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OSCILLATORY MODEL OF VORTEX-INDUCED OSCILLATIONS UNDER THE INFLUENCE OF WIDE-BAND RANDOM EXCITATION

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0. Introduction

The phenomenon of vortex-induced oscillations in elastic systems as a consequence of flow is an important engineering problem. The latest research has been concentrated on the determination of the fluctuating lift force on stationary models or on the laws of motion in elastic models i the case when the lift force is a deterministic value. Most often the oscillatory model is analyzed which is based on the concept that the model fluctuating lift is associated with vortex shedding which is the result of various velocities at the surfaces contacting points; namely, one from the stream flow, the other from the undisturbed flow part. This concept of excited oscillator was introduced by Birkhoff and Zarantanello [1] and reinforced by the experiments of Bishop and Hassan [2]. The case of a circular cylinder normal to the flow is considered. Cylinder motion is restricted to pure translation in the transverse direction perpendicular to the flow direction and cylinder axis. This oscillatory model is analysed in the paper of Hartlen and Currie [3] and their main objective was to determine the oscillatory system parameters and the differential equations of cylinder motion for linear characteristics of spring and damping. The external aerodynamic force is also determined and then the results obtained from the mathematical model are compared with phenomena in experimental studies. In the paper of Brückner and Lin [4] the oscillatory system from the paper [3] is analyzed under the influence of two random excitations, namely, the parametric and the external one by applying complex formulation of generalized equivalent linearization.

In this paper the elastic moments of the system response with two degrees of freedom from the paper [3] are determined under the influence of wide-band randomly fluctuating lift force by applying the Gaussian closure. The assumption is introduced that the non-linearity influence is small in the system and that the function of the solutions distribution approximately has the form of the Gaussian distribution. Then the statistic moments of the third, fourth and higher order are substituted with the statistic moments of the first and second order by the very well known relations. The resulting system of fourteen differential equations for the case of the stationary state is reduced to the system of non-linear algebraic equations the solution of which is used to determine the statistic characteristics of the solution process.

2. Equations of motion

Let's consider the oscillatory system from the paper [3] shown in Fig. 1. A cylinder of diameter D, length L and mass M is exposed to a flow of fluid of velocity V which is perpendicular to the cylinder axis. The cylinder is mounted on springs of constant stiffness $K/2$ and dampers of linear damping coefficient $R/2$. The external lift force acting on the cylinder as a result of water-shedding F_a. The cylinder motion is restricted to pure translation in the transverse direction perpendicular to the flow direction and cylinder axis. The law of motion is determined by solving two similar differential equations, one determining the cylinder motion in the direction x, the other one defining an instantaneous lift force $F_a = \rho V^2 DLC/2$.

