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THE EQUIVALENT JOINT LOADS OF THE COMPOSITE MEMBER

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1. Introduction

In the calculation of structures by the slope deflection method the external influences acting along particular members are replaced by the concentrated loads acting at the joints i.e. at the ends of the members. They are called the equivalent joint loads.

In the present paper the exact expressions for the equivalent joint loads will be derived due to uniformly distributed load and the shrinkage of concrete. They refer to two types of composite members: a member with two fixed ends, member "k", and a member having one fixed and another hinged end, member "g". The members are of constant cross sections.

The equivalent joint loads represent the negative values of the reactions appearing at the supports of the members.

The equivalent joint loads for the members "k" and "g" are introduced as components of vectors $Q_{ik,H}$ and $Q_{iq,H}$, respectively. Their positive values are shown in Fig.1.

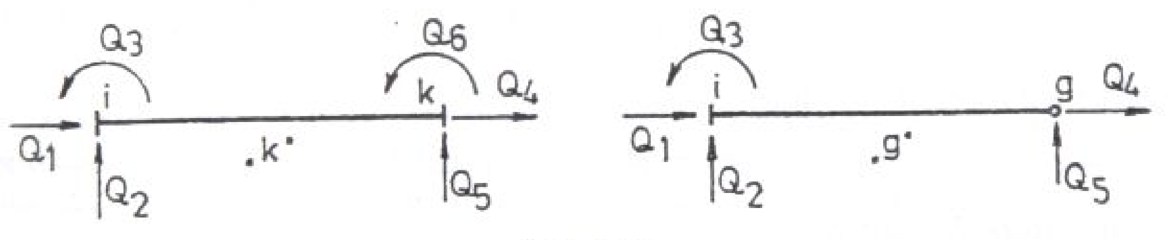

Figure 1

$$Q_{ik,H} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = - \begin{bmatrix} \mathcal{N}^*_{ik,H} \\ \mathcal{T}^*_{ik,H} \\ \mathcal{M}^*_{ik,H} \\ \mathcal{N}^*_{ki,H} \\ \mathcal{T}^*_{ki,H} \\ \mathcal{M}^*_{ki,H} \end{bmatrix}, \ (a) \quad \dot{Q}_{ig,H} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = - \begin{bmatrix} \mathcal{N}^*_{ig,H} \\ \mathcal{T}^*_{ig,H} \\ \mathcal{M}^*_{ig,H} \\ \mathcal{N}^*_{gi,H} \\ \mathcal{T}^*_{gi,H} \\ \mathcal{M}^*_{gi,H} \end{bmatrix} \ (b). \quad \text{(A)}$$

The quantities $\mathcal{N}^*_{ik,H}, \mathcal{T}^*_{ik,H}, \cdots \mathcal{M}^*_{ki,H}$ i.e. $\mathcal{N}^*_{ig,H}, \mathcal{T}^*_{ig,H}, \cdots \mathcal{M}^*_{gi,H}$ represent the relations of the fixed ends of the member and of the member whose one end is fixed and another hinged (Fig. 1), due to the influence H.

In this paper the equivalent joint load functions of the composite members "k" and "g" by the force method will be derived without mathematical negligences. The solution contains only the invenitable approximation concerning the descriptions of the reological properties of materials. Concrete is considered as an aging linear viscoelastic material. The relaxation of the prestressing steel is taken into account. The expressions are developed for any given concrete creep function.

In the theory of aging linear viscoelasticity, Mandel established a mathematical method using linear integro–differential operators. Since the operators obey the algebra of ordinary numbers, the problem is solved symbolically and formally, so that the mathematical operations are indicated only. Developing the theory of composite structures Lazić introduced linear integral operators for which the laws of algebra of ordinary numbers are valid, too. Then the problem was solved not only symbolically and formally but the expressions for stresses and displacements were reduced in the simplest form. The expressions concerned to any given concrete creep function and they were suitable for practical applications. Statically indeterminate composite structures were solved by the method of forces. The same procedure will be used in the present paper.

First, the necessary expressions related to the subject of the paper will be quoted so that the derivations could follow.

2. The basic expressions of the composite cross section

We consider the composite member where concrete (b), prestressing steel (p), steel member (n) and reinforcing steel (m) coact. Using linear integral operators the stress strain relation for concrete may be symbolically written in the following form:

$$\varepsilon - \varepsilon_S = \frac{1}{E_{bo}} \hat{F}' \sigma_b. \tag{1}$$

The solution of this integral equation is:

$$\sigma_b = E_{bo} \hat{R}' (\varepsilon - \varepsilon_S). \tag{2}$$

Operators $\hat{F}'$ and $\hat{R}'$:

$$\hat{F}' = \frac{1}{e} \hat{1}' + \hat{f}', \tag{3a}$$

$$\hat{R}' = e\hat{1}' - \hat{\psi}', \tag{3b}$$

$$\text{where:} \quad e = e(t) = \frac{E_b(t)}{E_{bo}} \tag{3c}$$

are inverse operators which obey:

$$\hat{R}' \hat{F}' = \hat{F}' \hat{R}' = \hat{1}'. \tag{4}$$

The concrete creep and relaxation functions F^* and R^* represent the following integrals:

