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INFLUENCE OF HALL-EFFECT ON HARTMANN FLOW — PART I

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1. Introduction. The influence of Hall-parameter on the electric field of Hartmann flow is discussed for different values of Hartmann number.

Sutton and Sherman [1] have studied the flow pattern, induced magnetic field and current distribution in modified Hartmann flow where Hall-current is taken into consideration. Todd [2] has given a detailed discussion of different conditions in Hartmann flow, which distinguish between the electromagnetic pump; electromagnetic brake, magneto-hydrodynamic generators and the magneto-hydrodynamic flow accelerators. His analysis is based on the values of the electric field parameter E, for different values of Hartmann number. Following Todd [2], we attempt here to study the influence of Hall-parameter on the electric field in the Hartmann flow.

2. Equations of motion. We consider flow of an incompressible, viscous fluid between two parallel planes at $x = \pm L$. A Cartesian coordinate system is chosen in such a way that the x-axis is parallel to the magnetic field $\vec{B}_0$. Induced magnetic field is neglected (see ref. [2–4]) and Hall-effect is taken into consideration. Assuming yz-plane as a complex plane with y and z axes as the real and imaginary axes respectively, and $\vec{e}$, the unit vector in the resultant-flow direction, the resultant-fluid velocity f is given as

$$f = U\left[1 - \frac{\cosh(M\sqrt{T}x/L)}{\cosh(M\sqrt{T})}\right] \tag{1}$$

where

$$f = u + iw, \qquad U = \frac{A}{T}, \qquad P^* = -\left(\frac{\partial}{\partial y} + i\frac{\partial}{\partial z}\right)p$$

$$E = E_y + iE_z, \qquad A = \frac{P^*}{\sigma B_0^2} - \frac{iET}{B_0}, \qquad T = \frac{1+im}{1+m^2}$$

$$M = (\sigma/\rho\nu)^{1/2}B_0 L = \text{Hartmann number},$$

$$m = \omega\tau = \text{Hall-parameter}.$$

We assume that P^* and B_0 are positive constant quantities and E is a parameter.

The average velocity is:

$$f_{\text{ave}} = U\left[1 - \frac{\tanh(M\sqrt{T})}{M\sqrt{T}}\right] \tag{2}$$

which tends to U as $M \to \infty$; and the flow rate Q is given by:

$$Q = 2Lf_{\text{ave}}. \tag{3}$$

Then, Q_0, the flow rate for a non-conducting fluid, is obtained from equation (3) (by keeping E fixed and letting $\sigma \to 0$) as:

$$Q_0 = 2L(P^* L^2/3\rho\nu). \tag{4}$$

Equations (3) and (4) show that the electromagnetic forces accelerate the flow if

$$Q > Q_0 \implies f_{\text{ave}} > \frac{P^* L^2}{3\rho\nu} \implies E < E^*, \tag{5}$$

where

$$\frac{E^* \rho\nu}{B_0 L^2 P^*} = -i\left[(M\sqrt{T})^{-2} - \frac{1}{3}M\sqrt{T}(M\sqrt{T} - \tanh M\sqrt{T})^{-1}\right]. \tag{6}$$

We assume, the resultant current density vector is $\vec{J} = \vec{e}J = \vec{e}(J_y + iJ_z)$, where

$$J = \sigma T(E - iB_0 f) \tag{7}$$

and that the not current is given by

$$I = \int_{-L}^{+L} J\,dx = 2L\sigma T(E - iB_0 f_{\text{ave}}) = 2LI_{\text{ave}}. \tag{8}$$

As the component of the Lorentz force is $-\vec{e}iJB_0$ and

$$J < 0 \iff E < E' = \frac{iP^*}{\sigma B_0}[\cosh(M\sqrt{T}) - 1] \tag{9a}$$

we get that the electromagnetic force aligns with $+\vec{e}$ and

$$J > 0 \iff E > 0 \tag{9b}$$

which implies that the opposite is true.

Equation (8) yields:

$$I \gtrless 0 \iff E \gtrless E^0 = -\frac{iP^* L^2 B_0}{\rho\nu}\left[(M\sqrt{T})^{-2} - (M\sqrt{T}\cdot\tanh M\sqrt{T})^{-1}\right]. \tag{10}$$

As $-\vec{e}i\int_{-L}^{+L} JB_0\,dx = -\vec{e}iIB_0$, the results discussed in equations (9) can be explained in terms of I and E.

