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THE STRESS STATE OF THE ELLIPTICAL-ANNULAR PLATE BY THE COMPLEX VARIABLE FUNCTION AND CONFORMAL MAPPING METHOD

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Introduction

An inclusive survey of the research on the problem of the plain stress state and of the plain strain state can be found in the paper [27] by P. P. Tedorescu. The application of the complex variable function first time appears in solving the problems of the plain elasticity theory in the papers of G. V. Kolosov (1909). N. I. Mushelishvili has given fundamental contribution in the field of the systematic development and application of the complex variable function to the theory of elasticity problems. These contributions are summarized in the well-known monograph [7] of the mathematical thery of elasticity. This monograph republished in 1966. gives a short survey of papers and authors who have given further contribution to the application of the complex variable function method to the elasticity theory problems. Among the listed authors we would like to emphasize the papers of D. I. Sherman in the period from 1949. to 1959. He dealt with the boundary conditions problesm for double-connected areas and gave an important contribution to the complex variable function to the study of the stress state of the multi-holed plates.

The paper of Hedrih, Jecić and Jovanović [19] and [21] give an analysis of the main stresses state at the points of the elliptical-annular plate contour stressed by one and two pairs of the concentrated forces. In this analysis the photoelastic experimental method is used by which the isochrome and isocline families are obtained for the tree cases of stress induced either by a pair or by pairs of concentrated forces. By these isochrome and isocline families the main stress distribution is determined at the points of the external and internal contours of the elliptical-annular plate and respective graphic displays are made.

In our expose [20] at the congress of the Yugoslav Society of Mechanics held in 1990. we gave our contribution to the application of the complex variable function and of the conformal mapping to the study of the stress state of the plain stressed plates whose contours can be expressed by means of the confocal ellipses and arches of the hyperboles from the respective families of the orthogonal curves. The stress tensor components are derived in the system of the hyperbolic-elliptical coordinates with analytical functions of the complex variable z in the conformally mapped plane ζ. By means of these expressions in this paper we have derived the expressions for the stress tensor components and displacement vector in the system of hyperbolic-elliptical coordinates at the points of the elliptical-annular plate segmentally stressed by the stress distributed along the external and internal contour.

1. Definition of the problem of the plain stressed elliptical-annular plate

The subject of our analysis is the stress state of the elliptical-annular plate segmentally stressed along the external and internal contour by the stress distributed in the middle plane in the form of pressure perpendicular to the contours of the plate as shown in the Fig. 1. Let $p_s(\varphi)$ and $p_u(\varphi)$ denote the pressures on the

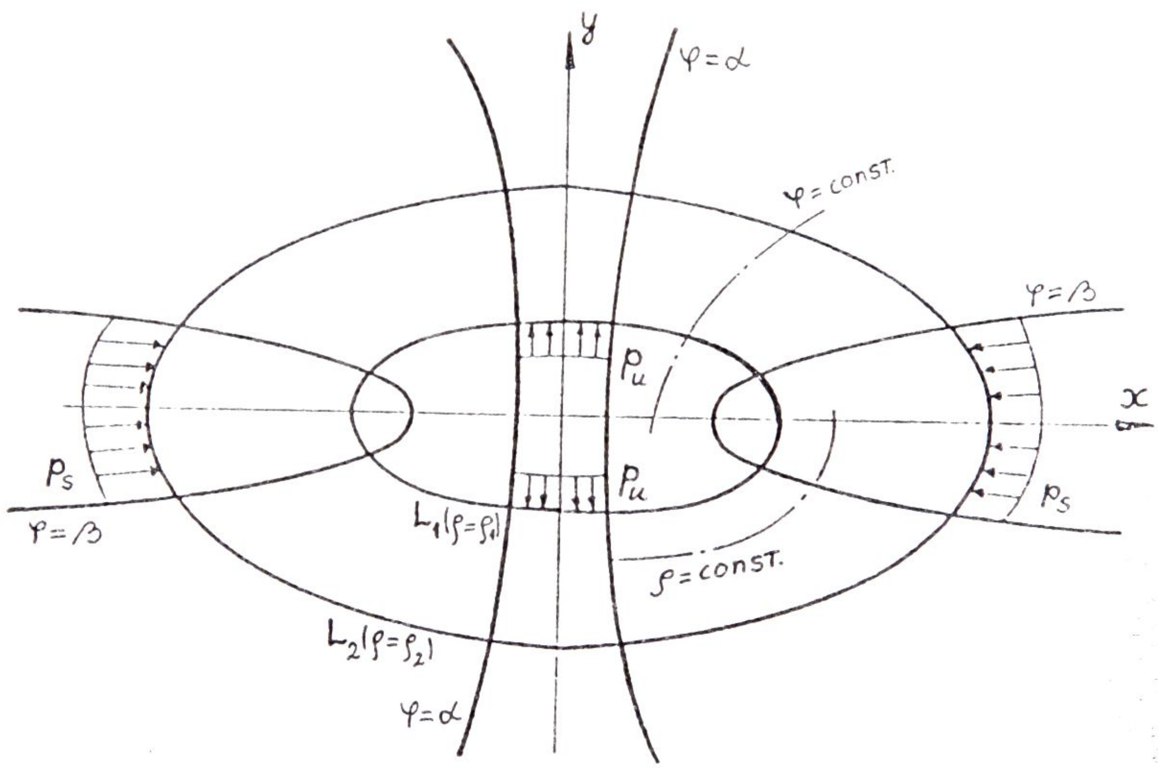

Fig. 1

external and internal contour and α and β parameters by which in the hyperbolic-elliptical coordinates we give the contour segments along which these pressures are normally distributed. Let's use the hyperbolic-elliptical coordinate system with the coordinates ρ and φ, where their relations with the Descartes coordinates is given in the form:

$$x = R\left(\rho + \frac{m}{\rho}\right)\cos\varphi, \qquad y = R\left(\rho + \frac{m}{\rho}\right)\sin\varphi$$

$$\left[\frac{x}{R(\rho + m/\rho)}\right]^2 + \left[\frac{y}{R(\rho - m/\rho)}\right]^2 = 1, \quad \left[\frac{x}{R\cos\varphi}\right]^2 - \left[\frac{y}{R\sin\varphi}\right]^2 = 1 \qquad (1)$$

therefore for $\rho =$ const. we obtain ellipses and for $\varphi =$ const. we obtain a hyperbola from the orthogonal curves family. Along the hyperbola the parameter p changes, whereas along the ellipse the parameter ρ changes. For $\rho = \rho_2$ the internal ellipse contour is defined. By using thus adopted coordinates of hyperbolic-elliptical coordinate system and according to the Fig. 1. the boundary conditions can be written in the form:

