TEORIJSKA I PRIMENJENA MEHANIKA 17, pp 45-51, 1991, UDK 532

ON THE MOTION OF SUSPENSIONS WITH NONSYMMETRIC STRESS TENSOR

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(Received 05.05.1991)

1. Introduction

In the classical continuum mechanics, the stress states are described by means of a symmetric stress tensor. Since the classical continuum model is not sufficient for the description of behavior of certain materials e.g., granular materials, fluid suspensions, liquid crystals, etc., the continuum model with microstructure has been introduced [1].

Eringen and Suhubi [2] introduced micropolar continuum and micropolar fluid models respectively, characterized by the couple stress and a nonsymmetric stress tensor. This theory comprises two independent kinematic quantities: the velocity vector v_i and the spin or microrotation vector ν_i.

Micropolar fluids can, among other applications, be used in describing the behavior of motion of suspensions as a mixture of two phases [3], [4], [5], [6] and [7]. The basic phase of the suspension is a fluid, wheras the dispersive phase are solid particles.

By the application of the basic equation of motion of the micropolar theory to the case of motion of the suspension between two parallel infinite plates, two effects of this flow are shown.

2. The balance equation

Let us consider an isotropic mixture consisting of $\alpha = 1, 2, \dots, n$, of chemically nonreactive constituents. We shall use the following balance equations:

a) The balance of mass of the α-th constituent and the mixture as a whole

(2.1) $$\frac{d\rho_{(\alpha)}}{dt} + \rho_{(\alpha)} \nabla \cdot v_{(\alpha)} = 0 \,,$$

(2.2) $$\frac{d\rho}{dt} + \rho \nabla \cdot v = 0 \,.$$

where $\rho_{(\alpha)}$, $v_{(\alpha)}$ and v are the mass density and velocity of the α-th constituent and the mixture as a whole, and where

$$\rho = \sum_{\alpha=1}^{n} \rho_{(\alpha)}, \qquad \rho v = \sum_{\alpha=1}^{n} \rho_{(\alpha)} v_{(\alpha)} \,. \tag{2.3}$$

Let us introduce the flux of the diffused mass of the α-th constituent:

$$J_{(\alpha)} = \rho_{(\alpha)}(v_{(\alpha)} - v) = \rho_{(\alpha)} u_{(\alpha)}, \tag{2.4}$$

where $u_{(\alpha)}$ is the diffusion rate of the α-the constituent. If the concentration of mass of the α-the constituent is

$$c_{(\alpha)} = \frac{\rho_{(\alpha)}}{\rho}, \tag{2.5}$$

then, by applying (2.1), it is possible to establish the relationship between the concentration of mass and the flux of the diffused mass of the α-th constituent

$$\rho \frac{dc_{(\alpha)}}{dt} = -\nabla \cdot J_{(\alpha)} \,. \tag{2.6}$$

b) The balance of momentum of the mixture

$$\rho \frac{dv}{dt} = \nabla \cdot t + \rho f \,, \tag{2.7}$$

where t and f are the nonsymmetric stress tensor and the body force.

c) The balance of moment of momentum

$$\rho I \frac{d\nu}{dt} = t \times I + \nabla \cdot m \,, \tag{2.8}$$

where ν is the microrotation vector, and m is the couple stress tensor, respectively.

d) The balance of the internal energy of the mixture

$$\rho \frac{du}{dt} = t : \nabla v - (t \times I)\nu + m : \nabla \nu - \nabla \cdot q + \sum_{\alpha=1}^{n} f_{(\alpha)} \cdot J_{(\alpha)}, \tag{2.9}$$

where u is the specific internal energy, and q is the heat flux vector.