$$F^* = \hat{F}'1^*, \quad R^* = \hat{R}'1^* \tag{5}$$

In the composite structure analysis the usual assumption is that the shrinkage of the concrete is time–wise similar to the concrete creep [1], [2]. Then the deformation function due to the shrinkage of concrete is given by the expression:

$$\varepsilon_S = r\,(F^* - 1^*)\,, \tag{6}$$

and for all observed times t and t_o the value of parameter r is constant:

$$r = \frac{\varepsilon_S(t,t_o)}{F^*(t,t_o) - 1^*}. \tag{7}$$

The relaxation of prestressing steel is taken into account. With satisfactory accuracy the stress strain relation is adopted as:

$$\sigma_p = E_p \hat{R}'_p \varepsilon, \tag{8}$$

where the relaxation function R^*_p linearly depends on the concrete relaxation function R^* [3]:

$$R^*_p = \hat{R}'_p 1^* = (1-\varrho)1^* + \varrho R^*. \tag{9}$$

These assumptions are experimentally approved. Besides, the corresponding operator $\hat{R}'_p$ is commutative with other operators.

For the observed times t and t_o and initial stress in steel σ_{po}, the constant value of parameters ϱ is adopted. It is determined form the expression:

$$\varrho = \frac{\zeta_p(t-t_o)}{1^* - R^*(t,t_o)}, \tag{10}$$

where $\zeta_p = \zeta_p(t-t_o)$ is the relaxation of prestressing steel.

The other kinds of steel: steel member (n) and reinforced steel (m) obey Hook's law:

$$\sigma_k = E_k \varepsilon, \quad k = n, m. \tag{11}$$

Starting from Bernoulli's hypothesis of plane composite cross section, the equations of equilibrium between the external and the internal forces and equations (2), (8), (11) the system of inhomogeneous integral equations is obtained:

$$\begin{aligned} E_u F_i \hat{R}'_{11}\eta + E_u S_i \hat{R}'_{12} k &= N, \\ E_u S_i \hat{R}'_{12}\eta + E_u J_i \hat{R}'_{22} k &= M, \end{aligned} \tag{12}$$

where $\eta = \eta(x,t,t_o)$ is the normal strain of the bar axis, $k = k(x,t,t_o)$ is the change in the curvature of the bar axis, E_u is the relative modulus of elasticity, F_i and J_i are the area and moment of inertia of the transformed cross section and $S_i = \sqrt{F_i J_i}$.

The elements of the symmetric operator matrix $\left[\hat{R}'_{hl}\right]_{2,2}$ are defined as follows:

$$\hat{R}'_{hl} = (\delta_{hl} - \gamma_{hl})\,\hat{1}' + \gamma_{hl}\hat{R}', \quad \delta_{hl} = \begin{cases} 1, & \text{for } h = l \\ 0, & \text{for } h \neq l \end{cases} \tag{13}$$

while the principal values of the matrix are:

$$\hat{R}'_h = \gamma'_h \hat{1}' + \gamma_h \hat{R}, \quad h = 1, 2. \tag{14}$$

The reduced geometrical properties of the composite cross section are contained in the symmetric matrix $[\gamma_{hl}]_{2,2}$. Its principal values are denoted by γ_h $(h = 1, 2)$. The following notation will be also used:

$$\gamma'_h = 1 - \gamma_h, \quad \delta\gamma_1 = \gamma_1 - \gamma_{11}, \quad \delta\gamma_2 = \gamma_{11} - \gamma_2, \quad \Delta\gamma = \gamma_1 - \gamma_2.$$

The operators have a commutativity property so that system (12) can be solved by Cramer's rule. The solution is:

$$\eta = \frac{1}{E_u F_i}\hat{F}'_{11} N + \frac{1}{E_u S_i}\hat{F}'_{12} M,$$

$$k = \frac{1}{E_u S_i}\hat{F}'_{12} N + \frac{1}{E_u J_i}\hat{F}'_{22} M. \tag{15}$$

The principal values of the operator matrix $\left[\hat{F}'_{hl}\right]_{2,2}$ are:

$$\hat{F}'_h = \frac{1}{e_h}\hat{1}' + \gamma_h \hat{\psi}'_h, \quad \left(e_h = \gamma'_h + \gamma_h e\right), \quad h = 1, 2. \tag{16}$$

It can be shown, that the next relations is valid:

$$\hat{R}'_h \hat{F}'_h = \hat{F}'_h \hat{R}'_h = \hat{1}'. \tag{17}$$

The operators $\hat{F}'_{hl}$ can be expressed by the principal values $\hat{F}'_1$ and $\hat{F}'_2$ of the operator matrix $\left[\hat{F}'_{hl}\right]_{2,2}$:

$$\hat{F}'_{11} = \frac{1}{\Delta\gamma}\left(\delta\gamma_2 \hat{F}'_1 + \delta\gamma_1 \hat{F}'_2\right), \quad \hat{F}'_{22} = \frac{1}{\Delta\gamma}\left(\delta\gamma_1 \hat{F}'_1 + \delta\gamma_2 \hat{F}'_2\right),$$

$$\hat{F}'_{12} = \hat{F}'_{21} = \frac{\gamma_{12}}{\Delta\gamma}\left(\hat{F}'_1 + \hat{F}'_2\right). \tag{18}$$