a) for the points on the internal contour

$$\sigma_\rho(\rho,\varphi) = \begin{cases} -p_u & \text{for } \varphi \in [\pi/2-\alpha, \pi/2+\alpha] \cup [3\pi/2-\alpha, 3\pi/2+\alpha] \\ 0 & \text{for } \varphi \in (0, \pi/2-\alpha) \cup (\pi/2+\alpha, 3\pi/2-\alpha) \cup (3\pi/2+\alpha, 2\pi) \end{cases}$$

$$\tau_{\rho\varphi}(\rho,\varphi) = 0 \qquad \text{for } \varphi \in (0, 2\pi) \qquad (2)$$

b) for the points on the external contour

$$\sigma_\rho(\rho,\varphi) = \begin{cases} -p_u & \text{for } 2\pi-\beta \le \varphi \le 2\pi,\ 0 \le \varphi \le \beta,\ \pi-\beta \le \varphi \le \pi+\beta \\ 0 & \text{for } \varphi \in (\beta, \pi/2+\beta) \cup (\pi+\beta, 2\pi-\beta) \end{cases}$$

$$\tau_{\rho\varphi}(\rho,\varphi) = 0 \qquad \text{for } \varphi \in (0, 2\pi) \qquad (3)$$

2. Essentials of the complex variable function method with interpretation within the hyperbolic-elliptical coordinate system

For determining the stress tensor components in the hyperbolic-elliptical system, that is, the normal stresses σ_h and σ_e at the points of plate for the sections with the normals in the direction of the tangent lines to the orthogonal family of hyperboles, that is, the ellipses and shear stresses τ_{he}, that is, τ_{eh} respective to these planes, we use the complex variable function method requiring that the stress biharmonic function $\Phi(x,y)$, should be expressed by means of the complex variable analytical function [10], [7], [5], [16] in the form:

$$\Phi = \mathrm{Re}\{\bar{z}F(z) + X(z)\} \qquad (4)$$

where z is a complex argument. Since the elliptical-annular plate contours in the plane z can be mapped in two concentric circles in the ζ plane by means of the mapping function:

$$z = \omega(\zeta) = R\left(\zeta + \frac{m}{\zeta}\right), \qquad R = \frac{a+b}{2}, \quad m = \frac{a-b}{a+b} \qquad (5)$$

in which then by means of the same mapping function the hyperbolic-elliptical coordinate system, coordinate lines — confocal ellipses and the hyperboles orthogonal to them, map in the concentric circles family and a beam of straight lines.

We use the following transformation formulas (15), (16) and (17) given in the paper of Hedrih and Jovanović [20]:

$$\sigma_\rho + \sigma_\varphi = 4\,\mathrm{Re}\left\{\frac{F_1'(\zeta)}{\omega'(\zeta)}\right\} = 2\left[\frac{F_1'(\zeta)}{\omega'(\zeta)} + \frac{\overline{F}_1'(\bar{\zeta})}{\overline{\omega}'(\bar{\zeta})}\right] \tag{6}$$

$$\sigma_\varphi - \sigma_\rho + 2i\tau_{\rho\varphi} = \frac{\zeta^2}{\rho^2}\frac{2}{\overline{\omega}'(\bar{\zeta})[\omega'(\zeta)]^2}\left[\overline{\omega}(\bar{\zeta})F_1''(\zeta)\omega'(\zeta) - \right.$$
$$\left. - \overline{\omega}(\bar{\zeta})F_1'(\zeta)\omega''(\zeta) + X_1''(\zeta)\omega'(\zeta) - X_1'(\zeta)\omega''(\zeta)\right] \tag{7}$$

$$u_\rho + iu_\varphi = \frac{\overline{\omega'(\zeta)}}{|\omega'(\zeta)|}\frac{\bar{\zeta}}{\rho}\left\{\frac{3-\mu}{E}F_1(\zeta) - \frac{1+\mu}{E}\left[\frac{\omega(\zeta)}{\overline{\omega}'(\bar{\zeta})}\overline{F}_1'(\bar{\zeta}) + \frac{\overline{X}_1'(\bar{\zeta})}{\overline{\omega}'(\bar{\zeta})}\right]\right\} \tag{8}$$

which are in this paper derived by means of the conformal mapping of the z plane into the plane by given function (5). These formula give the relation between the stress tensor components in the hyperbolic-elliptical coordinate system and analytical functions $F_1(\zeta)$ and $\chi_1(\zeta)$ complex variable ζ, and the mapping function $\omega(\zeta)$.

In the previous expressions (6), (7) and (8) the most prominent are the functions $F_1(\zeta) = F(\omega(\zeta))$ and $\chi_1(\zeta) = \chi(\omega(\zeta))$, and can be directly represented in the mapping plane ζ in the form of the Laurent series along the complex variable $\zeta = \rho e^{i\varphi}$ with the unknown coefficients A_n and B_n. The unknown coefficients of these series are determined from the boundary conditions (2) and (3), along the elliptical-annular plate whose stress state is being studied. By reason of an infinite number of coefficients the reduction of the number of the members of the series to the finite number in concrete calculations is defined with a desired approximate accuracy of the boundary conditions.

3. Determination of the boundary conditions development coefficients

Using the boundary conditions (2) and (3) for the sake of the comfortable application of the expressions (6), (7) and (8) let's write the boundary conditions by introducing the series:

$$[\sigma_\rho - i\tau_{\rho\varphi}]_{\rho=\rho_1} = \sum_{-\infty}^{\infty} C_n^{(1)} e^{in\varphi} = -p_u(\varphi) \tag{9}$$

$$[\sigma_\rho - i\tau_{\rho\varphi}]_{\rho=\rho_2} = \sum_{-\infty}^{\infty} C_n^{(2)} e^{in\varphi} = -p_s(\varphi) \tag{10}$$

with the development coefficients $C_n^{(1)}$ and $C_n^{(2)}$ in the form:

$$C_n^{(1)} = -\frac{1}{2\pi}\int_0^{2\pi} p_u(\varphi)\,d\varphi, \tag{11}$$

$$C_n^{(2)} = -\frac{1}{2\pi}\int_0^{2\pi} p_s(\varphi)\,d\varphi. \tag{12}$$

After calculating the integral in the expressions (11) and (12) according to the boundary conditions (2) for these development coefficients we obtain:

$$\begin{aligned} C_0^{(1)} &= -\frac{2p_u\alpha}{\pi}\,, \\ C_{2k}^{(1)} &= -\frac{(-1)^k p_u}{\pi k}\sin 2k\alpha = -\frac{p}{\pi k}e^{ik\pi}\sin 2k\alpha\,, \\ C_{2k+1}^{(1)} &= 0 \end{aligned} \tag{11'}$$

$$\begin{aligned} C_0^{(2)} &= -\frac{2p_s\beta}{\pi}\,, \\ C_{2k}^{(2)} &= -\frac{p_s}{\pi k}\sin 2k\alpha\,, \\ C_{2k+1}^{(1)} &= 0\,. \end{aligned} \tag{12'}$$

By analyzing we conclude that all the coefficients $C_{2n+1}^{(1)}$ and $C_{2n+1}^{(2)}$ with odd indexes are equal to zero, except the coefficient $C_{2n}^{(1)}$ and $C_{2n}^{(2)}$ with even indexes which are different then zero.

