flow computations. A few basic relationships will be recalled from the abundant literature that exists on the subject (for instance [5], [11], [12]), before showing how the results of the specific investigations can be used.

The friction coefficient of Bingham fluids (f_1) is generally dependent on the following values:

$$f_1 = \phi(D, V, \rho_m, \eta, \tau_c) \tag{6}$$

where D is the pipe diameter, V the mean cross-sectional velocity, and ρ_m, τ_c, η, are the already defined properties of the Bingham fluid. The function (6) can be expressed in a dimensionless form:

$$f_1 = \Phi(Re_1, He) \tag{7}$$

where Re_1 and He, are the Reynolds and Hedström numbers [5]:

$$Re_1 = \frac{\rho_m V D}{\eta} \quad He = \frac{\rho_m \tau_c D^2}{\eta^2} \tag{8}$$

Using the Buckingham relationship for Bingham fluids:

$$\frac{8V}{D} = \frac{\tau}{\eta}\left[1 - \frac{4}{3}\left(\frac{\tau_c}{\tau}\right) + \frac{1}{3}\left(\frac{\tau_c}{\tau}\right)^4\right], \tag{9}$$

the relationship (7) can be explicitly defined in the following manner [13]:

$$\frac{1}{Re_1} = \frac{f_1}{16} - \frac{1}{6}\frac{He}{Re_1^2} + \frac{1}{3}\frac{He^4}{f_1^3 Re^8} \tag{10}$$

The criterion for the laminar-turbulent flow transition of the Bingham fluids is the value of the "critical Reynolds number" (Re_{1c}), which can be determined from the diagram in Fig. 4.

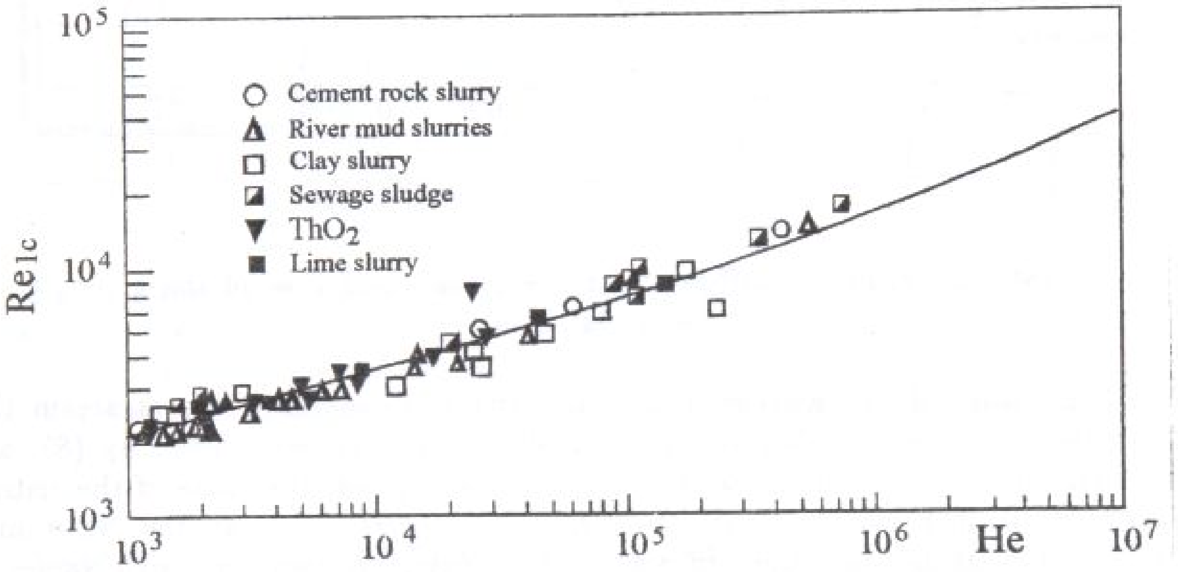

Fig. 4. The critical Reynolds numbers for Bingham fluids [4].

