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A MONOTONY METHOD IN QUASISTATIC RATE–TYPE VISCOPLASTICITY

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1. Introduction

Let Ω be a bounded domain in $\mathbb{R}^N$ $(N = 1,2,3)$ with a smooth boundary $\partial\Omega = \Gamma$ and let Γ_1 be an open subset of Γ. We denote by $\Gamma_2 = \Gamma - \overline{\Gamma}_1$, ν the outward unit normal vector on Γ and by S_N the set of second order symmetric tensor on $\mathbb{R}^N$. Let T be a real positive constant. We suppose *meas* $\Gamma_1 > 0$. Let us consider the following mixed problem: find the displacement function u: $\Omega \times [0,T] \rightarrow \mathbb{R}^N$ and the stress function σ: $\Omega \times [0,T] \rightarrow S_N$ such that:

$$\dot{\sigma} = \mathcal{E}(\varepsilon(\dot{u})) + F(\sigma, \varepsilon(u)) \quad in \quad \Omega \times (0,T) \tag{1.1}$$

$$Div\,\sigma + f = 0 \quad in \quad \Omega \times (0,T) \tag{1.2}$$

$$u = g \quad on \quad \Gamma_1 \times (0,T) \tag{1.3}$$

$$\sigma\nu = h \quad on \quad \Gamma_2 \times (0,T) \tag{1.4}$$

$$u(0) = u_0, \quad \sigma(0) = \sigma_0, \quad in \quad \Omega. \tag{1.5}$$

This problem represents a quasistatic problem for rate–type viscoplastic models of the form (1.1) where $\mathcal{E}$ is a nonlinear function, $\varepsilon(u) : \Omega \times [0,T] \rightarrow S_N$ is the small strain tensor (i.e. $\varepsilon(u) = \frac{1}{2}(\nabla u + \nabla^t u)$. In (1.1) $\mathcal{E}$ and F are given constitutive functions and, as well as everywhere in this paper, the dot above a quantity represents the derivate with respect to the time variable of that quantity. The equation (1.2) is the equilibrium equation in which f: $\Omega \times [0,T] \rightarrow \mathbb{R}^N$ is the given body force and $Div\,\sigma$ represents the divergence of vector–valued function σ; finally the functions g and h in (1.3), (1.4) are the given boundary data and the functions u_0, σ_0 in (1.5) are the initial data.

In the case when $\mathcal{E}$ is a linear function, existence and uniqueness results for problems of the form (1.1)–(1.5) were obtained by Duvaut and Lions [1], Djaoua and Suquet [2], Suquet [3], [4], Ionescu and Sofonea [5], Djabi and Sofonea [6] using different functional methods.

The purpose of this paper is to prove the existence and uniqueness of the solution for the problem (1.1)–(1.5) in the case when $\mathcal{E}$ is a nonlinear function using monotony arguments followed by a Cauchy–Lipschitz technique (theorem 3.1).

2. Notations and preliminaries

Everywhere in this paper we utilise the following notations:

"." – the inner product on the spaces $\mathbb{R}^N$ and S_N,

$|\cdot|$ – the Euclidean norms on $\mathbb{R}^N$ and S_N,

$$H = \left\{ v = (v_i) \,|\, v_i \in L^2(\Omega), \quad i = \overline{1,N} \right\},$$
$$H_1 = \left\{ v = (v_i) \,|\, v_i \in H^1(\Omega), \quad i = \overline{1,N} \right\},$$
$$\mathcal{H} = \left\{ \tau = (\tau_i) \,|\, \tau_i \in L^2(\Omega), \quad i = \overline{1,N} \right\},$$
$$\mathcal{H}_1 = \left\{ \tau = (\tau_i) \,|\, Div\,\tau \in H \right\}.$$

The spaces H, H_1, $\mathcal{H}$ and $\mathcal{H}_1$ are real Hilbert spaces with the canonical inner products denoted by $<\cdot,\cdot>_H$, $<\cdot,\cdot>_{H_1}$, $<\cdot,\cdot>_{\mathcal{H}}$ and $<\cdot,\cdot>_{\mathcal{H}_1}$ respectively.