Also the next relation is used:

$$\hat{F}'_{11}\hat{F}'_{22} - \hat{F}'_{12}\hat{F}'_{12} = \hat{F}'_1 \hat{F}'_2. \tag{19}$$

The following operators are introduced:

$$\hat{B}'_h = \hat{R}'\hat{F}'_h, \quad h = 1, 2, \tag{20}$$

which are simultaneously linearly dependent on the operators $\hat{F}_h'$:

$$\hat{B}_h' = \frac{1}{\gamma_h}\hat{1}' - \frac{\gamma_h'}{\gamma_h}\hat{F}_h', \quad h = 1, 2. \tag{21}$$

Functions B_h^*:

$$B_h^* = B_h^*(\gamma_h, t, t_o) = \hat{B}_h' 1^* = \hat{R}' F_h^* = \hat{F}_h' R^*, \quad h = 1, 2, \tag{22}$$

are the basic functions of the composite cross section [3]. The determination of the functions B_h^* is reduced to the solutions of the parametric inhomogeneous integral equation:

$$\hat{K}_h' B_h^* = 1, \quad h = 1, 2. \tag{23}$$

The functions K_h^* linearity depend on the concrete creep function F^* and on the reduced geometrical properties of the composite cross section:

$$K_h^* = K_h^*(\gamma_h, t, t_o) = \hat{K}_h' 1^* = \gamma_h 1^* + \gamma_h' F^*, \quad h = 1, 2. \tag{24}$$

Using the principle of virtual forces:

$$\xi = \int_L \left[\bar{M}(s,x)\, k(s,t,t_o) + \bar{N}(s,x)\, \eta(s,t,t_o)\right] \mathrm{d}s \tag{25}$$

and expression (15) the reduced generalized displacement will be:

$$\begin{aligned} \Delta^* = \Delta^*(s,t,t_o) = E_u I_u \xi = \\ \int_L \bar{M}(s,x)\, \hat{F}_{22}'(s,t,\tau)\, M(s,\tau,t_o) \frac{J_u}{J_i(s)}\, \mathrm{d}s + \\ \frac{J_u}{F_u} \int_L \bar{N}(s,x)\, \hat{F}_{11}'(s,t,\tau)\, N(s,\tau,t_o) \frac{F_u}{F_i(s)}\, \mathrm{d}s + \\ \int_L \hat{F}_{12}'(s,t,\tau) \left[\bar{M}(s,x)\, N(s,\tau,t_o) + \bar{N}(s,x)\, M(s,\tau,t_o)\right] \frac{J_u}{S_i(s)}\, \mathrm{d}s \end{aligned} \tag{26}$$

The solution procedure of statically indeterminate composite structures is analogous to the same procedure of homogeneous elastic structures [1]. If a structure is n times statically indeterminate, the redundants $X_{kH} = X_{kH}(t, t_o)$, $(k = 1, 2, \ldots, n)$ occur, which are time function. The subscript H denotes the influence.

The axial force N_H, the bending moment M_H, and the shear force T_H due to the influence H are:

$$N_H = N_{HO} + \sum_{k=1}^{n} N_k X_{kH}, \quad M_H = M_{HO} + \sum_{k=1}^{n} M_k X_{kH},$$

$$T_H = T_{HO} + \sum_{k=1}^{n} T_k X_{kH}. \tag{27}$$

We establish the geometrical conditions:

$$\Delta^*_{rH} = \Delta^*_{rH}(t, \tau^H) = 0, \quad r = 1, 2, \ldots, n \tag{28}$$

where, Δ^*_{rH} represent the reduced generalized displacement at the point $s = s_r$ due to generalized forces $X_{rH} = 1^*$.

Introducing expressions (27) into (26) and by realizing (28) we get n equations:

$$\sum_{k=1}^{n} \hat{\Delta}'_{rk} X_{kH} + \Delta^*_{kHo} = 0, \quad k, r = 1, 2, \ldots, n \tag{29}$$

where the redundants X_{kH} appear as unknown functions in the system of n inhomogeneous integral equations.

3. The equivalent joint loads of the composite member "g" due to uniformly distributed load

For obtaining the equivalent joint loads for the composite member "g" (A.b) (Fig. 2) the method of force is applied.

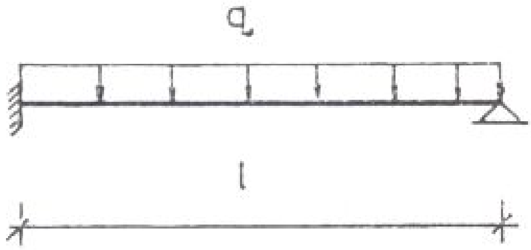

Figure 2.

The member is twice statically indeterminate. The primary system with redundants and corresponding diagrams are shown in Fig. 3.