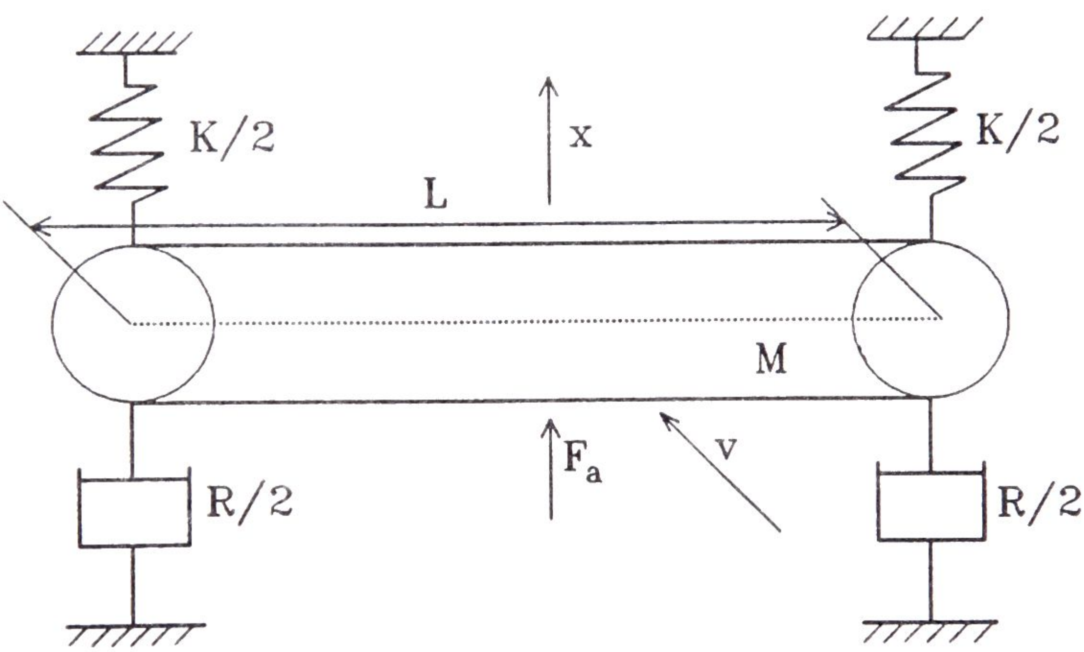

Fig. 1. Oscillatory system

The equation of dynamic equilibrium of the cylinder is,

$$M\ddot{x} + R\dot{x} + Kx = F_a. \tag{1}$$

Introducing dimensionless variables x_r and τ by the relations,

$$x_r = \frac{x}{D}, \quad \tau = t\left(\frac{K}{M}\right)^{1/2} = \omega_n t, \tag{2}$$

the equation (1) becomes,

$$x_r'' + 2\zeta x_r' + x_r = a\omega_0^2 C, \tag{3}$$

in which the prime denotes differentiation with respect to τ and dimension parameters are,

$$\zeta = \frac{R}{2M\omega_n}, \quad a = \frac{\rho D^2 L}{8\pi^2 S^2 M}, \quad \omega_0 = \frac{f_s}{f_n} = \frac{SV}{f_n D}, \quad \omega_n^2 = \frac{K}{M},$$

and $f_n = \omega_n/2\pi$ and $f_s = SVD^{-1}$ are natural frequency and vortex shedding frequency regarding the *Strouhal* relation (S is a constant depending on a flow of free-stream velocity V). The coefficient C appearing in the equation (3) has an instantaneous value influenced by the cylinder motion. Rigorous evaluation of the coefficient C would require theoretical treatment of the separated flows around the cylinder in the neighbourhood of a natural frequency of the flow a promidable task. Therefore, the approximation taken herein is the determination of the coefficient C on the basis af a simple equivalent self-exciting oscillator by which the values obtained in an experimential way are realized for certain values of constants α, γ and b and on the basis of the differential equation,

$$C'' - \alpha\omega_0 C' + \frac{\gamma (C')^3}{\omega_0} + \omega_0^2 C = b x_r'.$$

In oredr to include more accurately in the calculation the appearance of turbulence which is generally encompassed by the fluid flow nature a random process $f(t)$ is introduced which is a dimensionless velocity of the fluid flow turbulence. Let's consider the case when $f(t)$ is a stationary Gaussian process zero average value of constant spectral density S. Then the equations (3) and (4) are,

$$x_r'' + 2\zeta x_r' + x_r = a\omega_0^2 [1 + f(\tau)] C, \tag{5}$$

$$C'' - \alpha\omega_0 C' + \frac{\gamma (C')^3}{\omega_0} + \omega_0^2 [1 + f(\tau)] C = b x_r', \tag{6}$$

where the prime denotes derivatives with respect to dimensionless time τ.

3. Markov vector approach

If we introduce new variables $x_1 = x_r$, $x_2 = x_r'$, $x_3 = C$, $x_4 = C'$, the system of equations (3) and (4) can be represented by an equivalent system of four first-order differential equations,

$$\left.\begin{aligned} \frac{dx_1}{d\tau} &= x_2, \quad \frac{dx_2}{\tau} = -2\zeta x_2 - x_1 + a\omega_0^2 x_3 + a\omega_0^3 f(\tau) x_3, \\ \frac{dx_3}{d\tau} &= x_4, \quad \frac{dx_4}{d\tau} = \alpha\omega_0 x_4 - \frac{\gamma x_3^4}{\omega_0} - \omega_0^2 x_3 + b x_2 - \omega_0^2 f(\tau) x_3. \end{aligned}\right\} \tag{7}$$