Let $H_\Gamma = \left[H^{\frac{1}{2}}(\Gamma) \right]^N$ and $\gamma{:}H_1 \to H_\Gamma$ be the trace map. We denote by

$$V = \{ u \in H_1 \,|\, \gamma u = 0, \quad \text{on}\, \Gamma_1 \}$$

and

$$E = \gamma(V) = \{ \xi \in H_\Gamma \,|\, \gamma u = 0, \quad \text{on}\, \Gamma_1 \}.$$

The deformation operator $\varepsilon : H_1 \to \mathcal{H}$ definite above is linear and continuous. Moreover, since $meas\ \Gamma_1 > 0$, Korn's inequality holds:

$$|\varepsilon(v)|_{\mathcal{H}} \geq C |v|_{H_1} \quad \text{for all} \quad v \in V, \tag{2.1}$$

where C is a strictly positive constant wish depends only on Ω and Γ_1. Let $H'_\Gamma = \left[H^{\frac{1}{2}}(\Gamma) \right]^N$ be the strong dual of the space H_Γ and let $<\cdot,\cdot>$ denote the duality between H'_Γ and H_Γ. If $\tau \in \mathcal{H}_1$ there exists an element $\gamma_\nu \tau \in H'_\Gamma$ such that

$$< \gamma_\nu \tau, \gamma v > = < \tau, \varepsilon(v) >_{\mathcal{H}} + < Div\,\tau, v >_H \quad \text{for all} \quad v \in H_1. \tag{2.2}$$

By $\tau\nu\big|_{\Gamma_2}$ we shall understand the element of E' (the strong dual of E) that is the restriction of $\gamma_\nu \tau$ on E.

Let us now denote by $\mathcal{V}$ the following subspace of $\mathcal{H}_1$:

$$\mathcal{V} = \{ \tau \in \mathcal{H}_1 \,|\, Div\,\tau = 0 \,\text{in}\, \Omega, \quad \tau\nu = 0 \quad \text{on}\, \Gamma_2 \}.$$

Using (2.2) it may be proved that $\varepsilon(V)$ is the orthogonal complement of $\mathcal{V}$ in $\mathcal{H}$, hence

$$< \tau, \varepsilon(v) >_{\mathcal{H}} = 0, \quad \text{for all}\, v \in V, \quad \tau \in \mathcal{V}. \tag{2.3}$$

Finally, for every real Hilbert space X, we denote by $|\cdot|_X$ the norm on X and by $C^j(0,T,X)$ $(j=0,1)$ the spaces

$$C^0(0,T,X) = \{z : [0,T] \to X \mid z \quad \text{is continuous}\},$$

$$C^1(0,T,X) = $$
$$\{z : [0,T] \to X \mid \text{there exists } \dot{z} \text{ the derivate of } z \text{ and } \quad \dot{z} \in C^0(0,T,X)\}.$$

$C^j(0,T,X)$ are real Banach spaces endowed with the norms

$$|z|_{0,T,X} = \max_{t\in[0,T]} |z(t)|_X \tag{2.4}$$

and

$$|z|_{1,T,X} = |z|_{0,T,X} + |\dot{z}|_{0,T,X}$$

respectively.

Let us also recall that if K is a convex closed non empty set of X and $P : X \to K$ is the projector map on K, we have

$$y = Px \iff y \in K \quad \text{and} \quad < y - x,\, z - y >_X \geq 0 \quad \text{for all} \quad z \in K. \tag{2.5}$$

3. An existence and uniqueness result

In the study of the problem (1.1)–(1.5) we consider the following assumptions:

$$\left.\begin{array}{l}
\mathcal{E} : \Omega \times S_N \to S_N \quad \text{and} \\
(a)\ \text{there exist} \quad m > 0 \quad \text{such that} \\
\qquad < \mathcal{E}(\varepsilon_1) - \mathcal{E}(\varepsilon_2),\, \varepsilon_1 - \varepsilon_2 >\geq m|\varepsilon_1 - \varepsilon_2|^2 \\
\qquad \text{for all} \quad \varepsilon_1, \varepsilon_2 \in S_N, \quad a.e. \text{ in } \Omega \\
\\
(b)\ \text{there exist} \quad L^{'} > 0 \quad \text{such that} \\
\qquad |\mathcal{E}(\varepsilon_1) - \mathcal{E}(\varepsilon_2)| \geq L^{'}|\varepsilon_1 - \varepsilon_2|^2 \\
\qquad \text{for all} \quad \varepsilon_1, \varepsilon_2 \in S_N, \quad a.e. \text{ in } \Omega \\
(c)\ x \to \mathcal{E}(x,\varepsilon) \quad \text{is a mesurable function with respect to} \\
\qquad \text{the Lebesgue measure on } \Omega, \quad \text{for all} \quad \varepsilon \in S_N \\
\\
(d)\ x \to \quad \mathcal{E}(x,\varepsilon) \in \mathcal{H}
\end{array}\right\} \tag{3.1}$$