An alternative way for estimation of the friction coefficient would be to use the Moody diagram which is widely used in hydraulic engineering for Newtonian flow analysis [13]. In this case, the Reynolds number Re_2 needs to be expressed in terms of the effective viscosity μ_e:

$$Re_2 = \frac{\rho_m V D}{\mu_e} \tag{11}$$

$$\mu_e = \eta + \frac{\tau_c D}{6V} \tag{12}$$

If this approach is used, the friction coefficient for the laminar flow can be calculated from the well-known simple relationship

$$f_2 = 16/Re_2 \tag{13}$$

However, slightly different values would be obtained from those resulting from iterative solution of equation (10), because it is assumed that the last, fourth power term in the relationship (9) can be neglected. The exact and the approximate methods of solution are illustrated by the following numerical example.

Numerical example. The friction coefficient for the homogeneous water-clay mixture flow in a straight pipe is to be determined for the following characteristics of the mixture: density ρ_m=1389 kg/m^3, volumetric concentration C_v=23.6 %, and the parameters of the Bingham model τ_c=6.19 Pa, η=0.043 Pas (Tables 1 and 2). The pipe diameter is D=200 mm. The results are presented in Table 3.

V (m/s)	Re_1 (-)	f_1 (-) eq.(10)	μ_e (Pas) eq.(12)	Re_2 (-)	f_2 (-) eq.(13)	$\Delta = \frac{\lvert f_1 - f_2 \rvert}{f_1}$ (%)
0.50	3230	0.0280	0.456	305	0.0525	87.5
1.00	6460	0.0135	0.249	1114	0.0144	6.7
1.35	8721	0.0080	0.196	1915	0.0083	3.7

Table 3. Friction coefficient for the pressurized flow of the given water-clay mixture

The values of the Hedström and the critical Reynolds numbers are in this particular case He=186000, and $Re_{1c} \approx 9000$, the first determined by (8), and the latter from Fig. 3. If the second approach is applied, the value of the critical Reynolds number would be about 2100, which corresponds to the Newtonian fluids. Comparing the values of the critical Reynolds numbers with values of Re_1 and Re_2 in Table 3, the assumption of the laminar flow is verified. Notice that the relative difference (Δ) in calculated friction coefficients according to the two methods is extremely high for small velocities, when the exclusion of the fourth power term in equation (9), applied in the second approach, is obviously not justified.

5. Free-surface flow analysis

Contrary to the pipeline flow, the literature on the free-surface non-Newtonian flow is relatively scarce, and for this reason an original laboratory installation has been set up at the Hydraulic Laboratory of the Faculty of Civil Engineering in Belgrade, in order to investigate the free-surface steady and unsteady flow of the water-clay mixtures [7]. The experiments have been carried out in a 4.5 m long, 0.15 m wide laboratory flume with adjustable bottom slope (Fig. 5). The steady circulation of investigated mixtures has been ensured by a sludge pump. The concentration of any particular mixture has been constant throughout the experiment. Two kinds of electrical probes have been used for continuous measurement of water depth and velocity in the flume[2], and a system for automatic data acquisition and processing.

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[2] The membrane probes "DRUCK" have been used for water depth measurement, and the electromagnetic probes "UECM 200" for point velocity measurement.