4. Determination of the coefficients A_n and B_n of the development of the analytical functions $F_1(\zeta)$ and $\chi_1(\zeta)$ in the Laurent series

Let's represent the functions $F_1(\zeta)$ and $\chi_1(\zeta)$ in the form of their derivations along the complex variable ζ as the Laurent series:

$$F_1'(\zeta) = \sum_{n=0}^{\infty} A_n\zeta^n\,, \qquad X_1''(\zeta) = \sum_{n=0}^{\infty} B_n\zeta^n$$

$$F_1(\zeta) = A_0\zeta + A_{-1}\ln\zeta + \sum_{\substack{n=-\infty \\ n\neq -1,0}}^{\infty} \frac{A_n\zeta^{n+1}}{n+1} + c_1$$

$$X_1'(\zeta) = B_0\zeta + B_{-1}\ln\zeta + \sum_{\substack{n=-\infty \\ n\neq -1,0}}^{\infty} \frac{B_n\zeta^{n+1}}{n+1} + c_2 \tag{13}$$

with unknown coefficients A_n and B_n which would be determined from the boundary conditions by means of the relation between the stress at the points on the stressed elliptical-annular plate contour and assumed analytical functions of the complex variable. Therefore the expressions (6), (7) or (8) are written for the points on the external and internal contours by means of the series (13) and made equal with the expressions on the right side of the expressions (9) and (10) in which the development coefficients $C_n^{(1)}$ and $C_n^{(2)}$ are known by using the expressions (12). By transforming the expressions — sums according to the indexes n and by making equal the coefficients on the left and on the right sides with the equal degrees of the complex unit $e^{in\varphi}$ we obtain the desired relations between the coefficients A_n, $\overline{A}_n$,

B_n, $\overline{B}_n$ with the coefficients $C_n^{(1)}$ and $C_n^{(2)}$ as well as ρ_1 and ρ_2 defining contours. In order to simplify these relations according to the concrete defined stress problem of the elliptical-annular plate we carry the following analysis.

Let's now consider the fact that some of coefficients are equal to zero on the basis of the defined boundary conditions, the characteristics of a symmetry of the elliptical-annular plate and from the symmetry of the given stress, as well as from the limited value of the plate points displacements.

Since the displacements area in the hyperbolic-elliptical coordinates is given in the form:

$$u_\rho + i u_\varphi = \frac{1}{E}\left\{(3-\mu)\sum_{-\infty}^{\infty}\frac{A_{2k}\zeta^{2k+1}}{2k+1} - (1+\mu)\left[\frac{\zeta + m/\zeta}{1-m/\overline{\zeta}^2}\sum_{-\infty}^{\infty}\overline{A}_{2k}\overline{\zeta}^{2k} + \right.\right.$$
$$\left.\left. + R^{-1}\left(1-\frac{m}{\overline{\zeta}^2}\right)^{-1}\sum_{-\infty}^{\infty}\frac{B_{2k}}{2k+1}\zeta^{2k+1}\right]\right\}\frac{\overline{\zeta}}{\rho}\frac{1-m/\overline{\zeta}^2}{|1-m/\overline{?}^2|} \tag{14}$$

and in order to define unanimously the displacement vector from the previous expression (14) it follows that the coefficient next to the denoted member would be equal to zero, hence we conclude that $A_{-1} = 0$ and $B_{-1} = 0$.

From the condition that the elliptical-annular plate as well as the given external stress are with two symmetry axes, we conclude the that $A_{-n} = A_n$ and $B_{-n} = B_n$.

If we use the previous conclusions, and since the coefficients $C_n^{(1)}$ and $C_n^{(2)}$ are real, we can assume that the coefficients $A_n = \overline{A}_n$ are real, then from the boundary conditions we obtain the following relation:

$$\left[R\left(1-\frac{m}{\zeta^2}\right)\right]^{-1}\sum_{-\infty}^{\infty}A_n\zeta^n\Big|_{\zeta=\zeta_\circ} + \left[R\left(1-\frac{m}{\overline{\zeta}^2}\right)\right]^{-1}\sum_{-\infty}^{\infty}\overline{A}_n\overline{\zeta}^n\Big|_{\zeta=\zeta_\circ} -$$
$$-\left\{R\left(\overline{\zeta}+\frac{m}{\overline{\zeta}^2}\right)R^{-3}\left(1-\frac{m}{\zeta^2}\right)^{-3}\left[\left(\sum_{-\infty}^{\infty}nA_n\zeta^{n-1}\right)\times\right.\right.$$
$$\left.\times R\left(1-\frac{m}{\zeta^2}\right) - \left(\sum_{-\infty}^{\infty}A_n\zeta^n\right)\frac{2mR}{\zeta^3}\right] +$$
$$\left. + R^{-3}\left(1-\frac{m}{\zeta^2}\right)^{-3}\left[\left(\sum_{-\infty}^{\infty}B_n\zeta^n\right)R\left(1-\frac{m}{\zeta^2}\right) - \left(B_{-1}\ln\zeta + \sum_{-\infty}^{\infty}B_n\frac{\zeta^{n+1}}{n+1}\right)\frac{2mR}{\zeta^3}\right]\right\}\cdot$$
$$\cdot\frac{\zeta^2 R(1-m/\zeta^2)}{\rho^2 R(1-m/\overline{\zeta}^2)}\Big|_{\zeta=\zeta_\circ} = \sum_{-\infty}^{\infty}C_n^{(s)}e^{in\varphi} \tag{15}$$

