TEORIJSKA I PRIMENJENA MEHANIKA 15, pp 93–96, 1989, UDK 532

SECOND ORDER FLUID FLOW BETWEEN TWO PARALLEL PLATES WHEN ONE OF THEM MOVES WITH VELOCITY $U(t)$

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(Received 06.01.1989)

1. Introduction. The present study is devoted to the study of second order fluid flow which occurs between two parallel plates, one of them moving with the velocity $U(t)$. Sparrow and Gregg (1960) determined the time of establishment of the flow of a viscous incompressible fluid past an unsteady rotating-disk by a simple technique, namely by considering the deviation of the instantaneous value of the skin-friction from the quasi-steady value. Panigrahi and Bhunya (1988 a) studied the flow of viscous second order fluid flow between two parallel plates maintained at constant temperature and fluctuating temperature.

The present investigation does not appear to have engaged the attention of the researchers so far and is devoted to study the second order fluid flow between two parallel plates when one of them moves with the velocity $U(t)$ assumed to be a continuously differentiable function of time. By suggesting series solutions for the velocity and pressure functions in terms of certain time dependent parameters $\beta_1 = d\dot{U}/U^2$, $\beta_2 = d^2\ddot{U}/U^3$ we have obtained a set of linear ordinary differential equations. The equations have been exactly integrated and the values of the velocity function and skin-friction computed for different values of the pressure-gradient parameters.

2. Formulation of the Problem. In the present investigation we consider the two parallel infinite plates as lying in the plane $y = \pm d$ and a viscous incompressible second order fluid as occupying the space between the plates. The upper plate is assumed to move with a time dependent velocity $U(t)$, in its own plane in a direction which we have fixed as our x-direction. The flow of the fluid is assumed to be entirely due to the motion of the plate and a pressure gradient in the same direction.

The non-vanishing component of the velocity u in the x-direction satisfies the equation of continuity

$$\frac{\partial u}{\partial x} = 0 \tag{1}$$

and the momentum equation is

$$\frac{\partial u}{\partial t} - \frac{\mu_1}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{\mu_2}{\rho}\frac{\partial^3 u}{\partial y^2 \partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}. \tag{2}$$

The pressure p satisfies in addition to (1) the equation

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} = 0. \tag{3}$$

It would be proper to suggest solutions of the type

$$u = u(y,t), \qquad p = p(x,t). \tag{4}$$

It is obvious that

$$\frac{\partial u}{\partial t} - \frac{\mu_1}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{\mu_2}{\rho}\frac{\partial^3 u}{\partial y^2 \partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} = A(t). \tag{5}$$

The boundary conditions of the equation (5) are

$$y = -d, \quad u = 0; \qquad y = d, \quad u = U(t). \tag{6}$$

3. Solutions of the Equations. To solve the equations (5) subject to the boundary conditions (6), we non-dimensionalise them through the transformation

$$\begin{gathered}\eta = \frac{yU}{\nu}, \quad \eta_d = \frac{dU}{\nu}, \quad \frac{u}{U} = f(\eta, \beta_r) = f_0 + \beta_1 f_1 + \beta_2 f_2 + \cdots, \\ \frac{d^2 A(t)}{\nu U} = A_0 + \beta_1 A_1 + \beta_2 A_2 + \cdots, \quad \beta_r = \frac{d^r D^r U}{U^{r+1}}, \quad D^r U = \frac{d^r U}{dt^r},\end{gathered} \tag{7}$$

where $\nu = \mu_1/\rho$.

Using (7) in (5) and equating the co-efficient of $\beta_1, \beta_2, \ldots$, and independent terms, we get the following equations:

$$\begin{gathered}f_0'' = -\frac{A_0}{\eta_d}, \qquad f_1'' + \Lambda\eta_d(\eta f_0''' + 3f_0'') - \frac{\eta}{\eta_d}f_0' - \frac{f_0}{\eta_d} = -\frac{A_2}{\eta_d} \\ \text{and} \qquad f_2'' + \Lambda\eta_d f_1'' - \frac{f_1}{\eta_d} = -\frac{A_2}{\eta_d}\end{gathered} \tag{8}$$

where dashes denote differentiation with respect to η and $\Lambda = \mu_2/(\rho d^2)$. The boundary conditions of the equations (8) are

$$\eta = -\eta_d, \quad f_0 = f_1 = f_2 = 0 \quad \text{and} \quad \eta = \eta_d, \quad f_0 = 1, \quad f_1 = f_2 = 0. \tag{9}$$