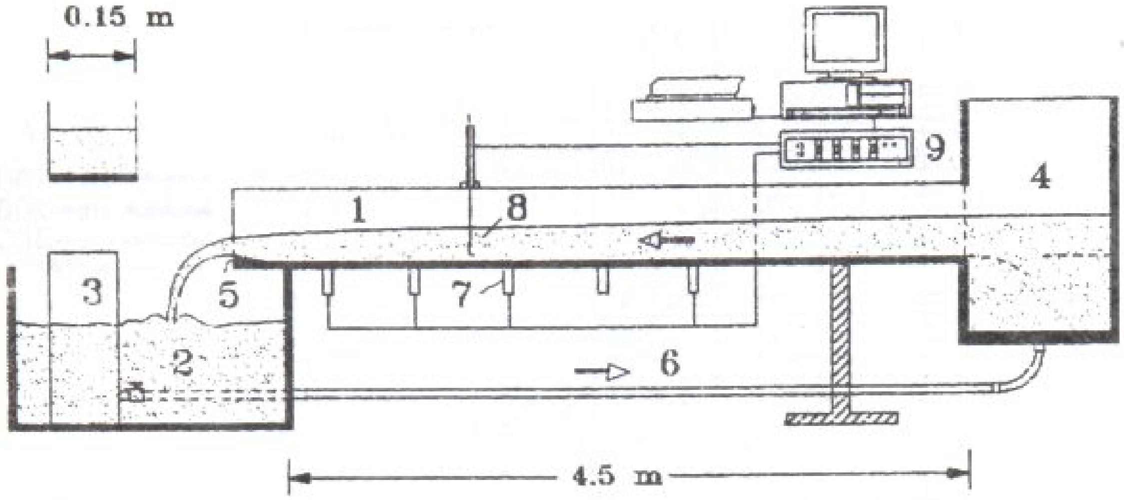

Fig. 5 Laboratory flume and measuring equipment: (1) flume (2) lower tank for preparation of the mixture (3) pump (4) upper tank (5) control weir (6) fluid supply rubber tube (7) depth-measurement probes (8) velocity measurement probe (9) system for data acquisition and processing

It can be shown by dimensional analysis that the friction coefficient for the steady, uniform, laminar flow of Bingham fluids depends (as in the case of the closed-conduit flow) on two dimensionless parameters:

$$Re_1 = \frac{\rho_m V h}{\eta} \quad He = \frac{\rho_m \tau_c h^2}{\eta^2} \tag{14}$$

where h is the flow depth, and V is the mean cross-sectional velocity of the channel flow. Using the expression for the free-surface flow similar to the Buckingham's formula (9):

$$\frac{3V}{h} = \frac{\tau}{\eta}\left[1 - \frac{3}{2}\left(\frac{\tau_c}{\tau}\right) + \frac{1}{2}\left(\frac{\tau_c}{\tau}\right)^3\right], \tag{15}$$

a relationship between the friction coefficient and the Reynolds and Hedström numbers can be derived:

$$\frac{1}{Re_1} = \frac{f_1}{6} - \frac{1}{2}\frac{He}{Re_1^2} + \frac{2}{3}\frac{He^3}{f_1^2 Re_1^6} \tag{16}$$

Fig. 6 depicts graphically this relationship for two investigated water-clay mixtures.