If we use the relation (15) then the relations between unknown and known coefficients can only be written in the function of six unknown and eight known coefficients A_n, A_{n+2}, A_{n-2}, A_{n+4}, B_{n-2}, B_n, and $C_n^{(1)}$, $C_n^{(2)}$, $C_{n+2}^{(1)}$, $C_{n+2}^{(2)}$, $C_{n-2}^{(1)}$, $C_{n-2}^{(2)}$, $C_{n+4}^{(1)}$, $C_{n+4}^{(2)}$:

$$\left\{A_n[(1-n)\rho^{n+2}+(3+n)m^2\rho^{n-2}+\rho^{-(n-2)}]+\right.$$
$$+mA_{n+2}[(n+3)\rho^{n+2}-2\rho^{-(n+2)}]-m(n-1)\rho^{n-2}A_{n-2}+$$
$$\left.+m^2\rho^{-(n+6)}A_{n+4}-\frac{1}{R}\rho^n B_{n-2}+\frac{m}{R}\frac{n+3}{n+1}\rho^n B_n\right\}\bigg|_{\rho=\rho_s}=$$
$$=R\left[\rho^2\left(1+2\frac{m^2}{\rho^4}\right)C_n^{(s)}+m\left(2+\frac{m^2}{\rho^4}\right)C_{n+2}^{(s)}+\frac{m^2}{\rho^2}C_{n+4}^{(s)}-mC_{n-2}^{(s)}\right]\bigg|_{\rho=\rho_s},$$
$$s=1,2. \quad (16)$$

If we introduce the following notations in the previous equations:

$$L_n^{(s)}(\rho)=R\left[\rho^{-(n-2)}\left(1+2\frac{m^2}{\rho^4}\right)C_n^{(s)}+m\rho^{-n}\left(2+\frac{m^2}{\rho^4}\right)C_{n+2}^{(s)}+\right.$$
$$\left.+\frac{m^2}{\rho^2}\rho^{-n}C_{n+4}^{(s)}-m\rho^{-n}C_{n-2}^{(s)}\right]\bigg|_{\rho=\rho_s} \quad (17)$$

and if expressions (16) and (17) are divided by ρ_1^n and ρ_2^n respectively then we write them in the following form:

$$\left\{A_n\left[(1-n)\rho^2+(3+n)\frac{m^2}{\rho^2}+\rho^{-2(n-1)}\right]+\right.$$
$$+mA_{n+2}[(n+3)\rho^2-2\rho^{-2(n+1)}]-m(n-1)\rho^{-2}A_{n-2}+$$
$$\left.+m^2\rho^{-2(n+3)}A_{n+4}-\frac{1}{R}B_{n-2}+\frac{m}{R}\frac{n+3}{n+1}B_n\right\}\bigg|_{\rho=\rho_s}=L_n^{(s)}(\rho)\big|_{\rho=\rho_s}. \quad (18)$$

If we introduce the following notation:

$$E_n(\rho_1,\rho_2)=L_n^{(2)}(\rho_2)-L_n^{(1)}(\rho_1) \quad (19)$$

and then we make subtract of the equations (18) we obtain the following equations along A_{n-2}, A_n, A_{n+2}, A_{n+4} in the following form:

$$m(1-n)(\rho_1^{-2}-\rho_2^{-2})A_{n-2}+$$
$$+A_n[(1-n)(\rho_2^{-2}-\rho_1^{-2})+(n+3)m^2(\rho_2^{-2}-\rho_1^{-2})+\rho_2^{-2(n-1)}-\rho_1^{-2(n-1)}]+$$
$$+mA_{n+2}[(n+3)(\rho_2^{-2}-\rho_1^{-2})-2(\rho_2^{-2(n+1)}-\rho_1^{-2(n+1)})]+$$
$$+A_{n+4}m^2(\rho_2^{-2(n+3)}-\rho_1^{-2(n+3)})=$$
$$=E_n(\rho_1,\rho_2) \qquad n=-\infty,\dots,-2,-1,0,1,2,\dots,\infty \quad (20)$$

If we introduce $n=-4,-2,0,2$ and if we take into consideration that $A_n=A_{-n}$, so that we can obtain four non-homogeneous algebraic equations with only four unknowns A_0, A_2, A_4 and A_6 in the form:

$$D\{A\}=E \quad (21)$$

where we have introduced the following notations

$$\{A\} = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_6 \end{bmatrix}, \qquad E = \begin{bmatrix} E_{-4}(\rho_1,\rho_2) \\ E_{-2}(\rho_1,\rho_2) \\ E_0(\rho_1,\rho_2) \\ E_2(\rho_1,\rho_2) \end{bmatrix} = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{bmatrix} \tag{22}$$

$$D = (a_{ik})\underset{k}{\downarrow}^{\rightarrow i} = \begin{bmatrix} a_{11} & a_{21} & a_{31} & a_{41} \\ a_{12} & a_{22} & a_{32} & a_{42} \\ a_{13} & a_{23} & a_{33} & a_{43} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} \tag{23}$$

$$\begin{aligned}
a_{11} &= m^2(\rho_2^2 - \rho_1^2) \\
a_{21} &= -m[(\rho_2^2 - \rho_1^2) + 2(\rho_2^6 - \rho_1^6)] \\
a_{31} &= 5(\rho_2^2 - \rho_1^2) - m^2(\rho_2^{-2} - \rho_1^{-2}) + (\rho_2^{10} - \rho_1^{10}) \\
a_{41} &= 5m(\rho_2^{-2} - \rho_1^{-2}) \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
a_{12} &= -m(\rho_2^2 - \rho_1^2) \\
a_{22} &= [3(\rho_2^2 - \rho_1^2) + 2m^2(\rho_2^{-2} - \rho_2^{-2}) + (\rho_2^6 - \rho_1^6)] \\
a_{32} &= 3m(\rho_2^{-2} - \rho_1^{-2}) \\
a_{42} &= 0 \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
a_{13} &= 2(\rho_2^2 - \rho_1^2) + 3m^2(\rho_2^{-2} - \rho_1^{-2}) \\
a_{23} &= m[3(\rho_2^2 - \rho_1^2) - (\rho_2^{-2} - \rho_1^{-2})] \\
a_{33} &= m^2(\rho_2^{-6} - \rho_1^{-6}) \\
a_{43} &= 0 \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
a_{14} &= -m(\rho_2^{-2} - \rho_1^{-2}) \\
a_{24} &= -(\rho_2^2 - \rho_1^2) + (5m^2 + 1)(\rho_2^{-2} - \rho_1^{-2}) \\
a_{34} &= m[5(\rho_2^2 - \rho_1^2) - 2(\rho_2^{-6} - \rho_1^{-6})] \\
a_{44} &= m^2(\rho_2^{-10} - \rho_1^{-10})
\end{aligned} \tag{23'}$$