$$\left.\begin{array}{l} F:\Omega\times S_N\times S_N\to S_N \quad \text{and} \\ \\ (a)\,\text{there exists} \quad L>0 \quad \text{such that} \\ \quad |F(x,\sigma_1,\varepsilon_1)-F(x,\sigma_2,\varepsilon_2)|\le L(|\sigma_1-\sigma_2|+|\varepsilon_1-\varepsilon_2|) \\ \quad \text{for all} \quad \sigma_1,\sigma_2,\varepsilon_1,\varepsilon_2\in S_N, \quad \text{a.e. in} \in \Omega \\ \\ (b)\,x\to F(x,\sigma,\varepsilon) \quad \text{is a mesurable function with respect to} \\ \quad \text{the Lebesgue measure on}\,\Omega, \quad \text{for all} \quad \sigma,\varepsilon\in S_N \\ \\ (c)\,x\to F(x,0,0)\in\mathcal{H} \end{array}\right\} \tag{3.2}$$

$$f\in C^1(0,T,H), \quad g\in C^1(0,T,H_\Gamma), \quad h\in C^1(0,T,E') \tag{3.3}$$

$$u_0\in H_1, \quad \sigma_0\in\mathcal{H}_1 \tag{3.4}$$

$$Div\,\sigma_0+f(0)=0 \quad \text{in}\,\Omega, \quad u_0=g(0) \quad \text{on}\,\Gamma_1, \quad \sigma_0\nu=h(0) \quad \text{on}\,\Gamma_2. \tag{3.5}$$

The main result of this section is the following:

Theorem 3.1. Let (3.1)–(3.5) hold. Then there exists a unique solution

$$u\in C^1(0,T,H_1), \quad \sigma\in C^1(0,T,\mathcal{H}_1)$$

of the problem (1.1)–(1.5).

In order to prove theorem 3.1 we need some preliminaries. Let

$$\tilde{u}\in C^1(0,T,H_1), \quad \tilde{\sigma}\in C^1(0,T,\mathcal{H}_1)$$

be two functions such that:

$$Div\,\tilde{\sigma}+f=0 \quad \text{in} \quad \Omega\times(0,T) \tag{3.6}$$

$$\tilde{u}=g \quad \text{on} \quad \Gamma_1\times(0,T) \tag{3.7}$$

$$\tilde{\sigma}\nu=h \quad \text{on} \quad \Gamma_2\times(0,T) \tag{3.8}$$

(the existence of this couple follows from (3.3) and the proprieties of the trace maps).

Considering the functions defined by

$$\bar{u}=u-\tilde{u}, \quad \bar{\sigma}=\sigma-\tilde{\sigma} \tag{3.9}$$

$$\bar{u}_0=u_0-\tilde{u}(0), \quad \bar{\sigma}_0=\sigma_0-\tilde{\sigma}(0) \tag{3.10}$$

it is easily to see that the pair $(u,\sigma)\in C^1(0,T,H_1\times\mathcal{H}_1)$ is a solution of (1.1)–(1.5) iff $(\bar{u},\bar{\sigma})\in C^1(0,T,V\times\mathcal{V})$ is a solution of the problem

$$\dot{\bar{\sigma}}=\mathcal{E}(\varepsilon(\dot{\bar{u}})+\varepsilon(\dot{\tilde{u}}))+F(\bar{\sigma}+\tilde{\sigma},\,\varepsilon(\bar{u})+\varepsilon(\tilde{u}))-\dot{\tilde{\sigma}} \quad \text{in}\,\Omega\times(0,T) \tag{3.11}$$

$$\bar{u}(0) = \bar{u}_0, \quad \bar{\sigma}(0) = \bar{\sigma}_0 \quad \text{in } \Omega. \tag{3.12}$$

Let $Z = \varepsilon(V) \times \mathcal{V}$; Z is a product Hilbert space endowed with the inner product

$$< z_1, z_2 >_Z = < x_1, x_2 >_{\mathcal{H}} + < x_2, y_2 >_{\mathcal{H}} \quad \forall z_i = (x_i, y_i) \in Z, \quad i = 1, 2. \tag{3.13}$$