Then the nonstationary *Fokker-Planck* equation is applied in order to determine the *Markov* vector of the solution $x = (x_1, x_2, x_3, x_4)$ which is,

$$\frac{\partial}{\partial \tau} p(x,\tau) = -\sum_{i=1}^{4} \frac{\partial}{\partial x_i}[a_i(x,\tau)p(x,\tau)] + \frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4} \frac{\partial^2}{\partial x_i \partial x_j}[b_{ij}(x,\tau)p(x,\tau)], \quad (8)$$

where $p(x,\tau)$ is a function of the joint probability density and $a_i(x,\tau)$ and $b_{ij}(x,\tau)$ are increments of the first and the second moments of the *Markov* process $x(\tau)$ which are determined for the given system as,

$$a_i(x,\tau) = \lim_{\delta\tau \to 0} \frac{1}{\delta\tau} E[x_i(\tau + \delta_\tau) - x_i(\tau)]. \quad (9)$$

Substituting the system equations (7), into (9) the increments of the first moments a_i follow,

$$\left.\begin{aligned} a_1 &= x_2, \quad a_2 = -2\zeta x_2 - x_1 + a\omega_0^2 x_3, \\ a_3 &= x_4, \quad a_4 = \alpha\omega_0 x_4 - \frac{\gamma x_4^3}{\omega_0} - \omega_0^2 x_3 + b x_2. \end{aligned}\right\} \quad (10)$$

The increment of the second moments b_{ij} are determined as

$$b_{ij} = \lim_{\delta\tau \to 0} \frac{1}{\delta\tau} E\{[x_i(\tau + \delta_\tau) - x_i(\tau)][x_j(\tau + \delta_\tau) - x_j(\tau)]\}, \quad (11)$$

using the system equations (7),

$$b_{12} = b_{21} = b_{13} = b_{31} = b_{14} = b_{41} = b_{23} = b_{32} = b_{34} = b_{43} = 0, \quad (12)$$
$$b_{11} = 0 \quad b_{22} = a^2\omega_0^4 x_3^2 S, \quad b_{33} = 0, \quad b_{44} = \omega_0^4 x_3^2 S, \quad b_{24} = b_{42} = -a\omega_0^4 x_3^2 S.$$

Substituting the values of the first and of the second moments increments denoted by the relations (10) and (12) into *Fokker-Planc* equation we obtain,

$$\begin{aligned} \frac{\partial p}{\partial \tau} = &- \frac{\partial}{\partial x_1}(x_2 p) - \frac{\partial}{\partial x_2}[(-2\zeta x_2 - x_1 + a\omega_0^2 x_3)p] - \frac{\partial}{\partial x_3}(x_4 p) \\ &- \frac{\partial}{\partial x_4}[(\alpha\omega_0 x_4 - \frac{\gamma x_4^3}{\omega_0} - \omega_0^2 x_3 + b x_2)p] + \frac{1}{2}\frac{\partial^2}{\partial x_2^2}(a^2\omega_0^4 x_3^2 S p) \\ &- \frac{\partial^2}{\partial x_2 \partial x_4}(a\omega_0^4 x_3^2 S p) + \frac{1}{2}\frac{\partial^2}{\partial x_4^2}(\omega_0^4 x_3^2 S p). \end{aligned} \quad (13)$$

The analytical solution $p(x,\tau)$, of the partial equation (13) is not possible to determine. However, it is possible to obtain differential equations with respect to statistical moments of any order N of the solution process $p(x,\tau)$ by multiplying the equation (13) with $(x_1^k x_2^l x_3^m x_4^n$, whwre $k+l+m+n = N$ and by partially integration in within the limits $-\infty < x_i < \infty$. By apllying this procedure for $N \leq 2$ we obtain the differential equations with respect to the first and second statistical moments in which the following notations are introduced for the different statistical moments,

$$m_{klmn}(\tau) = E\{x_1^k x_2^l x_3^m x_4^n\} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x_1^k x_2^l x_3^m x_4^n p(x,\tau)\, dx_1\, dx_2\, dx_3\, dx_4. \tag{14}$$

