TEORIJSKA I PRIMENJENA MEHANIKA 15, pp 45–53, 1989, UDK 531.39

PARAMETER IDENTIFICATION IN MATHEMATICAL MODELLING OF STEEL FRAME WITH BASE ISOLATION

Olivera Jovanović & Dimitar Jurukovski

(Received 30.03.1989; Revised 14.09.1989)

In this paper base isolate structures from earthquake atack by means of spring dashpot system are presented.

A mathematical model to predict the nonlinear behaviour of five story steel frame from earthquake attack by means of springdashpot system is also given. The paper has developed two stages: the development of the form of the mathematical model, and the establishment of the parameter functions appearing in the model both using optimization and experimental data. The experimental work was carried out using the shaking table of the Institut of Earthquake Engeenering, University of Skoplje, Yugoslavia. The base isolate system is consisted of helical springs and dashpots were manufactured in German company GERB. The experiments involved horizontal earthquake excitations.

1. Introduction. It is known that the main efforts of structural engineers, researches and scientists, specially in eartquake engineering have mostly been directed towards predicting the dynamic behaviour of structural systems subjected to strong ground mations. The importance of these efforts is how to define the mathematical model which enables determination of the dynamic response.

As a large number of mathematical models could be determined for each structural system whose solutions present the considered structural dynamic response with different precision, the resulting task becomes the selection of „the best" of all models.

Basic foundations of the system identification were given by P. Eukhoff [1]. Nonlinear behaviour of structures and application of the parametere indentification technique were studied by Distefano and Rath [2]. But as the systems with base isolation have not been studied before we taken them for the purpose of our research identifing dumping in a dashpot in versus of a path.

Mathematical models can be defined in two ways. The procedure in which the autor tries to incorporate all the essential effects in the model on the basis of his personal opinion is defined as „direct problem". Practical experience has proved that the mathematical model for a presented physical system is defined on the bases of the essential structural dynamic laws and structural analysis. The obtained results are evaluated by the researcher himself, and they depend on his own understanding of the physics of the given problem.

Opposite to the above procedure, which is widely applied on basis of the magnified volume of experimental data, a new approach has been developed recently. If could be simply called „inverse problem“, and that means nothing but the definition of the mathematical model using measured structural response data to unknown disturbances.

The problem of mathematical modelling is divided into two categories depending on the degree of the previous knowledge of the nature of the considered process. The modelling has a character of the „black box“ identification for a certain phenomena about and the physical aspect of the problem is unknown. Another category includes all problems whose physical aspects are known. The earthquake engineering problems fall into the second category because of einther their physical processes or their geometry, properties of the material or structural characteristics are known.

The mathematical modelling using system identifikaction is a process of the definition of mathematical equations for a given physical system whose input functions and responses of the physical system are known. The best interpretation of this identification technique for the application in engineering problems was given by Bekey and the system identification is consisted of the following three phases:

1. definition of the form of the model by selection of the differential equations of the model and isolation of the unknown parameters.

2. selection of a criterion by means of which the „goodness of fit“ of the model responses and physical system responses can be evaluated when both the mathematical model and the physical system have been excited by the same input.

3. selection of an algorithm or strategy for adjustment of parameters in such a way that the differences between model and system responses are minimized [3].

2. Mathematical modeling of the physical system. The first step of the parameter system identification as described above is the definition of the form of the mathematical model, i.e. the selection of differential equations for mathematically described presented physical system. A large number of different dynamic models, having different complexity level, can be applied for a certain physical model.

The experimental model is a five storey three bay steel frame mounted on two heavy (2×[30) base floor girders that are supported on a shaking table for simulation by four sets of spring-dashpot elements manufactured by the GERB company for simulation of base isolated model as shown on Fig. 1.

The aim of installing of viscous dashpots to absorbe the energy brought into the system and to decrease the amplitude of the system vibrations, with frame in the linear behaviour while the nonlinear behaviour of the system is connected with the dashpots.