$$\begin{aligned}
E_n(\rho_1,\rho_2) = R\Bigg[& C_n^{(2)}\rho_2^{2-n}\left(1 - \frac{2m^2}{\rho_2^4}\right) - C_n^{(1)}\rho_1^{2-n}\left(1 + \frac{2m^2}{\rho_1^4}\right) - \\
& - m\rho_2^{-n}\left(2 + \frac{m^2}{\rho_2^4}\right)C_{n+2}^{(2)} + m\rho_1^{-n}\left(2 + \frac{m^2}{\rho_1^4}\right)C_{n+2}^{(1)} + \\
& + \frac{m^2}{\rho_2^2}\rho_2^{-n}C_{n+4}^{(2)} - \frac{m^2}{\rho_1^2}\rho_1^{-n}C_{n+4}^{(1)} - m\rho_2^{-n}C_{n-2}^{(2)} + m\rho_1^{-n}C_{n-2}^{(1)}\Bigg]
\end{aligned} \tag{24}$$

$$l_1 = E_n(\rho_1, \rho_2)\big|_{n=-4} =$$
$$= R\Big\{ m^2(\rho_2^2 C_0^{(2)} - \rho_1^2 C_0^{(1)}) - C_4^{(1)}\rho_1^6\left(1 + \frac{2m^2}{\rho_1^4}\right) + C_4^{(2)}\rho_2^6\left(1 - \frac{2m^2}{\rho_2^4}\right) -$$
$$- m\left[\rho_2^4\left(2 + \frac{m^2}{\rho_2^4}\right)C_2^{(2)} + \rho_1^4\left(2 + \frac{m^2}{\rho_1^4}\right)C_2^{(1)}\right] - m(\rho_2^4 C_6^{(2)} + \rho_1^4 C_6^{(1)})\Big\}$$
$$l_2 = E_n(\rho_1, \rho_2)\big|_{n=-2} = R\Big\{ C_2^{(2)}\rho_2^4\left(1 + \frac{3m^2}{\rho_2^4}\right) - C_2^{(1)}\rho_1^4\left(1 + \frac{3m^2}{\rho_1^4}\right) -$$
$$- m\left[\rho_2^2\left(2 + \frac{m^2}{\rho_2^4}\right)C_0^{(2)} + \rho_1^2\left(2 + \frac{m^2}{\rho_1^4}\right)C_0^{(1)}\right] + m(\rho_2^2 C_4^{(2)} + \rho_1^2 C_4^{(1)})\Big\}$$
$$l_3 = E_n(\rho_1, \rho_2)\big|_{n=0} = R\Big\{ C_0^{(2)}\rho_2^2\left(1 + \frac{2m^2}{\rho_2^4}\right) - C_0^{(1)}\rho_1^2\left(1 + \frac{2m^2}{\rho_1^4}\right) -$$
$$- m\left[\left(3 + \frac{m^2}{\rho_2^4}\right)C_2^{(2)} + \left(3 + \frac{m^2}{\rho_1^4}\right)C_2^{(1)}\right] + m^2\left(\frac{1}{\rho_2^2}C_4^{(2)} - \frac{1}{\rho_1^2}C_4^{(1)}\right)\Big\}$$
$$l_4 = E_n(\rho_1, \rho_2)\big|_{n=2} = R\Big\{ C_2^{(2)}\left(1 + \frac{2m^2}{\rho_2^4}\right) - C_2^{(1)}\left(1 + \frac{2m^2}{\rho_1^4}\right) -$$
$$- m\left[\left(2 + \frac{m^2}{\rho_2^4}\right)\frac{1}{\rho_2^2}C_4^{(2)} + \left(2 + \frac{m^2}{\rho_1^4}\right)\frac{1}{\rho_1^2}C_4^{(1)}\right] +$$
$$+ m^2\left(\frac{1}{\rho_2^4}C_6^{(2)} - \frac{1}{\rho_1^4}C_6^{(1)}\right) - m\left(\frac{1}{\rho_2^2}C_0^{(2)} - \frac{1}{\rho_1^2}C_0^{(1)}\right)\Big\} \tag{24'}$$

Then the solution of the system of equations (21) according to the unknown coefficients A_0, A_2, A_4 and A_6 are obtained in the form:

$$A_0 = \frac{D_1}{D}, \qquad A_2 = \frac{D_2}{D}, \qquad A_4 = \frac{D_3}{D}, \qquad A_6 = \frac{D_4}{D} \tag{25}$$

in which for the sake of the simplification the following notation is introduced:

$$D = |a_{ik}| = \sum_{j=1}^{4} a_{ij}K_{ij} \qquad D = \sum_{j=1}^{4} l_j K_{ij} \tag{26}$$

of the system determinant (21) and of the determinant cofactors.

By means of these constants expressed in the function $C_0^{(1)}$, $C_0^{(2)}$, $C_2^{(1)}$, $C_2^{(2)}$, $C_4^{(1)}$, $C_4^{(2)}$, $C_6^{(1)}$, $C_6^{(2)}$ and m and R, ρ_1 and ρ_2, we express all the other coefficients A_n and B_n from the equations (20) and (16) and (18) by writing them further for $n = 2k$, $k = 2, 3, 4, 5, 6, \ldots$ so that we always obtain another new equation with another new unknown A_{2k+4} or B_{2k} determined by the four or five previously determined coefficients. The coefficients determined by the following recurrent formulas:

$$A_{n+4} = \frac{1}{m^2(\rho_2^{-2(n+3)} - \rho_1^{-2(n+3)})}\{[L_n^{(2)}(\rho_2) - L_n^{(1)}(\rho_1)] +$$

$$+ m(n-1)(\rho_2^{-2}-\rho_1^{-2})A_{n-2} - A_n[-(n-1)(\rho_2^2-\rho_1^2)+ + (n+3)m^2(\rho_2^{-2}-\rho_1^{-2}) + \rho_2^{-2(n-1)} - \rho_1^{-2(n-1)}] - - mA_{n+2}[(n+3)(\rho_2^2-\rho_1^2) - 2(\rho_2^{-2(n+1)} - \rho_1^{-2(n+1)})]\} \tag{27}$$

$$B_0 = \frac{R}{15m^2-3}\{5mL_0^{(s)}(\rho) + 3L_2^{(s)}(\rho) - mA_0[10\rho^2 + 3(5m^2-1)\rho^{-2}] - - A_2[3(5m^2-1)\rho^2 + (10m^2+3)\rho^{-2}] - - mA_4[(5m^2-6)\rho^{-6} + 15\rho^2] - 3m^2\rho^{-10}A_6\} \tag{28}$$

$$B_2 = \frac{R}{5m^2-1}\{L_0^{(s)}(\rho) + 3mL_2^{(s)}(\rho) - 2A_0\rho^2 - mA_2(2+15m^2)\rho^{-2} - - m^2(15\rho^2 - 5\rho^{-6})A_4 - 3m^3\rho^{-10}A_6\} \tag{29}$$

$$B_n = \frac{R(n+1)}{m(n+3)}\left\{ L_n^{(s)}(\rho) - A_n\left[(1-n)\rho^{-2} + (3+n)\frac{m^2}{\rho^2} + \rho^{-2(n-1)}\right] - - mA_{n+2}[(n+3)\rho^2 - 2\rho^{-2(n+1)}] + + m(n-1)\rho^2 A_{n-2} - m^2 A_{n+4}\rho^{-2(n+3)} + \frac{1}{R}B_{n-2}\right\}. \tag{30}$$

Thus we have formally solved the problem since we use these coefficients to determine the Laurent series by which we form the stress biharmonic function, hence, the stress tensor components, the small strain tensor and displacement vector.