For the first statistical moments the differential equations are,

$$\left.\begin{aligned}
&\frac{dm_{1000}}{d\tau} = m_{0100}, \quad \frac{dm_{0100}}{d\tau} = -2\zeta m_{0100} - m_{1000} + a\omega_0^2 m_{0010},\\
&\frac{dm_{0010}}{d\tau} = m_{0001}, \quad \frac{dm_{0001}}{d\tau} = \alpha\omega_0 m_{0001} - \frac{\gamma m_{0003}}{\omega_0} - \omega_0^2 m_{0010} + b m_{0100},
\end{aligned}\right\} \tag{15}$$

and for the second statistical moments we obtain the following by applying the same procedure,

$$\left.\begin{aligned}
\frac{dm_{2000}}{d\tau} &= 2m_{1100},\\
\frac{dm_{0200}}{d\tau} &= -4\zeta m_{0200} - 2m_{1100} + 2a\omega_0^2 m_{0110} + a^2\omega_0^4 S m_{0020},\\
\frac{dm_{0020}}{d\tau} &= 2m_{0011},\\
\frac{dm_{0002}}{d\tau} &= 2\alpha\omega_0 m_{0002} - 2\frac{\gamma m_{0004}}{\omega_0} - 2\omega_0^2 m_{0011} + \omega_0^4 S m_{0020} + 2b m_{0101},\\
\frac{dm_{1100}}{d\tau} &= m_{0200} - 2\zeta m_{1100} - m_{2000} + a\omega_0^2 m_{1010},\\
\frac{dm_{1010}}{d\tau} &= m_{0110} + m_{1001},\\
\frac{dm_{1001}}{d\tau} &= m_{0101} + \alpha\omega_0 m_{1001} - \frac{\gamma m_{1003}}{\omega_0} - \omega_0^2 m_{1010} + b m_{1100},\\
\frac{dm_{0110}}{d\tau} &= -2\zeta m_{0110} - m_{1010} + a\omega_0^2 m_{0020} + m_{0101},\\
\frac{dm_{0011}}{d\tau} &= m_{0002} + \alpha\omega_0 m_{0011} - \frac{\gamma m_{0013}}{\omega_0} - \omega_0^2 m_{0020} + b m_{0110},\\
\frac{dm_{0101}}{d\tau} &= (\alpha\omega_0 - 2\zeta) m_{0101} - m_{1001} + a\omega_0^2 m_{0011} - \frac{\gamma m_{0103}}{\omega_0}\\
&\qquad - \omega_0^2 m_{0110} - a\omega_0^4 S m_{0020} + b m_{0200}.
\end{aligned}\right\} \tag{16}$$

Analyzing the system of differential equations (15) and (16) with respect to the first and to the second statistical moments we can notice that the statistical moment of the third order appears in the first system of differential equations whereas the statistical moments of the third and of the fourth order appear in the system of differential equatins with respect to the other statistical moments. We assume that the solution process is the Gaussian one so that the statistical moments of higher order can be expressed by means of the statistical moments of the first and of the second order by relations deduced from the condition that all the cumulants of the third and of the higher order are equal to zero wherefrom the following relations follow,

$$\left.\begin{aligned} m_{0003} &= 3m_{0001}m_{0002} - 2(m_{0001})^3, \\ m_{0004} &= 4m_{0001}m_{0003} + 3(m_{0002})^2 - 12(m_{0001})^2 m_{0002} + 6(m_{0001})^4, \\ m_{1003} &= m_{1000}m_{0003} + 3m_{0001}m_{1002} + 3m_{1001}m_{0002} - 6m_{1000}m_{0001}m_{0002} \\ &\qquad - 6m_{1001}(m_{0001})^2 + 6m_{1000}(m_{0001})^3, \\ m_{0103} &= m_{0100}m_{0003} + 3m_{0001}m_{0102} + 3m_{0101}m_{0002} - 6m_{0010}m_{0001}m_{0002} \\ &\qquad - 6m_{0011}(m_{0001})^2 + 6m_{0010}(m_{0001})^3, \\ m_{0013} &= m_{0010}m_{0003} + 3m_{0001}m_{0012} + 3m_{0011}m_{0002} - 6m_{0010}m_{0001}m_{0002} \\ &\qquad - 6m_{0011}(m_{0001})^2 + 6m_{0010}(m_{0001})^3. \end{aligned}\right\} \tag{17}$$

If in the systems of differential equations (15) and (16) we substitute the statistical moments of the order higher than the second one by the relations (17) we obtain a closed system of fourteen differential equations. By numerical integration of this system we determine a nonstationary solution for the statistical moments of the solution process. A special problem in this numerical integration is the determination of the initial values of integration for the statistical moments. By making the right sides of these equations equal to zero we obtain a system of algebraic solution whose solution provides for the determination of stationary values of the solution process statistical moments.