The experimental model was instrumented by 30 channels which measured the accelerations, displacements and stresses. The displacements were recorded by linear potentiometers with respect to a reference beam located ont he foundation block. The horizontal displacements were measured on the base girder and each

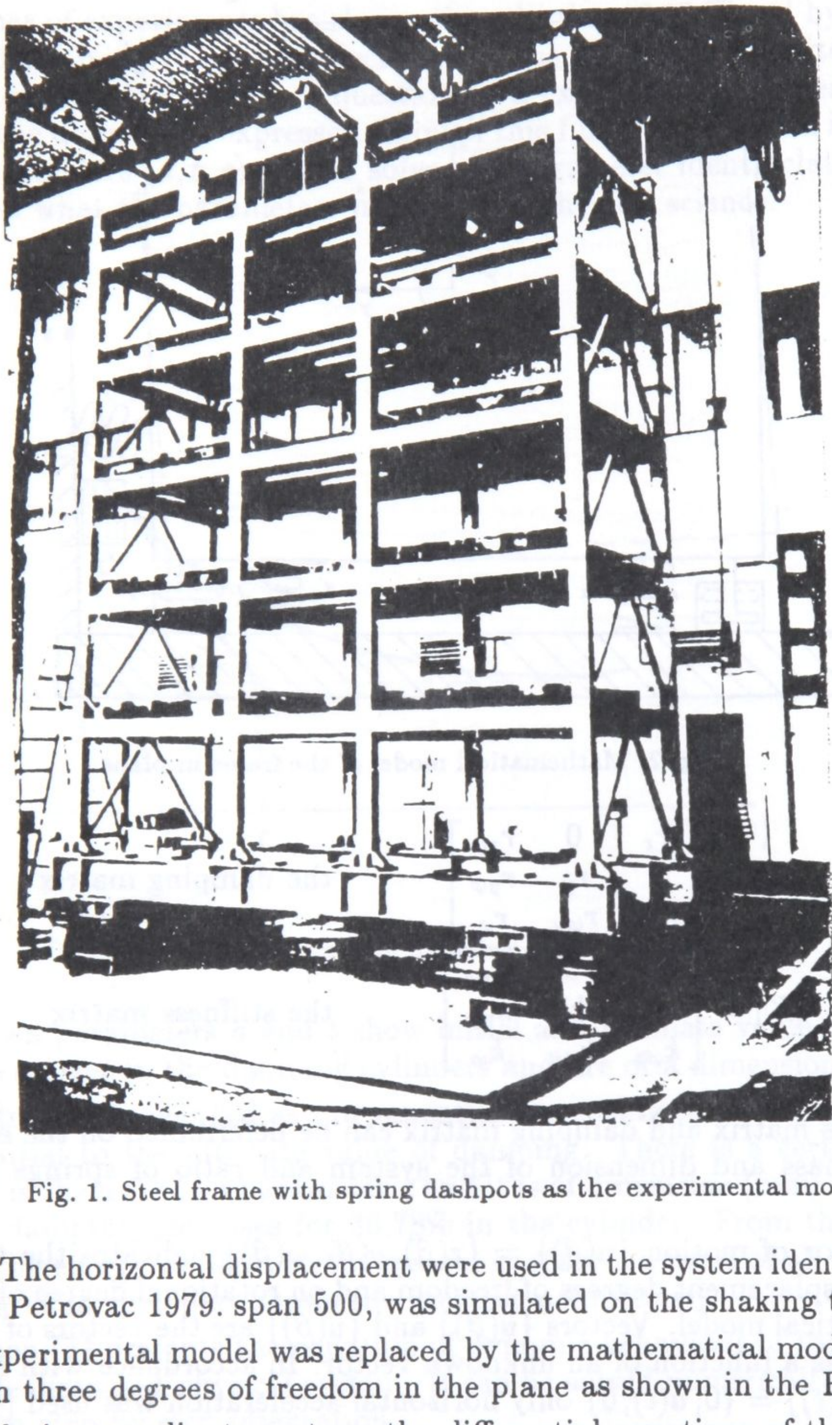

Fig. 1. Steel frame with spring dashpots as the experimental model

floor level. The horizontal displacement were used in the system identification. The earthquake Petrovac 1979. span 500, was simulated on the shaking table.

The experimental model was replaced by the mathematical model consisted of a mass with three degrees of freedom in the plane as shown in the Fig. 2.