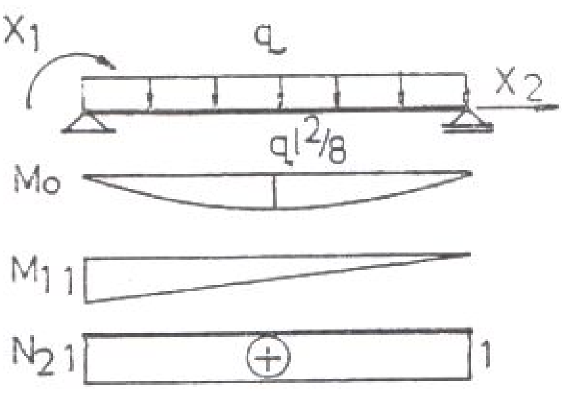

Figure 3.

On the basis of diagrams (Fig. 3) and (26) equations (29) in the following forms are obtained:

$$\frac{1}{3J_i}\hat{F}'_{22}X_1 + \frac{1}{2S_i}\hat{F}'_{12}X_2 + \frac{ql^3}{24J_i}F^*_{22} = 0,$$
$$\frac{1}{2S_i}\hat{F}'_{12}X_1 + \frac{1}{F_i}\hat{F}'_{11}X_2 + \frac{ql^3}{12S_i}F^*_{22} = 0. \tag{30}$$

This system can be solved by Cramer's rule.

Let the determinant of system (30) be denoted by $\hat{D}'$. After operator transformations it can be represented in the following form:

$$\hat{D}' = \frac{l^2}{F_iJ_i}\hat{F}'_1\hat{F}'_2\hat{C}', \tag{31}$$

where the operator $\hat{C}'$ is linear combination of operators $\hat{F}'_h$:

$$\hat{C}' = m\hat{1}' + h\left(\gamma_2\hat{F}'_1 + \gamma_1\hat{F}'_2\right). \tag{32}$$

The constants m and h depend on the reduced geometrical properties of the composite cross section:

$$m = \frac{4\gamma_1\gamma_2 + \gamma_{12}^2}{12\gamma_1\gamma_2}, \quad h = \frac{\gamma_{12}^2}{12\Delta\gamma\gamma_1\gamma_2}. \tag{33}$$

Using Cramer's rule D_1^* and D_2^* become:

$$D_1^* = \frac{ql^4}{24F_iJ_i}\hat{F}'_1F_2^*, \quad D_2^* = \frac{ql^4}{144F_iJ_i}\hat{F}'_{22}F_{12}^*, \tag{34}$$

so that the redundants are:

$$X_1 = -(\hat{D}')^{-1}D_1^*, \ (a) \quad X_2 = -(\hat{D}')^{-1}D_2^* \ (b). \tag{35}$$

The operators $(\hat{D}')^{-1}$ and $\hat{D}'$ are inverse. They satisfy:

$$\hat{D}'(\hat{D}')^{-1} = \hat{1}'. \tag{36}$$

Using relations (31) and (17) we get:

$$(\hat{D}')^{-1} = \frac{F_iJ_i}{l^2}\hat{R}'_1\hat{R}'_2(\hat{C}')^{-1}. \tag{37}$$

Now it is necessary to determine the operator $\hat{S}'$ inverse to the operator $\hat{C}'$:

$$\hat{S}'\hat{C}' = \hat{1}'. \tag{38}$$

Using expressions (17) relation (32) becomes:

$$\hat{C}' = \hat{F}_1'\hat{F}_2'\left[m\,\hat{R}_1'\hat{R}_2' + h\left(\gamma_2\hat{R}_2' - \gamma_1\hat{R}_1'\right)\right]. \tag{39}$$

Introducing (14) we get:

$$\hat{C}' = \hat{F}_1'\hat{F}_2'\,p\left[\hat{1}' + \frac{q}{p}\hat{R}' + \frac{r}{p}\hat{R}'\hat{R}'\right], \tag{40}$$

where:

$$p = \frac{4\gamma_1'\gamma_2' + \gamma_{12}^2}{12}, \quad r = \frac{4\gamma_1\gamma_2 + \gamma_{12}^2}{2}, \quad q = \frac{4\left(\gamma_1'\gamma_2 + \gamma_1\gamma_2'\right) - 2\gamma_{12}^2}{2}. \tag{41}$$

We defined the operators:

$$\hat{\mathcal{R}}_h' = \hat{1}' + \hat{\Theta}_h\hat{R}', \quad h = 1,2, \tag{42}$$

where:

$$\Theta_1 + \Theta_2 = \frac{q}{p}, \quad \Theta_1\cdot\Theta_2 = \frac{r}{p}. \tag{43}$$

We defined also the inverse operators $\hat{\mathcal{F}}_h'$:

$$\hat{\mathcal{R}}'\hat{\mathcal{F}}_h' = \hat{1}', \quad h = 1,2. \tag{44}$$

By analogy to expression (20), the operators $\hat{\mathcal{B}}_h'$ will be used:

$$\hat{\mathcal{B}}_h' = \hat{R}'\hat{\mathcal{F}}_h', \quad h = 1,2. \tag{45}$$

For the functions $\mathcal{B}_h^* = \hat{\mathcal{B}}_h' 1^*$ it can be shown too that they linearly depend on the functions $\mathcal{F}_h^*$:

$$\mathcal{B}_h^* = \frac{1}{\Theta_h}\left(1^* - \mathcal{F}_h^*\right). \tag{46}$$