4. Stationary solution

Let's consider the determination of stationary values of the system of differential equations (15) and (16) in which the statistical moments of the third and of the fourth order are substituted by the relations (17). As the result of this operation we obtain,

$$\left.\begin{aligned} & m_{1000} = m_{0100} = m_{0010} = m_{0001} = m_{1100} = m_{0011} = 0, \\ & -4\zeta m_{0200} + 2a\omega_0^2 m_{0110} + a^2\omega_0^4 S m_{0020} = 0, \\ & 2\alpha\omega_0 m_{0002} - 6\frac{\gamma m_{0002}^2}{\omega_0} + 2b m_{0101} + \omega_0^4 S m_{0020} = 0, \\ & m_{0200} - m_{2000} + a\omega_0^2 m_{1010} = 0, \quad m_{0110} + m_{1001} = 0, \\ & m_{0101} + \alpha\omega_0 m_{1001} - 3\frac{\gamma}{\omega_0} m_{1001}m_{0002} - \omega_0^2 m_{1010} = 0, \\ & -2\zeta m_{0110} - m_{1010} + a\omega_0^2 m_{0020} + m_{0101} = 0, \\ & (\alpha\omega_0 - 2\zeta)m_{0101} - m_{1001} - 3\frac{\gamma}{\omega_0} m_{0101}m_{0002} - \omega_0^2 m_{0110} \\ & \qquad - a\omega_0^4 S m_{0020} + b m_{0200} = 0, \\ & m_{0002} - \omega_0^2 m_{0020} + b m_{0110} = 0. \end{aligned}\right\} \tag{18}$$

After obvious algebraic transformations we obtain the following algebraic equation whose solutions are determined by the statistical moment m_{0002},

$$B_3 m_{0002}^3 + B_2 m_{0002}^2 + B_1 m_{0002} + B_0 = 0, \tag{19}$$

where the coefficients in the equation (19) B_0, B_1, B_2 and B_3 are equal to

$$B_3 = 54\frac{\gamma^3}{b\omega_0^2}, \tag{20}$$

$$B_2 = -9\frac{\gamma^2}{b\omega_0}[(2ab + S - 4\zeta)\omega_0^2 + 6\alpha\omega_0 - 4\zeta], \tag{21}$$

$$B_1 = \frac{3\gamma}{b}\Big[2(1-\omega_0^2)^2 - 2\zeta\omega_0(\alpha + \omega_0^3 S) + 2\omega_0(\alpha\omega_0 - \zeta)\times \tag{22}$$
$$\times\,(2ab\omega_0 - 4\zeta\omega_0 + \omega_0 S + 3\alpha) + \frac{ab\omega_0^2}{\zeta}\left(1 - \omega_0^2 - \frac{ab\omega_0^2 S}{2} + 2\zeta\omega_0^2 S\right)\Big],$$

$$B_0 = -\frac{\omega_0^2}{b}\Big\{\Big[1 - \omega_0^2 + \alpha\omega_0(\alpha\omega_0 - 2\zeta)$$
$$+\frac{ab\omega_0^2}{2\zeta}\Big][2ab\omega_0 - 4\zeta\omega_0 + 2\alpha + \omega_0 S(1-\omega_0^2)] - 2\omega_0(\alpha\omega_0 - 2\zeta)\times$$
$$\times\left[1 - \omega_0^2 - ab\omega_0^2 S + \frac{ab\omega_0^2}{4\zeta}(2 + abS) - \frac{\alpha\omega_0 - 2\zeta}{2}\omega_0^2 S\right]\Big\}, \tag{23}$$

and then by substitution in the following expressions we obtain the statistical moments of the second order,

$$m_{0020} = 2\frac{\omega_0(\alpha\omega_0 - 2\zeta)m_{0002} - 3\gamma m_{0002}^2}{2\omega_0^2(\alpha - 2\zeta\omega_0) + \omega_0^3(1-\omega_0^2)S + 2ab\omega_0^3 - 6\gamma m_{0002}}, \tag{24}$$