The solutions of the equations (8), satisfying the boundary conditions (9), are

$$\begin{aligned}f_0 &= (1/2)(1 + \eta/\eta_d) + (1/2)A_0\eta_d(1 - \eta^2/\eta_d^2), \\ f_1 &= (A_0/8)\eta_d^2(1 - \eta^4/\eta_d^4) - (\eta_d/12)(1 - \eta^3/\eta_d^3) \\ &\quad - (\Phi/2)\eta_d^2(1 - \eta^2/\eta_d^2) + (\eta_d/12)(1 - \eta/\eta_d) \\ f_2 &= (A_0\eta_d^3/240)(1 - \eta^6/\eta_d^6) + (\eta_d^2/240)(\eta/\eta_d)^2 \\ &\quad - (A_0 A/8 + \Phi/24)\eta_d^3(1 - \eta^4/\eta_d^4) - \eta_d^2(\Lambda/12 + 1/72)(\eta/\eta_d)^3 \\ &\quad + (\Lambda\Phi/2 - A_0/16 + \Phi/4 + A_2/(2\eta_d^2))\eta_d^3(1 - \eta^2/\eta_d^2) \\ &\quad + \eta_d^2(7/720 + \Lambda/12)(\eta/\eta_d)\end{aligned} \tag{10}$$

where $\Phi = (3\Lambda + 1/2 - 1/\eta_d)A_0 + 1/(2\eta_d)$.

4. Skin-Friction. The skin friction co-efficients are given by the relation $(\mu/\rho U^2)(\partial u/\partial y)_{y=\pm d} = f'(\pm\eta_d)$, the positive sign corresponding to the value at the upper wall and the negative sign at the lower one. The first term i.e., $f_0'(\eta_d)$ in the expansion of $f'(\eta_d)$ corresponds to the quasisteady state, at the upper wall and may be designated $\tau_{\text{qs}}^{\text{up}}$. When the strictly decreasing sequence of the parameter $\{\beta_n\}$ rapidly converges, one can neglect the terms involving β_2 and succeeding members down the sequence for a reasonable assessment of the time of establishment of the flow. The instantaneous value of the skin-friction, $\tau_{\text{ins}}^{\text{up}} = \tau_{\text{qs}}^{\text{up}}[1 + f_1'(\eta_d)/f_0'(\eta_d)]$. For a given velocity function, say, $U(t) = 100(1 - e^{-0.1t}))$ one could know $\beta_1 = \dot{U}(t)/[U(t)]^2$ and make an estimate of the time after which (from the instant considered) the flow would become effectively established near the upper wall. In the same way one is able to compute the time of establishment of the flow near the lower wall. One could reasonably take the greater of these two times, as the time after which the entire flow between the plates would ghet established.

5. Discussion of the Results. It is evident from the numerical calculation that f_0 represents the steady parabolic velocity profile of the generalised Couette flow. The unsteadyness in the flow which vanishes in time, is represented by the succeeding terms f_1, f_2 down the series for f. The flow is characterised by dimensionless numbers like the pressure-gradient parameters A_0, A_1 and the Reynold number, η_d based on the channel-width. The pressure gradient parameter occurs in the solutions in combination with the Reynolds number.

It is observed that the deviation from the steady value has parabolic distribution attaining its maximum in the midstream. Further in a positively accelerated flow ($\beta_1 > 0$) the unsteady velocity tends to attain the established value from below. It is found that initially with an increase in the pressure gradient from zero value the establishment is delayed more and more but beyond a certain value, the process of establishment is quickened, with further increase in the pressure-gradient. In the absence of a pressure-gradient, f_0 linearly increases with η from zero value to 1 and f_1 negatively attains its maximum at $\eta = 0.46$. A positively rising value of the pressure-gradient A_0, causes the flow to be deviated more and more from the established state. For a positive constant pressure gradient ($A_0 \neq 0$, $A_1 = A_2 = \ldots = 0$) f_1 is always negative. For a time variant pressure-gradient however f_1 can be made positive for suitable values of A_1.

The effect of heat transfer is currently under investigation.

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ТЕЧЕНИЕ ВТОРОГО ПОРЯДКА МЕЖДУ ДВУМЯ ПАРАЛЛЕЛЬНИМИ ТАРЕЛКАМИ, ОДНА ИЗ КОТОРЫХ ДВИГАЕТСЯ С СКОРОСТЬЮ $U(t)$

Пользуется техника изучения движения флюида между двумя бесконечными параллельными тарелками, одна из которых двигается с скоростю $U(t)$. Дифференциальные уравнения интегрированны и сделаны интересные выводы.

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