From equations (6), (9) and (10), we obtain

$$-\left|\frac{\rho\nu E'}{B_0 L^2 P^*}\right| < -\frac{1}{2} \tag{11}$$

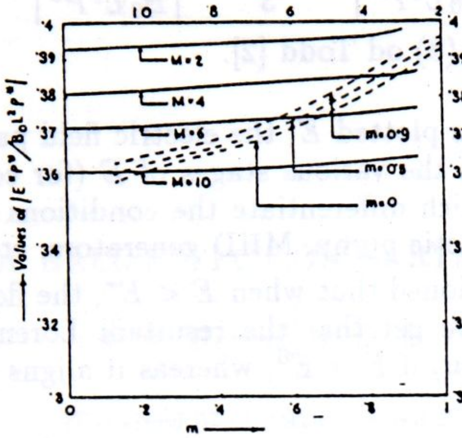
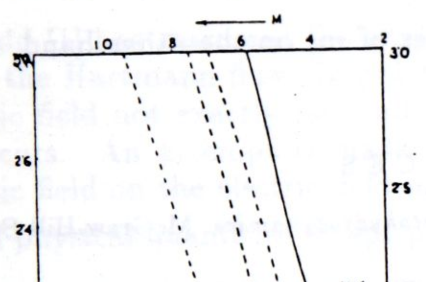


Fig. 1. Variation of $|E^* \rho^\nu / B_0 L^2 \rho^*|$ against m and M



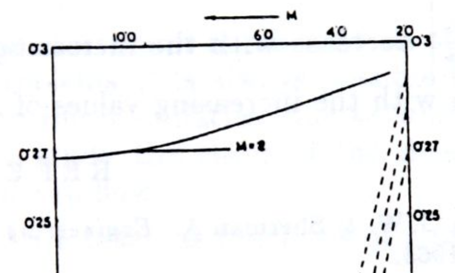


Fig. 2. Variation of $|E' \rho^\nu / B_0 L^2 \rho^*|$ against m and M

Fig. 3. Variation of $|E^0 \rho^\nu / B_0 L^2 \rho^*|$ against m and M

and

$$-\frac{2}{5} < -\left|\frac{\rho\nu E^*}{B_0 L^2 P^*}\right| < -\frac{1}{3} < -\left|\frac{\rho\nu E^0}{B_0 L^2 P^*}\right| < 0, \tag{12}$$

which are similar to equation (8) od Todd [2].

3. Results. Todd [2] has plotted E, the electric field parameter, graphically, against M. He has explained the various stages of E (for constant pressure gradient and magnetic field), which differentiate the conditions for different types of machines such as electromagnetic pump, MHD generators, etc.

It has already been mentioned that when $E < E^*$, the flow is accelerated (see equations (5)). From (10), we get that the resultant Lorentz force opposes the motion (i.e. in the $-\vec{e}$ direction) if $E > E^0$, whereas it aligns with the motion (i.e. in the $+\vec{e}$ direction) if $E < E^0$.

Figures (1) and (3) show the variations of $\left|\frac{\rho\nu E^*}{B_0 L^2 P^*}\right|$ and $\left|\frac{\rho\nu E^0}{B_0 L^2 P^*}\right|$ respectively, with m and M. Those absolute values increase with the increasing values of m and the decreasing values of M. Therefore the influence of these parameters on E^* and E^0 are similar in nature. We observe from figure (2) that the value of $\left|\frac{\rho\nu E'}{B_0 L^2 P^*}\right|$ decrease with the increasing values of m; on the other hand, its value increases with the increasing values of M.

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ВЛИЯНИЕ ЭФФЕКТА ХОЛЛА НА ПОТОК ХАРТМАННА

Вопрос о влиянии параметра Холла на электрическое поле в потоке Хартманна обсуждается для разных величин числа Хартманна.