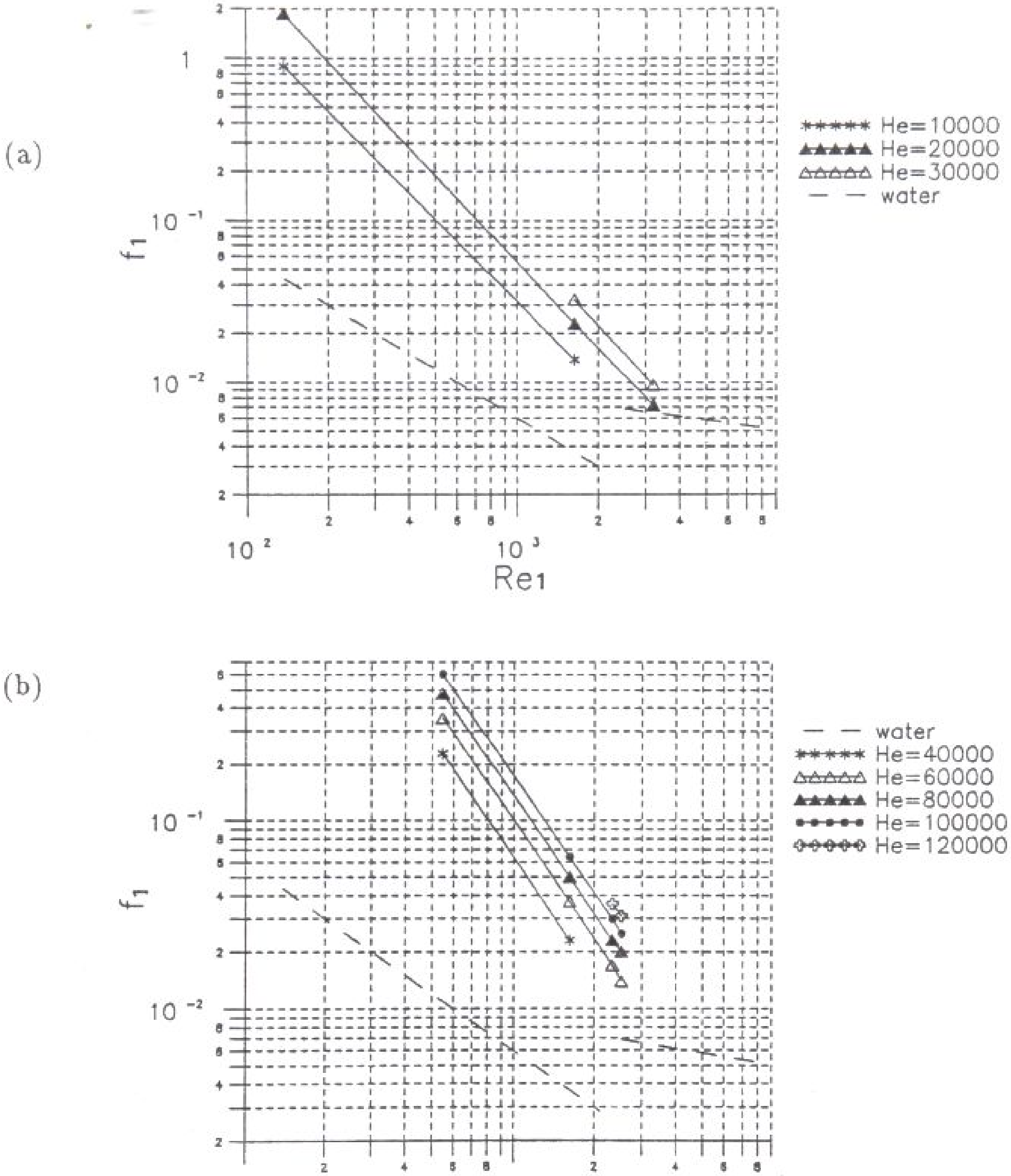

Fig. 6 Friction coefficient for heavily concentrated water-clay mixtures: (a) C_v=13.9%, (b) C_v=20.1%

The critical Reynolds number can be expressed in terms of the Hedström number according to the method presented in [13]:

$$Re_{1c} = 1050\left(1+\sqrt{1+\frac{He}{1050}}\right) \tag{17}$$

As in the case of the pressurized flow, an alternative method of friction coefficient estimation can be used, based on the standard procedure for Newtonian fluids, whereby the Reynolds number is expressed in terms of the effective viscosity:

$$Re_2 = \frac{\rho_m V h}{\mu_e} \tag{18}$$

$$\mu_e = \eta + \frac{\tau_c h}{2V} \tag{19}$$

For the laminar flow:

$$f_2 = \frac{6}{Re_2} \tag{20}$$

The relationship (20) for the considered water-clay mixtures is shown in Fig. 7.