In the case of the mixture, the entropy is a function of the specific internal energy u, the specific volume V ($V = 1/\rho$) and the concentration c_k, i.e.,

$$\eta = \eta(u, V, c_k) \,. \tag{2.10}$$

Now, the phenomenological equation can be derived in the following form [3]:

$$\begin{aligned} \rho D(\nabla c_p + k_p \nabla p) = {} & J_p + a_1 a_2 J_p \times (\nu - \Omega) + \\ & + a_1 a_3 (\nabla \times \nu) + a_1 a_4 [(\nabla \times \nu) \times (\nu - \Omega)] \end{aligned} \tag{2.11}$$

where D is the diffusion coefficient of the disperse phase, $k_p = a_1\gamma/(\rho D)$ and $\Omega = \frac{1}{2}\nabla \times v$, whereas a_1, a_2, a_3 and a_4 are the scalars which characterize the isotropic features of the medium.

3. The motion of the suspension between two parallel plates

In the case of incompressible suspensions, without body forces, couples and temperature influences, the balance equations can be written in the following forms:

$$v_{i,i} = 0\,, \tag{3.1}$$

$$\rho\frac{dv_i}{dt} = t_{ij,i}\,, \tag{3/2}$$

$$\rho\frac{d(I\nu_i)}{dt} = m_{ij,j} + \varepsilon_{ijk}t_{kj}\,, \tag{3.3}$$

$$\rho\frac{dc_p}{dt} = -J^p_{i,i}\,, \tag{3.4}$$

where the nonsymmetric stress tensor and the couple stress tensor are:

$$t_{kl} = (-\pi + \lambda v_{r,r})\delta_{kl} + \mu(v_{k,l} + v_{l,k}) + k(v_{l,k} - \varepsilon_{klr}\nu_r)\,, \tag{3.5}$$

$$m_{kl} = \alpha\nu_{r,r}\delta_{kl} + \beta\nu_{k,l} + \gamma\nu_{l,k}\,, \tag{3.6}$$

where k, α, β, and γ are the viscosity coefficients of the micropolar continuum.

Let us consider the case of the stationary motion of the suspension between two infinite parallel plates at the distance of $2b$.

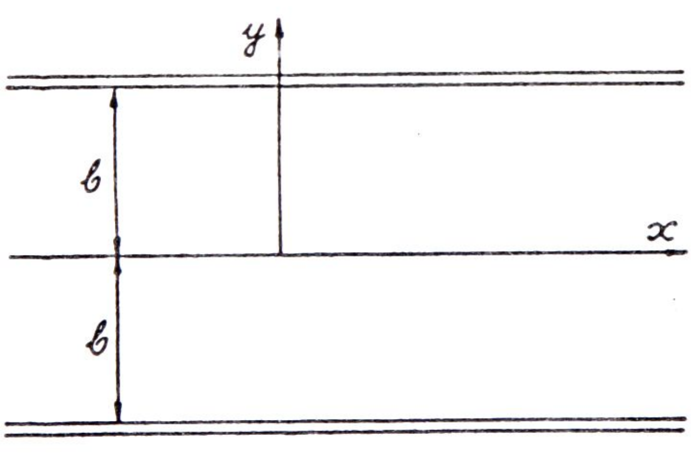

Fig. 1

The axis Ox overlaps the middle line. Oy axis is perpendicular to the plates, while Oz axis is perpendicular to the plates of the flow (Fig. 1). For the given coordinate system (Fig. 1) the components of the velocity, the velocity of microrotation, and the flux of the diffused mass of the dispersed phase (particles) are:

$$\begin{aligned} &v_y = v_z = 0\,, \qquad && v_x = v(y)\,, \\ &\nu_x = \nu_y = 0\,, && \nu_z = \nu(y)\,, \\ &J^p_y = J^p_z = 0\,, && J^p_x = J^p(x)\,. \end{aligned} \tag{3.7}$$

With the assumptions introduced, the balance of mass is identically satisfied, while the equations (3.2), (3.3), by using (3.5), (3.6) and (2.11), become:

$$\frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0, \qquad \frac{\partial p}{\partial x} = \text{const.}, \tag{3.8}$$

$$\frac{dp}{dx} = (\mu + k)\frac{d^2 v}{dy^2} + 2k\frac{d\nu}{dy}\,, \tag{3.9}$$

$$\frac{dv}{dy} = \frac{\gamma}{k}\frac{d^2 v}{dy^2} - 2\nu\,, \tag{3.10}$$

$$\rho D\frac{\partial c_p}{\partial y} = \left(-a_1 a_2 \rho D k_p \frac{\partial p}{\partial x} - a_1 a_4 \frac{\partial \nu}{\partial y}\right)(\nu - \Omega)\,. \tag{3.11}$$