The functions $\mathcal{B}_h^*$ are the solutions of the parametric inhomogeneous integral equations given here in the operator form:

$$\hat{\mathcal{K}}_h'\mathcal{B}_h^* = 1^*, \quad h = 1,2. \tag{47}$$

The known functions $\mathcal{K}_h^*$ linearly depend on the concrete creep function:

$$\mathcal{K}_h^* = \hat{\mathcal{K}}_h' 1^* = \Theta_h 1^* + F^*. \tag{48}$$

We return to the operator $\hat{C}'$ (40) where (43) and (42) are substituted:

$$\hat{C}' = p\,\hat{F}_1'\hat{F}_2'\hat{\mathcal{R}}_1'\hat{\mathcal{R}}_2'. \tag{49}$$

Using expressions (17) and (44) the inverse operator $\hat{S}'$ is:

$$\hat{S}' = (\hat{C}')^{-1} = \frac{1}{p}\,\hat{R}_1'\hat{R}_2'\hat{\mathcal{F}}_1'\hat{\mathcal{F}}_2'. \tag{50}$$

The operator $\hat{S}'$ is presented as four-fold operator product. By complex operator transformations it can be reduced to a sum of operators. In that way the operator $\hat{S}'$, i.e. function S^* has a considerably simplier form:

$$\hat{S}' = s\left(\hat{1}' + t\hat{\mathcal{F}}_1' + g\hat{\mathcal{F}}_2'\right), \tag{a}$$

(51)

$$S^* = \hat{S}'1^* = s\left(1^* + t\mathcal{F}_1^* + g\mathcal{F}_2^*\right), \tag{b}$$

where the constants:

$$s = \frac{\gamma_1\gamma_2}{p\Theta_1\Theta_2}, \quad g = \frac{-\Theta_1\left(\gamma_1 - \Theta_1\gamma_1'\right)\left(\gamma_2 - \Theta_2\gamma_2'\right)}{\Delta\Theta\gamma_1\gamma_2},$$

$$t = \frac{\Theta_2\left(\gamma_1 - \Theta_1\gamma_1'\right)\left(\gamma_2 - \Theta_2\gamma_2'\right)}{\Delta\Theta\gamma_1\gamma_2}, \tag{52}$$

depend on reduced geometrical properties of composite cross section.

Taking expressions (35), (37), (52) and (17) and relation $\hat{R}' = \hat{F}_{22}'\hat{R}_1'\hat{R}_2'$, obtained by solving the system of equations (12), the redundants are:

$$X_1 = -\frac{ql^2}{24}S^*, \quad (a) \quad X_2 = -\frac{ql^2F_i}{144S_i}\hat{R}_{11}'\hat{S}'F_{12}^* = -\frac{ql^2F_i}{144S_i}V^* \quad (b). \tag{53}$$

The new function is introduced:

$$V^* = \hat{V}'1^* = \hat{R}_{11}'\hat{S}'F_{12}^* = s\left[a1^* + b_1F_1^* - b_2F_2^* + c_1n_2\mathcal{F}_1^* + c_2n_3\mathcal{F}_2^*\right], \tag{54}$$

where a, b_1, b_2, c_1 and c_2 are constants:

$$a = \frac{\gamma_2 - \gamma_1}{\gamma_1\cdot\gamma_2}, \quad b_h = \frac{\gamma_{12}}{\Delta\gamma}\left[n_1 - \frac{\gamma_h'\gamma_{11}}{\gamma_h} + \frac{n_2\gamma_h}{\gamma_h - \gamma_h'\Theta_1} + \frac{n_3\gamma_h}{\gamma_h - \gamma_h'\Theta_2}\right], \quad h = 1,2$$

$$c_k = \frac{\gamma_{12}\Theta_k}{\Delta\gamma}\left[-\frac{1}{\gamma_1 - \gamma_1'\Theta_k} + \frac{1}{\gamma_2 - \gamma_2'\Theta_k}\right], \quad k = 1,2 \tag{55}$$

$$n_1 = \gamma_{11}' + \gamma_{11}\left(\frac{t}{\Theta_1} + \frac{g}{\Theta_2}\right), \quad n_2 = t\left(\gamma_{11}' - \frac{\gamma_{11}}{\Theta_1}\right)$$

$$n_3 = g\left(\gamma_{11}' - \frac{\gamma_{11}}{\Theta_2}\right).$$

It is necessary to notice that the function V^* is presented also as a linear combination of the function F_h^* and $\mathcal{F}_h^*$. The details related to the derivation of expressions (51) and (54) may be found in reference [4].