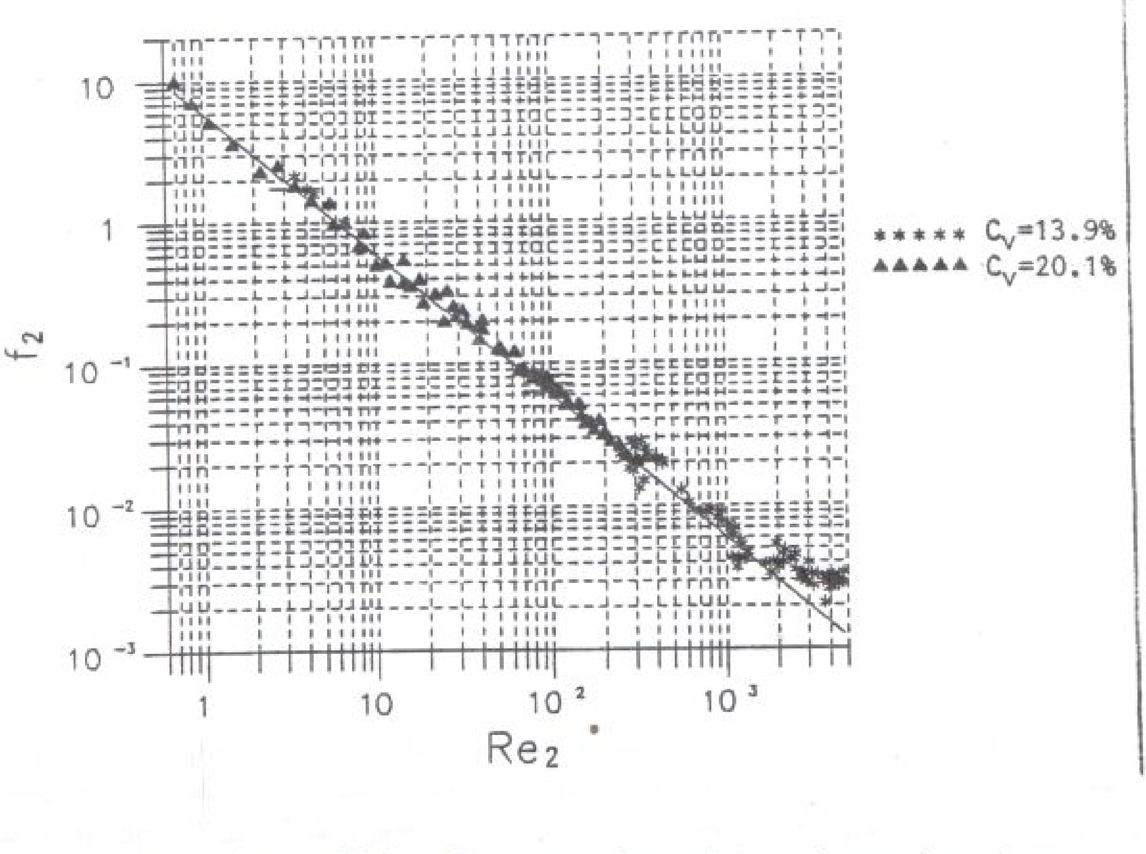

Fig. 7 Friction coefficient for water-clay mixture free-surface flow

Numerical example. Using the experimental data, estimate the friction coefficient for the free-surface flow of the water-clay mixture with the following characteristics: ρ_m=1230 kg/m^3, C_v=13.9 %, τ_c=0.2 Pa, and η=6.9 mPas (Tables 1 and 2). The bottom slope of the flume in this particular experiment was 0.1%. The results of calculation are presented in Table 4.

Q (l/s)	V (m/s)	h (m)	Re_1 (-) eq.(14)	He (-) eq.(14)	Re_c (-) eq.(17)	f_1 (-) eq.(16)
2.7	0.24	0.074	3157	28142	6586	0.0093
2.7	0.31	0.059	3244	17804	5499	0.0065
5.1	0.48	0.071	6118	26415	6420	0.0029

Table 4. Friction coefficient for the free-surface flow of the given water-clay mixture

μ_e (Pas) eq.(19)	Re_2 (-) eq.(18)	f_2 (-) eq.(20)	$\Delta = \frac{\lvert f_1 - f_2 \rvert}{f_1}$ (%)
0.0376	579	0.0103	11.5
0.0258	866	0.0069	6.5
0.0218	1937	0.0031	6.8

Table 4. (continued)

6. Conclusion

The water-clay mixtures exhibit non-Newtonian behaviour if the concentration of the solid phase is sufficiently high – for the analysed kaolinite clay suspensions, at the volumetric concentrations over about 10%. The rotary viscometer measurements have been used to define the viscosity-concentration correlations for the specific mixtures, and the obtained results indicated that the Bingham model can be used to describe such fluids. The two parameters of this model, the yield stress and the plastic viscosity, can be estimated by laboratory measurements in terms of the concentration of solid particles, and subsequently used to calculate the friction coefficient for the closed-conduit (pressurized), and free-surface laminar flows. The specific data obtained for the analysed mixtures, are presented and used in two numerical examples.