Суттон и Шерман исследовали структуру потока, индуцированное магнитное поле и распределение тока в модифицированном потоке Хартманна где рассматривается ток Холла. Тодд подробно обсуждал различные условия в потоке Хартманна, которые различаются между электромагнитным насосом, электромагнитным тормозом, магнито-гидродинамическими генераторами и ускорителями магнито-гидродинамического потока. Его анализ основан на величинах параметра E электрического поля, для разных величин числа Хартманна. Следуя Тодду, здесь мы стараемся исследовать влияние параметра Холла на электрическое поле в потоке Хартманна.

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INFLUENCE OF HALL-EFFECT ON HARTMANN FLOW — PART II

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1. Introduction. The effects of the deflection of the magnetic field in Hartmann flow between two parallel planes, particulalry to the electric field are discussed theoretically and plotted graphically for different values of the Hartmann number M.

Todd [1] has discussed the effect of the Hartmann number M on the electric field in the Hartmann flow. In practical problems, it is always possible to get the magnetic field not exactly perpendicular to the boundaries, due to which a cross flow occurs. An attempt is made here to study the effect of the deflections of magnetic field on the electric field in Hartmann flow.

All physical quantities except pressure are taken as functions of z only.

2. Equations of motion. Considering the magnetic field $\vec{B}_0 = (B_0\sqrt{1-\lambda^2}, 0, B_0\lambda)$ where $\lambda = \cos\theta$ and velocity $\vec{v} = (u, v, 0)$, the equations of motion reduce to

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{d^2u}{dz^2} + \frac{1}{\rho}J_y B_0\lambda \tag{1}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\frac{d^2v}{dz^2} - \frac{1}{\rho}J_x B_0\lambda \tag{2}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - \frac{1}{\rho}J_y B_0\sqrt{1-\lambda^2} \tag{3}$$

where

$$J_x = \sigma[E_x + vB_0\lambda], \tag{4}$$

$$J_y = \sigma[E_y - uB_0\lambda], \tag{5}$$

$$0 = J_z = \sigma[E_z - vB_0\sqrt{1-\lambda^2}] \implies E_z = vB_0\sqrt{1-\lambda^2}. \tag{6}$$

The boundary conditions are

$$u = 0, \quad v = 0, \quad w = 0 \qquad \text{at } z = \pm d. \tag{7}$$

Equations (1) and (2) yield $\partial p/\partial x$ = constant and $\partial p/\partial y$ = constant. The solution of equations (1) and (2) subject to (7) are obtained as

$$u = \frac{1}{B_0\lambda^2}\left(\frac{P_x}{\sigma B_0} + \lambda E_y\right)\left(1 - \frac{\cosh(M\lambda/d)z}{\cosh M\lambda}\right) \tag{8}$$

$$v = \frac{1}{B_0\lambda^2}\left(\frac{P_y}{\sigma B_0} - \lambda E_x\right)\left(1 - \frac{\cosh(M\lambda/d)z}{\cosh M\lambda}\right) \tag{9}$$

where $P_x = -\partial p/\partial x$ and $P_y = -\partial p/\partial y$. The flow rates Q_x and Q_y are given by

$$Q_x = \frac{2d}{B_0\lambda^2}\left(\frac{P_x}{\sigma B_0} + \lambda E_y\right)\left[1 - (M\lambda)^{-1}\tanh M\lambda\right], \tag{10}$$

$$Q_y = \frac{2d}{B_0\lambda^2}\left(\frac{P_y}{\sigma B_0} - \lambda E_x\right)\left[1 - (M\lambda)^{-1}\tanh M\lambda\right]. \tag{11}$$

For a nonconducting fluid, the flow rates Q_{x_0} and Q_{y_0} are

$$Q_{x_0} = \frac{2d^3 P_x}{3\rho\nu} \quad \text{and} \quad Q_{y_0} = \frac{2d^3 P_y}{3\rho\nu}. \tag{12}$$

The Lorentz force will accelerate the flow if

$$\left.\begin{matrix} Q_x > Q_{x_0} \\ Q_y > Q_{y_0} \end{matrix}\right\} \Longrightarrow \left\{\begin{matrix} E_y > E_y^* \\ -E_x > E_x^* \end{matrix}\right. \tag{13}$$

where

$$\frac{\sigma B_0 E_x^*}{M^2 P_y} = \frac{\sigma B_0 E_y^*}{M^2 P_x} = \lambda\left[\frac{1}{3}(M\lambda)(M\lambda - \tanh M\lambda)^{-1} - (M\lambda)^{-2}\right]$$
$$= \frac{-\sigma B_0 E^*}{M^2 P'} \qquad \text{(say)} \tag{14}$$

which shows that

$$-\frac{2\lambda}{5} < \frac{\rho\nu E^*}{P' d^2 B_0} < -\frac{\lambda}{3}. \tag{15}$$