Assuming that the fluid adheres to the plates, the boundary conditions for the velocity and the microrotation velocity are:

$$v = 0, \quad \nu = 0 \qquad \text{for} \;\; y = \pm b. \tag{3.12}$$

and the solutions of the equations (3.9) and (3.10) are:

$$v^* = \frac{v}{v_0} = 1 - (b^*)^2 - \frac{2k}{\mu + k}\,\frac{1}{\lambda}\,\frac{\operatorname{ch}\lambda + \operatorname{ch}\lambda b^*}{\operatorname{sh}\lambda}\,, \tag{3.13}$$

$$\nu^* = \frac{\nu b}{v_0} = b^* - \frac{\operatorname{sh}\lambda b^*}{\operatorname{sh}\lambda}\,, \tag{3.14}$$

where

$$v_0 = -\frac{1}{2}b^2\mu^{-1}\frac{dp}{dx}, \quad b^* = \frac{y}{b}, \quad \lambda = kb, \quad k = \left(\frac{2\mu + k}{\mu + k}\,\frac{k}{\gamma}\right)^{1/2}. \tag{15}$$

Fig. 2 shows a velocity graph for different values of (k/μ) (at $\lambda = 1$). It demonstrates that the velocity distribution is different from the classical case.

Fig. 3 is a graph of the microrotation velocity $\nu^* = \nu b/v_0$ for different values of λ.

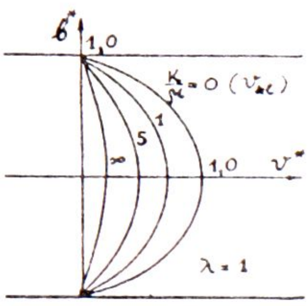

Fig. 2

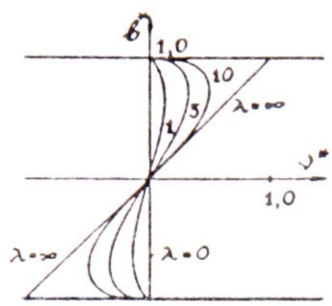

Fig. 3

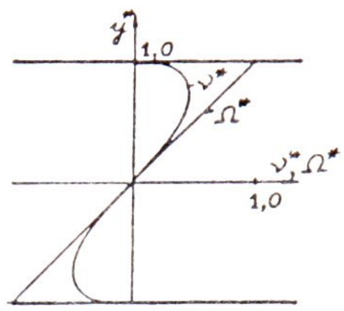

Fig. 4

In order to examine the concentration during the motion of suspension, we shall use the relationship (3.11) written in the dimensionless form:

$$\frac{dc_p}{dy^*} = \left(A - B\frac{d\nu^*}{dy^*}\right)(\nu^* - \Omega^*)\,, \tag{3.16}$$

where

$$A = \frac{1}{8}a_1 a_2 (2b)^2 k_p \frac{1}{\eta}\left(\frac{dp}{dx}\right)^2\,, \tag{3.17}$$

$$B = \frac{1}{64}a_1 a_4 \frac{(2b)^2}{\rho D \eta^2}\left(\frac{dp}{dx}\right)^2\,,$$

$$\Omega^* = \frac{\Omega b}{v_0} = b^* - \frac{k}{\mu + k}\,\frac{\operatorname{sh}\lambda b^*}{\operatorname{sh}\lambda}$$

and where A and B are the diffusion coefficients.

Fig. 4 represents a graph of the functions $\nu^*(y^*)$ and $\Omega^*(y^*)$ for $\lambda = 5$ and $k/\mu = 0$.

From equation (3.16) it can be seen that the diffusion coefficient influences the concentration distribution.