REFERENCES

[1] Chong, J.S., Christiansen, E.B., Bear, A.D., *Rheology of Concentrated Suspensions*, Journal of Applied Polymer Science, **15** (1971).

[2] Churchill, S.W., *Viscous Flows - The Practical Use of Theory*, Butteworths, (1988).

[3] HAAKE Viscometers - Introduction to Practical Viscometry, Karlsruhe, (1981).

[4] Hanks, R.W., Pratt, D.R., *On the Flow of Bingham Plastic Slurries in Pipes and Between Paralel Plates*, Journal of Society of Petroleum Engineers, Dec. (1967).

[5] Hedström, B.O.A., *Flow of Plastic Materials in Pipes*, Industrial and Engineering Chemistry, **44**, No.3, (1952).

[6] Jovanović, M., Komatina, D., *On Non-Newtonian Flow: Theoretical Considerations and Results of Laboratory Investigations*, Current Problems of Sanitary Engineering, Faculty of Civil Engineering publication, Belgrade, (1993) (in Serbian).

[7] Komatina, D., *Open-Channel Flow of Hyperconcentrated Mixtures*, Master of science dissertation, Faculty of Civil Engineering, Belgrade, (1993) (in Serbian).

[8] O'Brien, J.S., Julien, P.Y., *Laboratory Analysis of Mudflow Properties*, Journal of Hydraulic Engineering, ASCE, **114**, Aug. (1988).

[9] Ogihara, Y., Miyazawa, N., *Laws of Resistance of Pipe Flow of Solid-Liquid Mixtures*, Journal of IAHR, 29, No. 2 (1991).

[10] Skelland, A., *Non-Newtonian Flow and Heat Transfer*, John Wiley, New York (1967).

[11] Task Committee *Sediment Transportation Mechanics: Transportation of Sediment in Pipes*, Journal of the Hydraulics Division, ASCE, July (1970).

[12] Thomas, D.G.,*Progress in International Research on Thermodynamic and Transport Properties*, ASME, Chapter 61, New York (1962).

[13] Wasp, E.J., Kenny, J.P., Gandhi, R.L., *Solid-Liquid Flow Slurry Pipline Transportation*, Gulf Publishing Co., Houston (1978).

ЕКСПЕРИМЕНТАЛЬНОЕ ИЗУЧЕНИЕ ПОТОКА ОДНОГО НЕНЬЮТОНОВСКОГО ФЛУИДА

В настоящей статье приводятся результаты лабораторных исследований глинистых гидросмесей. На основании измерений, проводимых на ротационном вискозиметре, замечается что смеси с высокой концентрацией твёрдых частиц проявляют свойства неньютоновских флуидов. Рассматриваются возможности описания таких смесей известным реологическим моделом Бингама. В статье определяются параметры модели (напряжение при начальном сдвиге и структурная вязкость) и устанавливаются зависимости параметров от концентрации твёрдой фазы. После этого приводятся методы определения коеффициента сопротивления исследованых смесей в напорном и безнапорном потоках. В конце, данные приёмы проиллюстрированны численными примерами.

EKSPERIMENTALNO ISPITIVANJE JEDNOG NENJUTNOVSKOG FLUIDA

U ovom radu su prikazani rezultati laboratorijskih ispitivanja mešavina vode i kaolinske gline, pri različitim koncentracijama čvrstih čestica. Utvrđeno je da ove mešavine pokazuju svojstva nenjutnovskih fluida pri zapreminskim koncentracijama iznad 10%. Viskoznost ispitanih mešavina određena je pomoću rotacionog viskozimetra i dobijeni rezultati su ukazali da se na date mešavine može primeniti Bingham-ov reološki model. Parametri ovog modela – granična nosivost i plastična viskoznost, utvrđeni su u funkciji koncentracije čvrste faze, a zatim su korišćeni za određivanje koeficijenata trenja za tečenje pod pritiskom i tečenje sa slobodnom površinom. Postupak je ilustrovan sa dva brojna primera.

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