Let us assume that

$$\int_{-d}^{+d} \vec{J}_x \vec{B}_0 \, dz = \vec{e}_x F_x - \vec{e}_y F_y,$$

where

$$F_x = \lambda B_0 I_y = \lambda B_0 \int_{-d}^{+d} J_y \, dz = \sigma\lambda B_0 \int_{-d}^{+d} (E_y - \lambda B_0 u) \, dz,$$

$$F_y = \lambda B_0 I_x = \lambda B_0 \int_{-d}^{+d} J_x \, dz = \sigma\lambda B_0 \int_{-d}^{+d} (E_x + \lambda B_0 v) \, dz.$$

Therefore

$$\left.\begin{matrix} F_x \gtreqless 0 \\ F_y \gtreqless 0 \end{matrix}\right\} \Longrightarrow \left.\begin{matrix} I_y \gtreqless 0 \\ I_x \gtreqless 0 \end{matrix}\right\} \Longrightarrow \left\{\begin{matrix} E_y \gtreqless E_y^0 \\ -E_x \gtreqless E_x^0 \end{matrix}\right.$$

Hence

$$\frac{\sigma B_0 E_y^0}{M^2 P_x} = \frac{\sigma B_0 E_x^0}{M^2 P_y} = \lambda\left[(M\lambda \cdot \tanh M\lambda)^{-1} - (M\lambda)^{-2}\right] = \frac{-\sigma B_0 E^0}{M^2 P'} \quad \text{(say)} \tag{16}$$

and

$$-\frac{\lambda}{3} < \frac{\rho\nu E^0}{P' d^2 B_0} < 0.$$

Now $J_y > 0 \implies E_y > uB_0\lambda \implies E_y > E'_y = \frac{P_x}{\sigma B_0 \lambda}(\cosh M\lambda - 1)$ and $J_y < 0 \implies E_y < uB_0\lambda \implies E_y < 0$. Also, $J_x > 0 \implies E_x > -vB_0\lambda \implies E_x > 0$ and $J_x < 0 \implies E_x < -vB_0\lambda \implies E_x < -E'_x = \frac{-P_y}{\sigma B_0\lambda}(\cosh M\lambda - 1)$, where

$$\frac{\sigma B_0 E'_y}{P_x} = \frac{\sigma B_0 E'_x}{P_y} = \frac{1}{\lambda}(\cosh M\lambda - 1) = \frac{-\sigma B_0 E'}{P'} \quad \text{(say)}. \tag{17}$$

This shows that

$$\frac{\rho\nu E'}{P' d^2 B_0} < -\frac{\lambda}{2}.$$

From equation (3), we get

$$u = (P_z + \sigma B_0\sqrt{1-\lambda^2}\,E_y)/(\sigma B_0^2 \lambda\sqrt{1-\lambda^2}), \tag{18}$$

where $P_z = \partial p/\partial z$.

Using the expression for u, we have

$$E_y = -\frac{1}{\sigma B_0}\left[\frac{P_z}{\sqrt{1-\lambda^2}} \cdot \frac{\cosh M\lambda}{\cosh(M\lambda^2/d)} - \frac{P_x}{\lambda}\left(\frac{\cosh M\lambda}{\cosh(M\lambda^2/d)} - 1\right)\right]. \tag{19}$$

For all values of M

$$\frac{\sigma B_0 E_y}{P_x} < 0, \quad \text{if} \quad \frac{P_z}{P_x} > \frac{\sqrt{1-\lambda^2}}{\lambda}\left[1 - \frac{\cosh(M\lambda^2/d)}{\cosh M\lambda}\right] > \frac{\sqrt{1-\lambda^2}}{\lambda} \quad \text{and}$$

$$\frac{\sigma B_0 E_y}{P_x} > 0, \quad \text{if} \quad \frac{P_z}{P_x} < \frac{\sqrt{1-\lambda^2}}{\lambda}\left[1 - \frac{\cosh(M\lambda^2/d)}{\cosh M\lambda}\right] \leq 0. \quad \text{i.e.} \quad P_z < 0.$$