The norm on Z will be denoted by $|\cdot|_Z$. We have:

Lemma 3.1. Let $x \in \varepsilon(V)$, $y \in \mathcal{V}$ and $t \in [0, T]$. Then there exists a unique element $z = (\varepsilon(v), \tau) \in Z$ such that:

$$\tau = \mathcal{E}(\varepsilon(v) + \varepsilon(\dot{\bar{u}}(t))) + F(y + \bar{\sigma}(t), x + \varepsilon(\bar{u}(t))) - \dot{\bar{\sigma}}(t).$$

Proof. The uniqueness part is a consequence of (3.1); indeed, if $z_1 = (\varepsilon(v_1), \tau_1)$, $z_2 = (\varepsilon(v_2), \tau_2)$ are such that:

$$\tau_1 = \mathcal{E}(\varepsilon(v_1) + \varepsilon(\dot{\bar{u}}(t))) + F(y + \bar{\sigma}(t), x + \varepsilon(\bar{u}(t))) - \dot{\bar{\sigma}}(t)$$

$$\tau_2 = \mathcal{E}(\varepsilon(v_2) + \varepsilon(\dot{\bar{u}}(t))) + F(y + \bar{\sigma}(t), x + \varepsilon(\bar{u}(t))) - \dot{\bar{\sigma}}(t),$$

using (3.1a) we have:

$$\begin{aligned} &< \tau_1 - \tau_2, \varepsilon(v_1) - \varepsilon(v_2) >_{\mathcal{H}} = \\ &\qquad < \mathcal{E}(\varepsilon(v_1) + \varepsilon(\dot{\bar{u}}(t))) - \mathcal{E}(\varepsilon(v_2) + \varepsilon(\dot{\bar{u}}(t))), \varepsilon(v_1) - \varepsilon(v_2) >_{\mathcal{H}} \\ &\qquad\qquad \geq m |\varepsilon(v_1) - \varepsilon(v_2)|^2_{\mathcal{H}}. \end{aligned}$$

Using now the orthogonality in $\mathcal{H}$ of $(\tau_1 - \tau_2) \in \mathcal{V}$ and $(\varepsilon(v_1) - \varepsilon(v_2)) \in \varepsilon(V)$ (see (2.3)) we deduce $\varepsilon(v_1) = \varepsilon(v_2)$ which implies $\tau_1 = \tau_2$.

For the existence part let us consider the map $G(t, x, y, \cdot) : \varepsilon(V) \to \mathcal{H}$ defined by

$$G(t, x, y, q) = \mathcal{E}(q + \varepsilon(\dot{\bar{u}}(t))) + F(y + \bar{\sigma}(t), x + \varepsilon(\bar{u}(t))) - \dot{\bar{\sigma}}(t) \tag{3.14}$$

and let $S(t, x, y, \cdot) : \varepsilon(V) \to \varepsilon(V)$ be given by $S(t, x, y, \cdot) = PG(t, x, y, \cdot)$ where $P : \mathcal{H} \to \varepsilon(V)$ is the projector map on $\varepsilon(V)$. Using (2.5), (3.1) and (3.2) we get that the operator $S(t, x, y, \cdot) : \varepsilon(V) \to \varepsilon(V)$ is a strongly monotone and Lipschitz operator. Indeed, for all q_1, $q_2 \in \varepsilon(V)$ we get

$$\left.\begin{aligned} &< S(t, x, y, q_1) - S(t, x, y, q_2), q_1 - q_2 >_{\mathcal{H}} = \\ &< G(t, x, y, q_1) - G(t, x, y, q_2), q_1 - q_2 >_{\mathcal{H}} \geq m |q_1 - q_2|^2_{\mathcal{H}} \end{aligned}\right\} \tag{3.15}$$

which implies that $S(t, x, y, \cdot)$ is a strongly monotone operator. Moreover, from (3.1.b) and the proprieties of the projector map, we get:

$$\begin{aligned} &|S(t, x, y, q_1) - S(t, x, y, q_2)|_{\mathcal{H}} \leq \\ &\leq |G(t, x, y, q_1) - G(t, x, y, q_2)|_{\mathcal{H}} \leq \\ &\leq L' |q_1 - q_2|_{\mathcal{H}} \end{aligned} \tag{3.16}$$