By expression (53) the relations X_1 and X_2 are given. Using the equations of equilibrium, the other relations can be found. The vector of the equivalent joint loads (Ab)(Fig. 1b) of the member "g" due to uniformly distributed load has the form:

$$Q^T_{ig,q} = -\left[\frac{ql^2F_i}{144S_i}V^*,\ \frac{ql}{24}S^* + \frac{ql}{2}1^*,\ \frac{ql^2}{24}S^*,\ -\frac{ql^3F_i}{144S_i}V^*,\ -\frac{ql}{24}S^* + \frac{ql}{2}1^*\right]. \quad (56)$$

When the concrete creep function F^* is chosen according to (48) we form the function $\mathcal{K}^*_h$ where Θ_h is a parameter. Than the solution of parametric equation (47) for two values of Θ_1 and Θ_2 represents the functions $\mathcal{B}^*_1$ and $\mathcal{B}^*_2$, respectively. The functions $\mathcal{F}^*_h$ are obtained from (46) and finally the function S^* (51) is determined. To define the function V^* we apply the similar procedure using (24), (23), (21) and (54).

4. The equivalent joint loads of the composite member "g" due to the shrinkage of concrete

The influence of the shrinkage of concrete is introduced in the usual manner [1], [2].

$$N^*_S = E_u F_{br} r \,|\, 1^* - R^* \,| = n_S \,|\, 1^* - R^* \,|, \quad \text{(a)}$$

(57)

$$M^*_S = E_u F_{br} y_b r \,|\, 1^* - R^* \,| = m_S \,|\, 1^* - R^* \,|, \quad \text{(b)}$$

where assumption (6) and expression (2) are contained.

The system of equations for the determination of the redundants is the same as (30) except for the third members. They are:

$$\frac{M^*_S l}{2J_i}\hat{F}'_{22} - \frac{N^*_S l}{2S_i}\hat{F}'_{12} \quad \text{and} \quad -\frac{N^*_S l}{F_i}\hat{F}'_{11} + \frac{M^*_S l}{S_i}\hat{F}'_{12}. \quad (58)$$

Using Cramer's rule we get:

$$D^*_1 = \frac{M^*_S l}{2J_iF_i}\hat{F}'_1\hat{F}'_2, \quad D^*_2 = -\frac{N^*_S l^2}{J_iF_i}\hat{F}'_1\hat{F}'_2\hat{C}' + \frac{M^*_S l^2}{12J_iS_i}\hat{F}'_{12}\hat{F}'_{22}. \quad (59)$$

The redundants are determined from expressions (35), (37), (50) and (54) and they are:

$$X_1 = -(\hat{C}')^{-1}\frac{M^*_S}{2} = -\hat{S}'\frac{M^*_S}{2}, \quad \text{(a)}$$

(60)

$$X_2 = N^*_S - \frac{F_i}{12S_i}\hat{F}'_{12}\hat{R}'_{11}(\hat{C}')^{-1}M^*_S = N^*_S - \frac{F_i}{12S_i}\hat{V}'M^*_S. \quad \text{(b)}$$

We introduced relation (57b) in expression (60):

$$X_1 = -\frac{1}{2}m_S\hat{S}' \,|\, 1^* - R^* \,| = -\frac{1}{2}m_S \,|\, S^* - \hat{S}'R^* \,|, \quad \text{(a)}$$

(61)

$$X_2 = N^*_S - \frac{F_i m_S}{12S_i}\hat{V}' \,|\, 1^* - R^* \,| = N^*_S - \frac{F_i m_S}{12S_i}m_S \,|\, V^* - \hat{V}'R^* \,|. \quad \text{(b)}$$

The following functions are also introduced:

$$S_S^* = S^* - \hat{S}' R^* \quad \text{and} \quad V_S^* = V^* - \hat{V}' R^* . \tag{62}$$

For transformation of the members $\hat{S}' R^*$ and $\hat{V}' R^*$ we used (51), (54), (22) and (45). Then S_S^* and V_S^* can be expressed as a linear combination of the functions R^*, F_h^*, $\mathcal{F}_h^*$, B_h^* and $\mathcal{B}_h^*$.

$$S_S^* = s\,[1^* - R^* + t\,(\mathcal{F}_1^* - \mathcal{B}_1^*) + g\,(\mathcal{F}_2^* - \mathcal{B}_2^*)]\,, \tag{a}$$

$$V_S^* = s\,[a\,(1^* - R^*) + b_1\,(F_1^* - B_1^*) - \tag{63}$$

$$b_2\,(F_2^* - B_2^*) + c_1 n_2\,(\mathcal{F}_1^* - \mathcal{B}_1^*) + c_2 n_3\,(\mathcal{F}_2^* - \mathcal{B}_2^*)]\,. \tag{b}$$

According to (61) and (62) the redundants have the forms:

$$X_1 = -\frac{1}{2} m_S S_S^*, \quad X_2 = N_S^* - \frac{F_i m_S}{12 S_i} V_S^* . \tag{64}$$

The other reactions can be found using the equations of equilibrium. The vector of the equivalent joint loads of the member "g" due to the shrinkage of concrete has the following form:

$$Q_{ig,s}^T = -\left[-N_S^* + \frac{m_S F_i}{12 S_i} V_S^*,\ \frac{m_S}{2l} S_S^*,\ \frac{m_S}{2} S_S^*,\ N_S^* - \frac{m_S F_i}{12 S_i} V_S^*,\ -\frac{m_S}{2l} S_S^*\right] . \tag{65}$$

The functions S_S^* and V_S^* are determined in the same manner as the functions S^* (51 b) and V^* (54) adding the function R^*. The function R^* is the solution of the inhomogeneous integral equation (4).