In a relative coordinate system, the differential equations of the dynamic behaviour of the physical system in a matrix form are expressed in Eq (1), as:

$$[M]\{\ddot{u}(\beta)\} + [R(\dot{u})]\{\dot{u}(\beta)\} + [K]\{u(\beta)\} = -[M]\{\ddot{n}(t)\} \tag{1}$$

with:

$$[M] = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_c \end{bmatrix} \quad \text{the mass matrix} \tag{2}$$

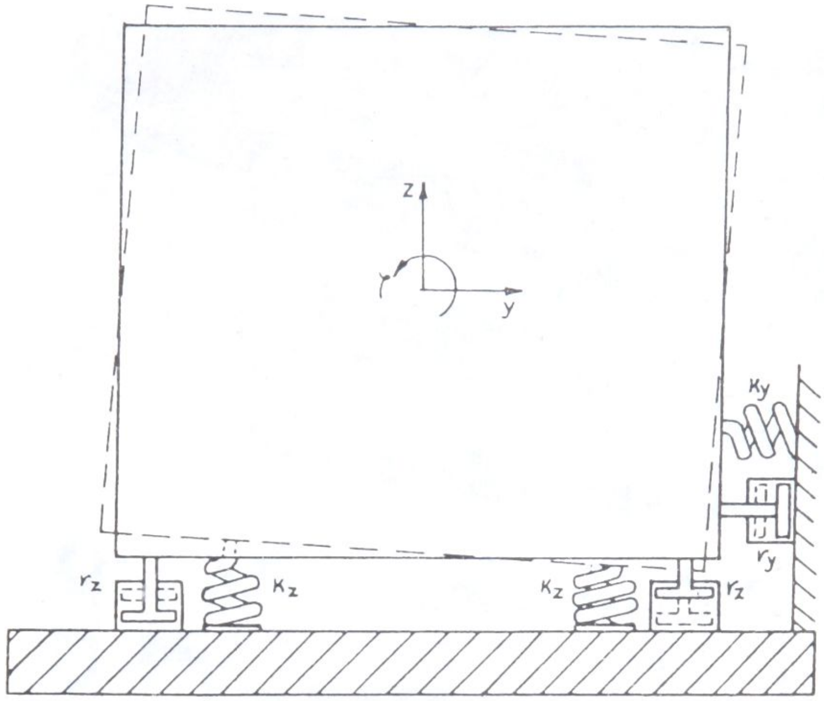

Fig. 2. Mathematical model of the frame in plane

$$[R] = \begin{bmatrix} r_z & 0 & r_{z\varphi} \\ 0 & r_y & r_{y\varphi} \\ r_{\varphi z} & r_{\varphi y} & r_\varphi \end{bmatrix} \qquad \text{the damping matrix} \tag{3}$$

$$[K] = \begin{bmatrix} k_z & 0 & k_{z\varphi} \\ 0 & k_y & k_{y\varphi} \\ k_{\varphi z} & k_{\varphi y} & k_\varphi \end{bmatrix} \qquad \text{the stiffness matrix} \tag{4}$$

The mass matrix and damping matrix can be defermined on the experimental base if the mass and dimension of the system and ratio of springs stiffness are known.

The vector of motion $\{u(\beta)\} = \{z(\beta), y(\beta), \varphi(\beta)\}$ indicates the total motion for the two displacement degrees of freedom and on rotational degree of freedom for the mathematical model. Vectors $\{\ddot{u}(\beta)\}$ and $\{\dot{u}(\beta)\}$ are the vectors of acceleration and velocity as a function of an unknown vector. In accordance with the vector of excitation $\{\ddot{n}(t)\} = \{0, \ddot{u}(t), 0\}$ only horizontal acceleration was used [4].

The damping matrix is versus of the dimensions of the system and ratio of the vertical damping.

It has been known so far that the damping matrix was calculated as a linear combinations of the mass matrix and stiffness, or as a function of the square of velocity of a body in a dashpot, but this work presents another point of view i.e. damping is given through three unknown parameters in the function of the path of the body in cylinder.

$$r_z = b + (a - b)\exp\left(-c^{-1} \int |v|\, dt\right) \tag{5}$$

$\{\beta\} = \{a, b, c\}$ — vector of unknown parameters.

A number of experiments has shown that the damping caused by constant displacement of the body in cylinder decreases in the physically permited limits what brings the conclusion that it is a question of a exponentially decreasing function that is why the damping is expressed through this function as shown in Eq (5). The unknown parameters (a, b, c) will be solved by parameter identificlation of system. Fig. 3. shows what the parameters mean in the physical science.