5. Stress tensor components in the hyperbolic-elliptical coordinates

By splitting up the real and imaginary parts of the expressions (5), (6) and (7) in the following form:

$$\sigma_\rho + \sigma_\varphi = 2\left\{R^{-1}\left(1-\frac{m}{\zeta^2}\right)^{-1}\sum_{-\infty}^{\infty} A_{2k}\zeta^{2k} + R^{-1}\left(1-\frac{m}{\bar{\zeta}^2}\right)^{-1}\sum_{-\infty}^{\infty} A_{2k}\bar{\zeta}^{2k}\right\} = = \frac{2}{gR}\sum_{-\infty}^{\infty} A_{2k}\rho^{2k}\left[\cos 2k\varphi - \frac{m}{\rho^2}\cos 2(k+1)\varphi\right] \tag{31}$$

$$\sigma_\rho - i\tau_{\rho\varphi} = R^{-1}\left(1-\frac{m}{\zeta^2}\right)^{-1}\sum_{-\infty}^{\infty} A_{2k}\zeta^{2k} + R^{-1}\left(1-\frac{m}{\bar{\zeta}^2}\right)^{-1}\sum_{-\infty}^{\infty} A_{2k}\bar{\zeta}^{2k} - - \frac{\zeta^2}{\rho^2}\frac{1-m/\zeta^2}{1-m/\bar{\zeta}^2}\left\{R\left(\bar{\zeta}+\frac{m}{\bar{\zeta}}\right)R^{-3}\left(1-\frac{m}{\zeta^2}\right)^{-3}\times \times\left[\sum_{-\infty}^{\infty} 2kA_{2k}\zeta^{2k-1}R(1-m/\zeta^2) - \sum_{-\infty}^{\infty} A_{2k}\zeta^{2k}(2m/\zeta^3)R\right] + + R^{-3}\left(1-\frac{m}{\zeta^2}\right)^{-3}\left[\sum_{-\infty}^{\infty} B_{2k}\zeta^{2k}R\left(1-\frac{m}{\zeta^2}\right) - \sum_{-\infty}^{\infty}\frac{B_{2k}}{2k+1}\zeta^{2k-1}\frac{2m}{\rho^3}R\right]\right\} \tag{31'}$$

$$\sigma_\rho - i\tau_{\rho\varphi} = \frac{1}{gR}\sum_{-\infty}^{\infty} A_{2k}\rho^{2k}\Bigg\{\Bigg[e^{2ik\varphi} + e^{-2ik\varphi} - \frac{m}{\rho^2}\left(e^{2i(k+1)\varphi} + e^{-2i(k+1)\varphi}\right)\Bigg] -$$
$$- \frac{2k}{g}\left[e^{2ik\varphi} - \frac{m}{\rho^2}e^{2i(k-1)\varphi} + \frac{m^3}{\rho^6}e^{2i(k+1)\varphi} - \frac{m^2}{\rho^4}e^{2i(k+2)\varphi}\right] +$$
$$+ \frac{2m}{g^2\rho^2}\Bigg[\left(1 + 3\frac{m^2}{\rho^4}\right)e^{i2(k-1)\varphi} - \frac{m}{\rho^2}e^{i2(k-2)\varphi} -$$
$$- 3\frac{m}{\rho^2}\left(1 + \frac{m^2}{\rho^4}\right)e^{i2(k+1)\varphi} + \frac{m^2}{\rho^4}\left(3 + \frac{m^2}{\rho^4}\right)e^{i2(k+1)\varphi} - \frac{m^3}{\rho^6}e^{i2(k+2)\varphi}\Bigg]\Bigg\} +$$
$$+ \frac{1}{g^2R^2}\sum_{-\infty}^{\infty} B_{2k}\rho^{2k}\Bigg\{\Bigg[\frac{m}{\rho^2}e^{i2k\varphi} + \frac{m}{\rho^2}e^{i2(k+2)\varphi} - \left(1 + \frac{m^2}{\rho^4}\right)e^{i2(k+1)\varphi}\Bigg] +$$
$$+ \frac{2m}{g\rho^2(2k+1)}\Bigg[\left(1 + 2\frac{m^2}{\rho^4}\right)e^{i2k\varphi} - \frac{m}{\rho^2}e^{i2(k-1)\varphi} -$$
$$- \frac{m}{\rho^2}\left(2 + \frac{m^2}{\rho^4}\right)e^{i2(k+1)\varphi} + \frac{m^2}{\rho^4}e^{i2(k+2)\varphi}\Bigg]\Bigg\} \tag{32}$$

Considering certain values for the coefficients A_{2k} and B_{2k} from the previous expressions we determine the stress tensor components in the elliptical-hyperbolic coordinate system in the form:

a) the expression for the normal stress $\sigma_\rho(\rho,\varphi)$ at the plate points for the plane with the normal unit $\vec{h}_0$ in the tangent direction to the hyperbolic coordinate line:

$$\sigma_\rho(\rho,\varphi) = \sigma_h = \frac{1}{gR}\Bigg\{\sum_{-\infty}^{\infty}\rho^{2k}\Bigg(\cos 2k\varphi\Bigg\{A_{2k}\left[1 - \frac{2k}{g} - \frac{6m^2}{\rho^4 g^2}\left(1 + \frac{m^2}{\rho^4}\right)\right] +$$
$$+ \frac{1}{gR}B_{2k}\frac{m}{\rho^2}\left[1 + \frac{2}{(2k+1)g}\left(1 + 2\frac{m^2}{\rho^4}\right)\right]\Bigg\} +$$
$$+ \cos 2(k-1)\varphi\Bigg\{A_{2k}\left[\frac{2km}{g\rho^2} + \frac{2m}{g^2\rho^2}\left(1 + 3\frac{m^2}{\rho^4}\right)\right] - \frac{1}{gR}B_{2k}\frac{2m^2}{\rho^4 g(2k+1)}\Bigg\} +$$
$$+ \cos 2(k-2)\varphi\left[A_{2k}\left(-2\frac{m^2}{\rho^4 g^2}\right)\right] +$$
$$+ \cos 2(k+1)\varphi\Bigg\{A_{2k}\left[-\frac{m}{\rho^2} - \frac{2km^3}{g\rho^6} + \frac{2m^3}{g^2\rho^6}\left(3 + \frac{m^2}{\rho^4}\right)\right] +$$
$$+ \frac{1}{gR}B_{2k}\left[-\left(1 + \frac{m^2}{\rho^4}\right) - \frac{2m^2}{\rho^4 g(2k+1)}\left(2 + \frac{m^2}{\rho^4}\right)\right]\Bigg\} +$$
$$+ \cos 2(k+2)\varphi\Bigg\{A_{2k}\left[-\frac{m^2}{\rho^4} - \frac{2m^4}{g^2\rho^6}\right] + \frac{1}{gR}B_{2k}\left[\frac{m}{\rho^2} + \frac{2m^3}{\rho^6 g(2k+1)}\right]\Bigg\}\Bigg)\Bigg\} \tag{33}$$