$$m_{0200} = \frac{\omega_0^4}{4\zeta}\left(\frac{2a}{b} + a^2 S\right)m_{0020} - \frac{a\omega_0^2}{2b\zeta}m_{0002}, \tag{25}$$

$$m_{2000} = -\frac{a}{2b}\left(2\alpha\omega_0 m_{0002} - 6\frac{\gamma}{\omega_0}m_{0002}^2 + \omega_0^4 S m_{0020}\right) + \frac{a\omega_0^4}{4\zeta}\left(\frac{2}{b} + aS\right)m_{0020}$$
$$-\frac{a\omega_0^2}{2b\zeta}m_{0002} - \frac{a}{b}\left(\alpha\omega_0^3 m_{0020} - \alpha\omega_0 m_{0002} - 3\gamma\omega_0 m_{0002}m_{0020} + \frac{3\gamma}{\omega_0}m_{0002}^2\right). \tag{26}$$

As it has been shown for the oscillatory model in Fig. 1. it is possible to determine the stationary values for the statistical moments of the solution process. This comes to the solution of the equation (19), and then to the substitution in the equations (24), (25) and (26); thus we can simply determine the second order moments of the solution process. The numerical solution of the equation (19) is carried out for the following dimensionless system parameters: $\omega_0 = 1.1$, $a = 0.002$, $b = 0.4$, $\alpha = 0.02$, $\gamma = 0.667$ and $\zeta = 0.0015$ for which only one positive root exists for various values of the spectral density S of the velocity process of the fluid flow turbulence $f(t)$. Fig. 2 shows the results obtained for the statistical moments of displacement and velocity in the direction of x-axis (m_{2000} and m_{0200}) and Fig. 3 shows the values obtained for the statistical moments of the lift force and its fluctuation velocity (m_{0020} and m_{0002}).