hence $S(t,x,y,\cdot)$ is a Lipschitz operator. Using now Browder's surjectivity theorem we get that there exists $\varepsilon(v) \in \varepsilon(V)$ such that $S(t,x,y,\varepsilon(v)) = 0_{\varepsilon(V)}$. It results that the element $G(t,x,y,\varepsilon(v))$ belongs to $\mathcal{V}$ and we finish the proof taking $z = (\varepsilon(v), \tau)$ where

$$\tau = G(t,x,y,\varepsilon(v)) = \mathcal{E}(\varepsilon(v) + \varepsilon(\dot{\tilde{u}}(t))) + F(y + \tilde{\sigma}(t),\, x + \varepsilon(\dot{\tilde{u}}(t))) - \dot{\tilde{\sigma}}(t).$$

The previous lemma allows us to consider the operator $A : [0,T] \times Z \to Z$ defined as follows:

$$A(t,\omega) = z \iff \omega = (x,y),\ z(\varepsilon(v),\tau) \quad \text{and} \quad \tau = G(t,x,y,\varepsilon(t)). \tag{3.17}$$

We have:

Lemma 3.2. The operator $A : [0,T] \times Z \to Z$ is continuous and there exists $C > 0$ which depends on $\mathcal{E}$ and F such that

$$|A(t,\omega_1) - A(t,\omega_2)|_Z \le C\,|\omega_1 - \omega_2|_Z \quad \text{for all} \quad t \in [0,T],\ \omega_1, \omega_2 \in Z. \tag{3.18}$$

Proof. Let $t_i \in [0,T]$, $\omega_i = (x_i, y_i) \in Z$ and $z_i = (\varepsilon(v), \tau_i) = A(t_i, \omega_i)$, $i = 1,2$. Using (3.17) and (3.14) we get

$$\begin{aligned} \tau_i &= G(t,x,y,\varepsilon(v_i)) = \\ &= \mathcal{E}(\varepsilon(v_i) + \varepsilon(\dot{\tilde{u}}(t_i))) + F(y_i + \tilde{\sigma}(t_i),\, x_i + \varepsilon(\dot{\tilde{u}}(t_i))) - \dot{\tilde{\sigma}}(t_i) \quad i = 1,2 \end{aligned} \tag{3.19}$$

which implies

$$S(t_i, x_i, y_i, \varepsilon(v_i)) = 0_{\varepsilon(V)}, \quad i = 1,2. \tag{3.20}$$

From (3.15) and (3.20) we get

$$\begin{aligned} & m|\varepsilon(v_1) - \varepsilon(v_2)|^2_{\mathcal{H}} \le \\ &\le < S(t_1,x_1,y_1,\varepsilon(v_1)) - S(t_1,x_1,y_1,\varepsilon(v_2)),\, \varepsilon(v_1) - \varepsilon(v_2) >_{\mathcal{H}} \le \\ &\le < S(t_2,x_2,y_2,\varepsilon(v_2)) - S(t_1,x_1,y_1,\varepsilon(v_2)),\, \varepsilon(v_1) - \varepsilon(v_2) >_{\mathcal{H}} \le \\ &\le |G(t_2,x_2,y_2,\varepsilon(v_2)) - G(t_1,x_1,y_1,\varepsilon(v_2))|_{\mathcal{H}}\, |\varepsilon(v_1) - \varepsilon(v_2)|_{\mathcal{H}} \end{aligned}$$

which implies

$$|\varepsilon(v_1) - \varepsilon(v_2)|_{\mathcal{H}} \le \frac{1}{m} |G(t_2,x_2,y_2,\varepsilon(v_2)) - G(t_1,x_1,y_1,\varepsilon(v_2))|_{\mathcal{H}} \tag{3.21}$$