b) the expression for the normal stress $\sigma_\varphi(\rho,\varphi)$ at the plate points for the plane with the normal unit $\vec{e}_0$ in the tangent direction to the elliptical coordinate line:

$$\sigma_\varphi(\rho,\varphi) = \sigma_e = \frac{1}{gR}\sum_{-\infty}^{\infty} A_{2k}\rho^{2k}\left\{\frac{2k}{g}\left[\cos 2k\varphi - \frac{m}{\rho^2}\cos 2(k-1)\varphi + \right.\right.$$

$$\left. + \frac{m^3}{\rho^6}\cos 2(k+1)\varphi - \frac{m^2}{\rho^4}\cos 2(k+2)\varphi\right] -$$

$$- \frac{2m}{g^2\rho^2}\left[\left(1+3\frac{m^2}{\rho^4}\right)\cos 2(k-1)\varphi - \frac{m}{\rho^2}\cos 2(k-2)\varphi - \right.$$

$$\left.\left. - 3\frac{m}{\rho^2}\left(1+\frac{m^2}{\rho^4}\right)\cos 2k\varphi + \frac{m^2}{\rho^4}\left(3+\frac{m^2}{\rho^4}\right)\cos 2(k+1)\varphi - \frac{m^3}{\rho^6}\cos 2(k+2)\varphi\right]\right\} -$$

$$- \frac{1}{g^2R^2}\sum_{-\infty}^{\infty} B_{2k}\rho^{2k}\left\{\left[\frac{m}{\rho^2}\cos 2k\varphi + \frac{m}{\rho^2}\cos 2(k+2)\varphi - \left(1+\frac{m^2}{\rho^4}\right)\cos 2(k+1)\varphi\right] + \right.$$

$$+ \frac{2m}{\rho^2 g(2k+1)}\left[\left(1+2\frac{m^2}{\rho^4}\right)\cos 2k\varphi - \frac{m}{\rho^2}\cos 2(k-1)\varphi - \right.$$

$$\left.\left. - \frac{m}{\rho^2}\left(2+\frac{m^2}{\rho^4}\right)\cos 2(k+1)\varphi + \frac{m^2}{\rho^4}\cos 2(k+2)\varphi\right]\right\} \tag{34}$$

c) the expression for the shear stress $\tau_{\rho\varphi}(\rho,\varphi)$ or $\tau_{\varphi\rho}(\rho,\varphi)$ at the points $N(\rho,\varphi)$ of the plate for the plane with the normal unit $\vec{h}_0$ in the tangent direction to the hyperbolic line, and in the unit direction $\vec{e}_0$ which is tangent to the elliptical coordinate line, hence, for the plane with normal unit $\vec{e}_0$ and in the unit direction $\vec{h}_0$:

$$\tau_{\rho\varphi}(\rho,\varphi) = \tau_{he} = \frac{1}{gR}\left\{\sum_{-\infty}^{\infty} A_{2k}\rho^{2k}\left\{\frac{2k}{g}\left[\sin 2k\varphi - \frac{m}{\rho^2}\sin 2(k-1)\varphi - \right.\right.\right.$$

$$\left. - \frac{m}{\rho^2}\sin 2(k-1)\varphi + \frac{m^3}{\rho^6}\sin 2(k+1)\varphi - \frac{m^2}{\rho^4}\sin 2(k+2)\varphi\right] +$$

$$+ \frac{2m}{g^2\rho^2}\left[\frac{m}{\rho^2}\sin 2(k-2)\varphi - \left(1+3\frac{m^2}{\rho^4}\right)\sin 2(k-1)\varphi + \right.$$

$$\left.\left.\left. + 3\frac{m}{\rho^2}\left(1+\frac{m^2}{\rho^4}\right)\sin 2k\varphi - \frac{m^2}{\rho^4}\left(3+\frac{m^2}{\rho^4}\right)\sin 2(k+1)\varphi + \frac{m^3}{\rho^6}\sin 2(k+2)\varphi\right]\right\}\right\} +$$

$$+ \frac{1}{g^2R^2}\left\{\sum_{-\infty}^{\infty} B_{2k}\rho^{2k}\left\{\left[\left(1+\frac{m^2}{\rho^4}\right)\sin 2(k+1)\varphi - \frac{m}{\rho^2}\sin 2k\varphi - \right.\right.\right.$$

$$\left. - \frac{m}{\rho^2}\sin 2(k+2)\varphi\right] + \frac{2m}{\rho^2 g(2k+1)}\left[\frac{m}{\rho^2}\sin 2(k-1)\varphi - \left(1+2\frac{m^2}{\rho^4}\right)\sin 2k\varphi + \right.$$

$$\left.\left.\left. + \frac{m}{\rho^2}\left(2+\frac{m^2}{\rho^4}\right)\sin 2(k+1)\varphi - \frac{m^2}{\rho^4}\sin 2(k+2)\varphi\right]\right\}\right\}. \tag{35}$$

6. Conclusion

The introductory part of this paper gives a brief survey of the research of the stress state in the plain-strained plates with double connected domains.

Our concrete contribution in this paper is the analysis of the stress state of the elliptical-annular plate strained by pairs of two opposing segmentally distributed forces along the external and internal contours by means of the complex variable function and conformal mapping method. In these cases, the expressions for the stress tensor components at an arbitrary point of the elliptical-annular plate, have been derived.

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REFERENCES

[1] Alfirević I., Jecić S., (1982), *Fotoelasticimetrija*, Sveučilišna nakl. Liber, Zagreb.

[2] Brčić V., Čukić R., (1988), *Eksperimentalne metode u projektovanju konstrukcija*, Građevinska knjiga, Beograd.

[3] Banić M., (1975), *Optička analiza napona*, Sinopsis predavanja na postdiplomskim studijama, Mašinski fakultet, Beograd.

[4] England A., (1971), *Complex Variable Methods in Elasticity*, London.