3. Results. The deflection of the magnetic field is associated with three mutually perpendicular components (E_x, E_y, E_z) of the electric field, where E_z vanishes automatically when $\lambda = 1$ (see equation (6)). Here we study the effect of λ on E_x and E_y only. In figures (1), (2) and (3), the effects on the electric fields i.e. $-\sigma B_0 E^0/P'$, $-\sigma B_0 E'/P'$ and $-\sigma B_0 E^*/P'$ are shown for various values of λ against fixed values of M i.e. $(M = 2, 6, 10)$. It is clear from the figures that the electric field will increase or decrease due to increase or decrease of λ respectively for constant values of Hartmann number M.

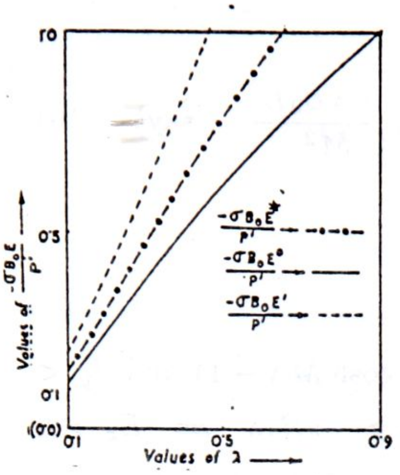

Fig. 1. Graphs for $M = 2$

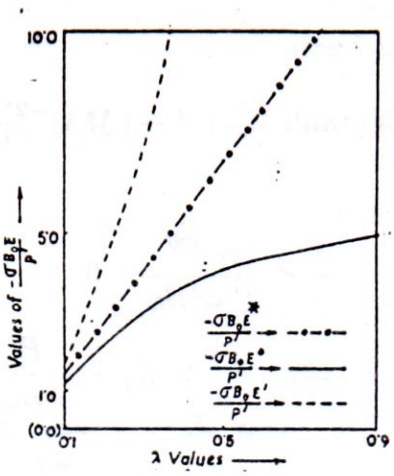

Fig. 2. Graphs for $M = 6$

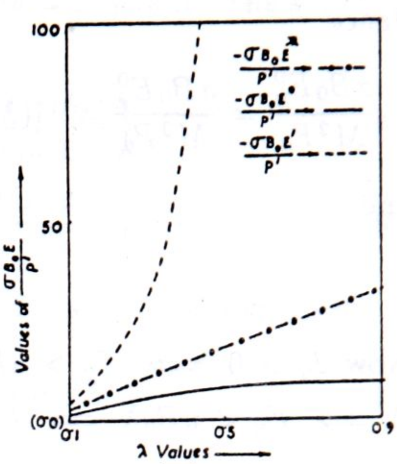

Fig. 3. Graphs for $M = 10$

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ОТКЛОНЕНИЕ МАГНИТНОГО ПОЛЯ В ПОТОКЕ ХАРТМАННА

Эффекты отклонения магнитного поля в потоке Хартманна между двумя параллелными плоскостями, особенно к электрическому полю обсуждается графически для разных величин числа Хартманна M.

Тодд обсуждал эффект числа Хартманна M на электрическое поле в потоке Хартманна. В практических проблемах, всегда можно получить магнитное поле не точно перпендикулярное к границам, благодаря чему происходить перекрёстный поток. Здесь мы стараемся исследовать эффект отклонения магнитного поля на электрическое поле в потоке Хартманна.

ODSTUPANJE MAGNETNOG POLJA U HARTMANOVOM TOKU

Grafički se prikazuju efekti magnetnog polja u Hartmann-ovom toku između dve paralelne ravni, posebno za električno polje, za razne vrednosti Hartmann-ovog broja M.

Todd je razmatrao efekt Hartmann-ovog broja M na električno polje u Hartmann-ovom toku. U praktičnim problemima uvek se može dobiti magnetno polje koje nije tačno normalno na granicu, zbog čega nastaje unakrsni tok. Ovde se izučava efekt odstupanja magnetnog polja na električno polje u Hartmann-ovom toku.

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