5. The equivalent joint loads of the composite member "k" due to uniformly distributed load and the shrinkage of concrete

It is known that the composite bar with fixed ends affected by uniformly distributed load has the time independent reactions. That means that the equivalent joint loads for member "k" due to the same load do not depend on time. The vector (Aa) has the following form:

$$Q_{ik,q}^T = -\left[0,\ \frac{ql}{2} 1^*,\ \frac{ql^2}{12} 1^*,\ 0,\ \frac{ql}{2} 1^*,\ -\frac{ql^2}{12} 1^*\right] . \tag{66}$$

By analogy with development of expression (65) the system of equations corresponding to the shrinkage of concrete is created. The relations i.e. the equivalent joint loads are calculated.

$$Q_{ik,s}^T = -\left[-n_S |\, 1^* - R^* \,|, 0,\ m_S |\, 1^* - R^* \,|,\ n_S |\, 1^* - R^* \,|, 0,\ m_S |\, 1^* - R^* \,|\right] \tag{67}$$

In this case it is necessary to solve only one inhomogeneous integral equation (4) to determine the concrete relaxation function R^*, i.e. the equivalent joint loads.

6. The equivalent joint loads of the composite member at time $t = t_o$

The equivalent joint loads of composite member due to uniformly distributed load at time $t = t_o$ is equal to the equivalent joint loads of the corresponding homogeneous elastic member having the modulus of elasticity E_u and the cross section geometrical properties, F_i and J_i:

$$Q^{T_o}_{ig,q} = -\left[0,\ \frac{5ql}{8}1^*,\ \frac{ql^2}{8}1^*,\ 0,\ \frac{3ql}{8}1^*\right],$$

$$Q^{T_o}_{ik,q} = -\left[0,\ \frac{ql}{2}1^*,\ \frac{ql^2}{12}1^*,\ 0,\ -\frac{ql}{2}1^*,\ -\frac{ql^2}{12}1^*\right], \tag{68}$$

At the time $t = t_o$ the equivalent joint loads due to the shrinkage of concrete are equal to zero.

7. The numerical example

Consider the composite member "g" (Fig. 4) loaded with a uniformly distributed load q. We shall obtain the equivalent joint loads at times $t \Rightarrow \infty$ and t_o due to the load and the shrinkage of concrete.

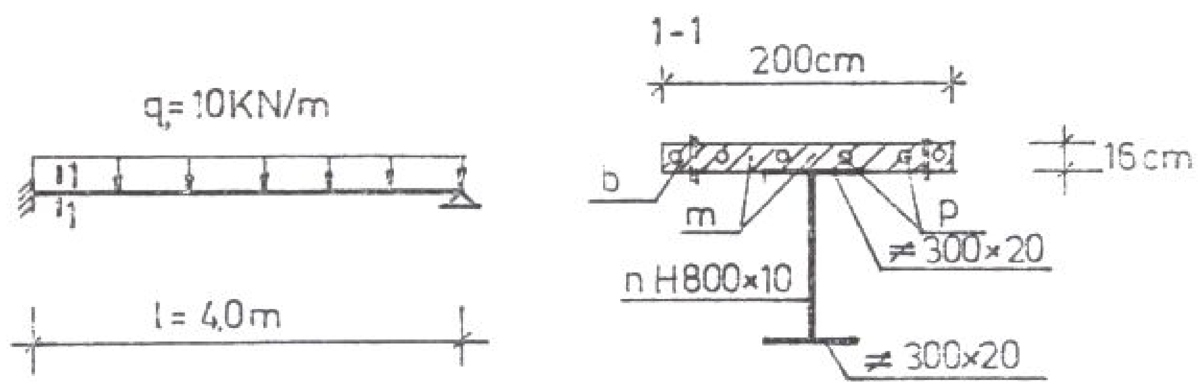

Figure 4

Data: $l = 4\,m,\ q = 10\,kN/m$

Concrete (b): $E_b = 30\,GPa,\ \varphi_r = 3.5,\ \varepsilon_S = -30 \times 10^{-5}$

Prestressing steel (p): $E_p = 210\,GPa,\ F_p = 100\,cm^2,\ \xi_p = 8\%$

Steel member (n): $E_n = 200\,GPa = E_u$

Reinforced steel (m): $E_m = 200\,GPa,\ F_m = 80\,cm^2$

The concrete creep function of the aging theory with constant modulus of elasticity of concrete is used, so that the application of the Laplace transforms can be possible. For the concrete aging creep function of constant elastic modulus the basic functions B_h^* (22) [1] and the functions B_h^* (46) are as follows:

$$B_h^* = e^{-\gamma_h' \varphi_r}, \quad B_h^* = \frac{1}{\Theta_h + 1} e^{-\left(\frac{1}{\Theta_h + 1}\right)\varphi_r}.$$

Following the procedures described earlier we calculate required the values at times $t \Rightarrow \infty$ (TM) and t_o, given in table 1.