[5] Kojić M., (1975), *Teorija elastičnosti*, Mašinski fakultet, Kragujevac.

[6] Love A.E.H., (1952), *A Treatise on the Mathematical Theory of Elasticity*, Cambridge at University Press.

[7] Mushelishvili N.I., (1953), *Some Basic problems of the Mathematical Theory of Elaticity*, P. Noordhof LTD., Groningen, Holland.

[8] Nowacki W., (1970, 1975), *Teoria Sprezystošći*, Panstwowe Wydawnictwo Naukowe, Warszawa.

[9] Pearson C.E., (1959), *Theoretical Elasticity*, Harward University Press, Cambridge.

[10] Rašković D., (1968, 1985), *Teorija elastičnosti*, Naučna knjiga, Beograd.

[11] Sokolnikoff I.S., (1956), *Mathematical Theory of Elasticity*, McGraw-Hill Book Company, London.

[12] Shedon J.S. and Bery D.S., (1958), *The Classical Theory of Elasticity*, Handbuch der Physik, S. Flügge, Band VI, Springer-Verlag.

[13] Timoshenko S.P. and Goodier J.N., (1951), *Theory of Elasticity*, New York.

[14] Josifović M., (1964), *Izabrana poglavlja iz teorije elastičnosti*, Mašinski fakultet, Beograd.

[15] Radojković M., (1970), *Ispitivanje konstrukcija I i II*, Beograd.

[16] Hedrih K., (1988), *Izabrana poglavlja teorije elastičnosti*, Mašinski fakultet, Niš.

[17] Hedrih K., (1983), *Optička analiza naponskog stanja*, štampana predavanja, Mašinski fakultet, Niš.

[18] Hedrih K., Mitić S., (1988), *Ravno naprezanje kružno-prstenaste ploče*, Zbornik radova XVIII kongresa teorijske i primenjene mehanike, Vrnjačka Banja.

[19] Hedrih K., Jecić S., Jovanović D., (1990), *Glavni naponi u tačkama konture eliptično-prstenaste ploče ravno napregnute parovima koncentrisanih sila*, Mašinstvo 39, Beograd.

[20] Hedrih K., Jovanović D., (1990), *Primena funkcije kompleksne promenljive i konformnog preslikavanja za određivanje eliptično-hiperboličnih koordinata tenzora napona ravno napregnutih ploča*, XIX Jugoslovenski kongres teorijske i primenjene mehanike, Ohrid.

[21] Jovanović D., (1990), *Primena metode fotoelastičnosti za ispitivanje naponskog stanja eliptično-prstenaste ploče opterećene koncentrisanim silama*, XIX Jugoslovenski kongres teorijske i primenjene mehanike, Ohrid.

[22] Milne-Thompson L.M., (1960), *Plane Elastic Systems*, Springer, Göttingen-Heidelberg-Berlin.

[23] Filon, L.N.G., (1903), *On an approximative solution for the bending of a beam of rectangular cross section etc.*, Philos. Roy. Soc. ser. A **201**, London.

[24] Filon L.N.G., (1922), *On stresses in multiply-conected plates*, British Associations for the advancement of science.

[25] Frocht M.M., (1969), *Photoelasticity — The Selected Scientific Papers*, Oxford.

[26] Frocht M.M., (1941), *Photoelasticity I and II*, New York.

[27] Teodorescu P.P., (1964), *One hundred years of investigations on the plane problem of the theory of elasticity*, Appl. Mech. Rev. **17** (3).

[28] Weighardt K., (1915), Sitzungsberichte der Akademie der Wissenschaften in Wien **124**.

[29] Filon L.N.G., (1924), *The Stress in a Circular Rings*, Selected Engineering Papers, No. 12, London.

[30] Shilkrut D. and Bengad F., (1985), *Elastic stress concentration phenomena in an axially stressed rectangular plate with a central circular hole and related problems*, Trans. ASME, J. Appl. Mech. **52** (1).

[31] Sato Rakio and Isida Macoto, (1964), *Analysis of finite regions containing a circular hole or inclusion with applications to their periodic arrays*, Trans. Jap. Soc. Mech. Eng. A 50 (457).

[32] Caulk, D.A., (1984), *Special boundary integral equations for potential problems in regions with circular holes*, Trans. ASME. J. Appl. Mech. **51** (4).

[33] Wolf H., (1961), *Spannungsoptik*, Springer-Verlag, Berlin.

[34] Zandman F., (1977), *Photoelastic Coatings*, Westport Society for experimental stress analysis.

[35] Dally J.W. and Riley W.F., (1970), *Introduction in Photomechanic*, Mir, Moskva.

[36] Dally J.W. and Riley W.F., (1965), *Experimental stress analysis*, McGraw-Hill Book Company, New York.

[37] Aben H., (1979), *Integrated Photoelasticity*, McGraw-Hill Book Company, New York.

[38] Heywood R.B., (1969), *Photoelasticity for designers*, Pergamon Press, New York.

[39] Holister G.S., (1967), *Experimental Stress Analysis*, Cambridge University Press, Cambridge.

[40] Parton V.Z. and Perlin P.I., (1953), *Integral Equations in Elasticity*.

АНАЛИЗ НАПРЯЖЕННОГО СОСТОЯНИЯ ЭЛЛИПТИЧЕСКО-КОЛЬЦЕВОЙ ПЛАСТИНИ ПРИМЕНЕНИЕМ ФУНКЦИЙ КОМПЛЕКСНОГО ПЕРЕМЕННОГО И КОНФОРМНОГО ОТОБРАЖЕНИЯ

В вводной части этой работы сделано короткое представление исследовании напряженного состояния плоско нагруженних пластин. Наше приложение есть в изучении эллиптическо-кольцевой пластине сегментно нагруженной по внешней и внутренной контуре постоянным распределеним давлением. Для этого случая построенни выражениа для компоненты тензора напряжений пользуясь методом функции комплексной переменной и конформного отображения.

ANALIZA NAPONSKOG STANJA ELIPTIČNO-PRSTENASTE PLOČE POMOĆU METODE FUNKCIJE KOMPLEKSNE PROMENLJIVE I KONFORMNOG PRESLIKAVANJA

U uvodnom delu rada dat je kratak pregled istraživanja naponskog stanja ravno-napregnutih ploča oblika konture dvostruko povezane oblasti. Naš konkretan doprinos u ovom radu su izvedeni analitički izrazi za komponente tenzora napona u proizvoljnoj tački eliptično-prstenaste pčloče napregnute u svojoj srednjoj ravni pomoću dva para suprotno smernih jednako raspodeljenih pritisaka po segmentima konture, korišćenjem metode funkcije kompleksne promenljive i konformnog preslikavanja.

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