1) Let $B = 0$. Since A is a positive value, and $(\nu^* - \Omega^*) \leq 0$ (Fig. 4), it follows from (3.16) that $dc_p/dy^* \leq 0$ with $y^* \geq 0$. On the axis of symmetry, $\nu^* = \Omega^* = 0$, and $dc_p/dy^* = 0$. When approaching the plates, the difference $\nu^* - \Omega^*$ increases in the absolute value, which means that the dispersive phase concentration (the particles) decreases, on that position.

2) Let $A = 0$. Since B is a positive value, it gives

$$\frac{dc_p}{dy^*} = -B\frac{d\nu^*}{dy^*}(\nu^* - \Omega^*)\,.$$

It is obvious that:

$$\frac{dc_p}{dy^*} > 0 \quad \text{at} \quad 0 < y^* < y^*_m, \qquad \left(\frac{d\nu^*}{dy^*} > 0, \quad \nu^* - \Omega^* < 0\right),$$

$$\text{for} \quad y^* = y^*_m \implies \frac{d\nu^*}{dy^*} = 0\,,$$

$$\frac{dc_p}{dy^*} < 0 \quad \text{at} \quad y^*_m < y^* < 1, \qquad \left(\frac{d\nu^*}{dy^*} < 0, \quad \nu^* - \Omega^* < 0\right),$$

$$\frac{dc_p}{dy^*} = 0 \quad \text{at} \quad y^* = 0 \quad (\nu^* = \Omega^* = 0), \quad \text{or} \quad y^* = y^*_m \quad \left(\frac{d\nu^*}{dy^*} = 0\right).$$

In this case, the dispersive phase concentration increases in the vicinity of the axial plane.

4. Conclusion

In this paper, the micropolar continuum theory has been applied, in which a nonsymmetric stress tensor has been used for describing the stress states. The theory has been applied to the case of the motion of suspension between two parallel plates. The analysis of the results obtained shows that the suspension concentration distribution of the dispersive phase depends upon the diffusion coefficient. It can be concluded that the suspension separates into the solid and the liquid phase. Namely, it has been demonstrated that during the motion of the suspension the following two effects are present: an accumulation of the dispersive phase (the particles) around the axial plane, and a decrease of the dispersive phase concentration in the vicinity of the parallel plates.

Acknowledgement. This research was supported by a grant from the National Science Foundation of Serbia, Grant No. 7709.

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ДВИЖЕНИЕ СУСПЕНЗИИ С НЕСИММЕТРИЧНЫМ ТЕНЗОРОМ НАПРЯЖЕНИЯ

В работе применяется теория микрополярного континуума, в которой для описания напражённого состояния пользуется несимметричный тензор напряжения. Эта теория применяется в случае движения суспензии между двумя параллельними плоскостями. Анализ полученных результатов показывает, что распределение концентрации твёрдой фазы суспензии зависит от коэффициента диффузии. Можно сделать заключение, что при суспензии происходит расслоение на твёрдую и жидкую фазы. Показано, что при движении суспензиии существуют два эффекта: нагромеждение твёрдой фазы (частицы) вблизи осевой плоскости и уменьшение концентрации твёрдой фазы вблизи параллельных плоскостей.

O KRETANJU SUSPENZIJA SA NESIMETRIČNIM TENZOROM NAPONA

U radu je primenjena teorija mikropolarnog kontinuuma kod koje se za opisivanje naponskog stanja koristi nesimetrični tenzor napona. Ta teorija je primenjena u slučaju kretanja suspenzije između dve paralelne ravni. Analiza dobijenih rezultata pokazuje da raspodela koncentracije disperzne faze suspenzije zavisi od koeficijenta difuzije. Može se zaključiti da kod suspenzije dolazi do raslojavanja na čvrstu i tečnu fazu. Naime, pokazano je da pri kretanju suspenzije postoje dva efekta: nagomilavanje disperzne faze u okolini osne ravni i smanjenje koncentracije disperzne faze u okolini paralelnih ravni.

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