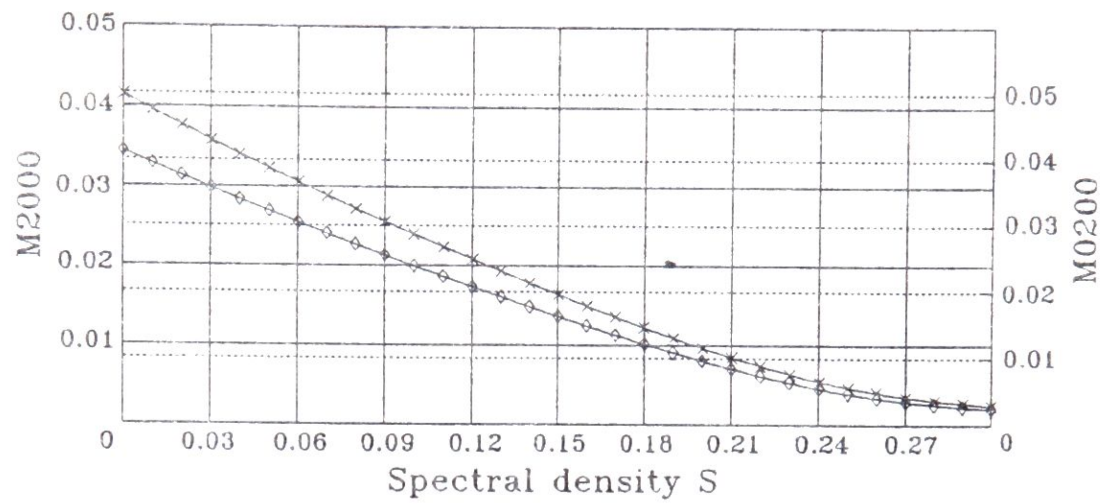

Fig. 2. Statistical moments displacement and velocity
—×— M2000 —◇— M0200

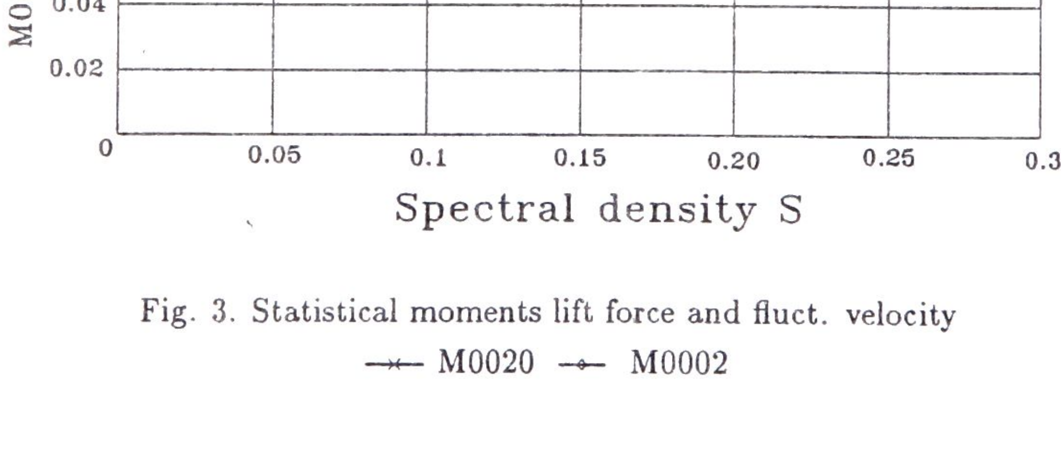

Fig. 3. Statistical moments lift force and fluct. velocity
—×— M0020 —◇— M0002

5. Conclusion

An oscillatory system with two degrees of freedom is analysed when influenced by stationary wide-band *Gaussian* random process by introducing the *Markov* approximation vector. The differential equations with respect to the first and to the second statistical moments of the solution process comprise the statistical moments of the third and of the fourth order and they are determined by using the *Fokker-Planck* equations. Introducing the assumption that the statistical moments of the order higher than the third one can be expressed by the moments of the lower order, the system of differential equations with respect to the statistical moments is reduced to the closed system. The stationary values of the solution process are numerically determined as well as the moments of the first and of the second order and these values are graphically shown in Figs. 2 and 3 as the functions of the spectral density S of the velocity process of the fluid flow turbulence.

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ERSATZMMODELL DER DURCH WIRBELSTROM HERVORGERUFENEN SCHWINGUNGEN BEI DER WIRKUNG EINER BREITBAENDIGEN ZUFALLSERREGUNG

Es wurde ein Schwingungssystem mit zwei Bewegungsfreicheitsgraden bei der Wirkung eines stationaeren breitbaendigen *Gauss'schen* Zufallprozesses analysiert in dem einen Aproksimationsvektor nach *Markow* eingezogen wurde. Die Diferentialgleichungen nach einen und zweitem statistischen Momente des Loesungsprozesses erhalten auch die statistische Momente der driten und vierten Ordnung und werden an hand der *Fokker-Planck* Gleichung bestimmt. Mit der Voraussetzung dass sich die statistische Momente hoeherer Ordnung (hoeher 3) mit den Momenten niedriger Ordnung darstellen koennen, dann wird das System der Diferentialgleichungen auf das sgn. "geschlossene System" reduziert. Die stationaere Werte des Loesungprozesses koennen dann numerisch mit Hilfe von Momenten der ersten und

zweiten Ordnung ermittelt werden. Diese Werte lassen sich grafisch (Bild 2 und 3), als Funktionen der Spektraldichte S des turbulenten Geschwindigkeitsprozesses des Fluidstroms darstellen.

OSCILATORNI MODEL VRTLOGOM-IZAZVANIH OSCILACIJA PRI DEJSTVU ŠIROKOPOJASNE SLUČAJNE POBUDE

Analiziran je oscilatorni sistem sa dva stepena slobode pri dejstvu stacionarnog širokopojasnog *Gauss*-ovog slučajnog procesa, uvođenjem *Markov*-ljevog vektora aproksimacije. Diferencijalne jednačine po prvim i drugim statističkim momentima procesa rešenja sadrže statističke momente trećeg i četvrtog reda, a određene su korišćenjem *Fokker-Planck*-ove jednačine. Uvodeći pretpostavku da se statistički momenti višeg reda od trećeg mogu izraziti momentima nižeg reda, sistem diferencijalnih jednačina po statističkim momentima redukuje se na "zatvoreni sistem". Određene su numerički, stacionarne vrednosti procesa rešenja, momentima prvog i drugog reda i ove vrednosti su grafički prikazane na slikama br. 2 i br. 3 u funkciji od spektralne gustine S procesa brzine turbulencije struje fluida.

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