Using now (3.19), (3.14) and (3.1.b) we get

$$\left.\begin{aligned} |\tau_1 - \tau_2|_{\mathcal{H}} &= |G(t_1,x_1,y_1,\varepsilon(v_1)) - G(t_2,x_2,y_2,\varepsilon(v_2))|_{\mathcal{H}} \le \\ &\le |G(t_1,x_1,y_1,\varepsilon(v_1)) - G(t_1,x_1,y_1,\varepsilon(v_2))|_{\mathcal{H}} + \\ &+ |G(t_1,x_1,y_1,\varepsilon(v_2)) - G(t_2,x_2,y_2,\varepsilon(v_2))|_{\mathcal{H}} \le \\ \le L'\,|\varepsilon(v_1) - \varepsilon(v_2)|_{\mathcal{H}} &+ |G(t_1,x_1,y_1,\varepsilon(v_2)) - G(t_2,x_2,y_2,\varepsilon(v_2))|_{\mathcal{H}} \end{aligned}\right\} \tag{3.22}$$

hence by (3.21) it results

$$|\tau_1 - \tau_2|_{\mathcal{H}} \leq \left(\frac{L'}{m} + 1\right) |G(t_1, x_1, y_1, \varepsilon(v_2)) - G(t_2, x_2, y_2, \varepsilon(v_2))|_{\mathcal{H}}. \quad (3.23)$$

Using now (3.14) and (3.2.b) we get

$$\left.\begin{aligned} &|G(t_1, x_1, y_1, \varepsilon(v_2)) - G(t_2, x_2, y_2, \varepsilon(v_2))|_{\mathcal{H}} \leq \\ &\leq L\,(|x_1 - x_2|_{\mathcal{H}} + |y_1 - y_2|_{\mathcal{H}}) \\ &+|G(t_1, x_2, y_2, \varepsilon(v_2)) - G(t_2, x_2, y_2, \varepsilon(v_2))|_{\mathcal{H}} \end{aligned}\right\} \quad (3.24)$$

Moreover, by (3.14), (3.1), (3.2), (3.24) and the regularities $\bar{u} \in C^1(0,T,V)$, $\bar{\sigma} \in C^1(0,T,\mathcal{V})$, we get

$$\left.\begin{aligned} &|G(t_1, x_1, y_1, \varepsilon(v_2)) - G(t_2, x_2, y_2, \varepsilon(v_2))|_{\mathcal{H}} \to 0 \\ &\text{when } t_1 \to t_2 \quad \text{in } [0,T], \quad x_1 \to x_2 \quad \text{in } \mathcal{H} \quad \text{and} \quad y_1 \to y_2 \quad \text{in } \mathcal{H}. \end{aligned}\right\} \quad (3.25)$$

Using now (3.21), (3.13) we get

$$|A(t_1, \omega_1) - A(t_2, \omega_2)|_Z \leq \tilde{C}|G(t_1, x_1, y_1, \varepsilon(v_2)) - G(t_2, x_2, y_2, \varepsilon(v_2))|_{\mathcal{H}} \quad (3.26)$$

where $\tilde{C} > 0$, hence by (3.25) we obtain that A is a continuous operator. Taking $t_1 = t_2 = t$ in (3.26) and using (3.24), (3.13) we get (3.18).

Proof of theorem 3.1. Using the definition of the operator A we get that $\bar{u} \in C^1(0,T,\varepsilon(V))$ and $\bar{\sigma} \in C^1(0,T,\mathcal{V})$ is a solution of (3.11), (3.12) iff $z = (\varepsilon(\bar{u}), \bar{\sigma}) \in C^1(0,T,Z)$ is a solution of the problem

$$\dot{z} = A(t, z(t)) \quad \text{for all} \quad t \in [0,T] \quad (3.27)$$

$$z(0) = (\varepsilon(\bar{u}_0), \bar{\sigma}_0). \quad (3.28)$$

In order to study (3.27), (3.28) let us remark that by (3.4)–(3.8) $\varepsilon(\bar{u}_0) \in \varepsilon(V)$, $\bar{\sigma}_0 \in \mathcal{V}$ hence $(\varepsilon(\bar{u}_0), \bar{\sigma}_0) \in Z$. Using now lemma 3.2 and the classical Cauchy–Lipschitz theorem we get that (3.27), (3.28) has a unique solution $z \in C^1(0,T,Z)$. It results that (3.11), (3.12) has a unique solution $\bar{u} \in C^1(0,T,V)$, $\bar{\sigma} \in C^1(0,T,\mathcal{V})$ and we get the statement of theorem 3.1.

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UNE METHODE DE MONOTONIE EN VISCOPLASTICITE QUASISTATIQUE

Dans cet article on considère un problème initial et aux limites décrivant l'évolution quasistatique pour quelques modèles viscoplastiques semi-linéaires. On prouve un résultat d'existence et d'unicité de la solution en utilisant des arguments de monotonie suivis d'une technique de type Cauchy–Lipschitz.

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