This example is also solved by the well-known effective modulus (EM) method suggested by Faber. This procedure is based on the algebraic relation between stress and strain for concrete:

$$\sigma_{b\infty} = E_{b\infty}(\varepsilon_\infty - \varepsilon_{s\infty}), \quad E_{b\infty} = \frac{E_{b0}}{1 + \varphi_r^\infty}$$

which is accurate only for the hereditary function at time $t \Rightarrow \infty$. $E_{b\infty}$ is the asymptotic modulus of elasticity.

The values obtained by the (EM) method are given in the Table 1.

Influen.	kN/kNm	t_o	EM	TM
q	Q_1	0.000	2.675	3.351
	Q_2	-25.000	-24.908	-24.881
	Q_3	-20.000	-19.634	-19.524
	Q_4	0.000	-2.675	-3.351
	Q_5	-15.000	-15.0920	-15.119
S	Q_1		654.845	818.729
	Q_2		-27.237	-33.991
	Q_3		-108.949	-135.964
	Q_4		-654.845	-818.729
	Q_5		27.237	33.991

Table 1.

It is known that the values obtained by the TM i.e. EM method represent their upper or lower limits. All other values calculated by any approximate method must lie in these intervals.

8. Conclusion

The exact functions of the equivalent joint loads of the composite members "k" and "g" are derived due to the uniformly distributed load and the shrinkage of concrete by the method of forces using linear integral operators.

Applying this mathematical procedure it is possible to reduce the complex integrations appearing in the functions to the simplest form. They are linear combinations of the concrete relaxation function and two pairs of functions which are solutions of the two corresponding parametric inhomogeneous integral equations.

When we determine the redundants by the slope deflection method of the composite structures the integrations of the functions of the equivalent joint loads occur. That is why it is very important for them to be of a simple form. We emphasize that this form refers to any given concrete creep function.

In order to show the simplicity of the equivalent joint loads functions an example is solved.

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ЭКВИВАЛЕНТНАЯ УЗЛОВАЯ НАГРУЗКА ДЛЯ СОПРЯЖЁННОЙ БАЛКИ

При расчёте сопряжённых конструкций методом деформации влияния наружной нагрузки на балку заменяютця эквивалентной концентрической нагрузкой в узелам балки. В этой статьи произведенны точные виражения для эквивалентной узловой нагрузки сопряажонной балки константнйх сечений, вследствие распределённой нагрузкой и сжатости бетона. Они представлены для сопряжённой балки са обоими защемленными концами, тип "к", и для сопряжённой балки с одним защемленным концом и шарнирно опертой другим, тип "г". Балки имеют константные сечения. С неизбежными апроксимациями реологических свойств материялов каторие существуют в сопряженном сечении этые виражения произведены без математических отступлений. Бетон считаем линейной вязкой упругим материялом с свойствием старения. При расчёте введена и релаксация предворительно напряжённой арматури. Виражења произведенны для орнитировчной функции ползучести бетона. Пользуються линейные интегральные операторы.

Подходящими трансформациями многосложные произведения операторов, которие являются при образовании этых виражений, предоставляются как линейные комбинации операторов. Таким образом уменьшается

число интеграции а виражения имеют самую простую форму подходящую для применения. Для знакомую функцию ползучести бетона довольно решить три интегральные уравнения от которых две функции параметрические. Их решением являюця функции от которых линейно зависят произведены виражения.

EKVIVALENTNO ČVORNO OPTEREĆENJE SPREGNUTOG ŠTAPA

Pri proračunu konstrukcija metodom deformacija uticaje spoljašnjeg opterećenja na štap prevodimo u ekvivalentna koncentrisana opterećenja u čvorovima štapa. U ovom radu izvedeni su tačni izrazi za ekvivalentna čvorna opterećenja usled ravnomerno raspodeljenog opterećenja i skupljanja betona. Oni su dati za obostrano uklješten spregnuti štap tj. štap tipa "k" i spregnut štap tipa "g", koji je na jednom kraju uklješten, a na drugom kraju zglobno vezan. Štapovi su konstantnog poprečnog preseka. Uz neizbežne aproksimacije reoloških osobina materijala, koji sadejstvuju u spregnutom preseku, ovi izrazi su izvedeni bez matematičkih zanamarenja. Beton se posmatra kao linearno viskoelastičan materijal sa osobinom starenja. Uzeta je u obzir relaksacija čelika za prethodno naprezanje. Izrazi su izvedeni za proizvoljnu funkciju puzanja betona. Koriste se linearni integralni operatori. Pogodnim transformacijama višestruki proizvodi operatora koji se javljaju pri izvođenju ovih izraza prikazuju se kao linearne kombinacije operatora. Na taj način je smanjen broj integracija i izrazi imaju najjednostavniji oblik prikladan za primenu. Za poznatu funkciju puzanja betona dovoljno je rešiti tri integralne jednačine of kojih su dve parametarske. Njihova rešenja su funkcije of kojih linearno zavise izvesni izrazi.

Pri određivanju deformacijski neodređenih veličina spregnutog nosača u metodi deformacija nad izvedenim izrazima za ekvivalentna čvorna opterećenja vrše se integracije. To je razlog što su ovi izrazi operatorskim transformacijama svedeni na najjednostavniji oblik. Da bi se pokazala jednostavnost primene ovih izraza urađen je brojni primer.

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