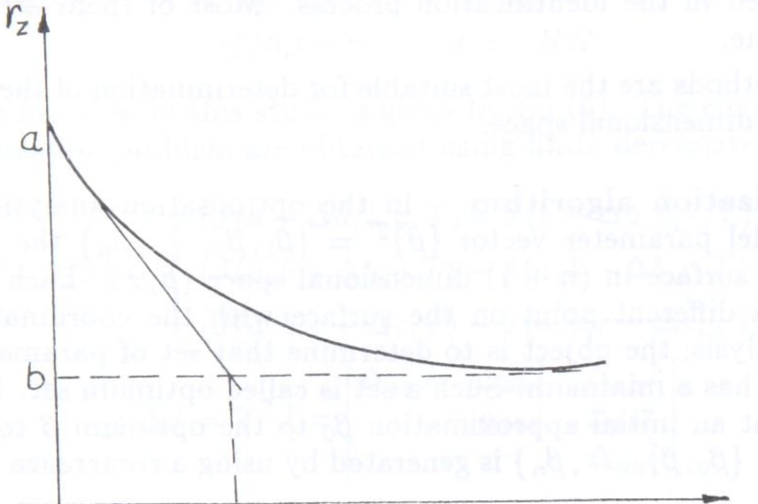

Fig. 3. Curve of damping in the dashpot

Unknown parameters a and b show initial and ultimate value of the damping ratio of the matter in the damping cylinders and are of a dimension [Ns/m].

The damping is changed as a exponential decreasing curve depending of a path from the initial to the ultimate value of damping. There is a certain of value of path and it is the unknown parameter c and it is of dimension [mm]. After a passed path c the damping decreases for 36,78% in the cylinder. From the beginning to that point the energy brought to the system is apsorbed most of all that is why the damping curve in that part shows the greatest nonlinearity.

Equation (1) presents the equation of dynamic behaviour of the mathematical model of the discussed physical system. It can be solved numerically by Newmark's method with step by step integration.

The solution of this equation presenting the horizontal displacement of the system is compared with experimental measure of the horizontal displacement of the system and is used for determining the error function.

The next is the selection of a criterion function or error function. It is expressed as the integrated least squares error between the theoretical and experimental horizontal displacement as:

$$J(\beta) = \int_0^T W_k [u(\beta, t) - u(t)][u(\beta, t) - u(t)] \, dt \tag{6}$$

where T time intervals defining the sensitive part of the model responses, while $u(\beta, t)$ is the vector of displacement obtained from the mathematical model, and $u(t)$ is the vector of displacement experimentally recorded from the physical model. W_k represents the weight coeficients for evaluation of the various physical variable influence. W_k has the value of 1 because the error function is form only by displacement that is by one and same physical value.

The third step in identification technique is very important and represent selection of the algorithm for parameters adjustment to minimize the criterion function (6). There is a large number of methods in the mathematical optimization theory which can be used in the identification process. Most of them are based on the iterative technique.

Gradient methods are the most suitable for determination of the function minimum in a multi-dimensional space.

3. Optimization algorithm. In the optimisation analysis involving n-dimensional model parameter vector $\{\beta\}^T = \{\beta_1, \beta_2, \dots, \beta_n\}$ the error function $J(\beta)$ describes a surface in $(n+1)$ dimensional space (β, J). Each set of parameters β defines a different point on the surface with the coordinates (β, J). In optimization analysis, the object is to determine that set of parameters for which the error surface has a minimum. Such a set is called optimum set. In an iterative algorithm, as first an initial approximation β_0 to the optimum β to be is chosen. Then, a sequence $\{\beta_0, \beta_1, \dots, \beta_n\}$ is generated by using a recurrence formula of the form:

$$\beta_{i+1} = \beta_i + \Delta\beta_i \tag{7}$$

where β_{i+1} designates the updated set at the $(i+1)$-th iteration.

The expression defining $\Delta\beta_i$ in Eq (7) has different form in different iterative algorithms. If the algorithm generates a convergent process, the error J decreases at each iteration, i.e.

$$J(\beta_{i+1}) < J(\beta_i), \tag{8}$$

and reaches its minimum value, within a limit of a specified accuracy, in a finite number of iterations.

Gauss Newton's method, which is also refered to as the second order gradient method, is based on Taylor's expansion of the error function in the neighborhood of a set β, which gives

$$J\beta, T = J\beta_{i-1}, T + \nabla J^T(\beta_{i-1}, T)(\beta_i - \beta_{i-1}) + (1/2)(\beta_i - \beta_{i-1})\nabla^2 J(\beta_i - \beta_{i-1}) \\ + \text{higher other terms} \tag{9}$$

where $\nabla J(\beta_i, T)$ is a gradient vector, $\nabla^2 J(\beta_i, T)$ is the symmetric Hessian matrix and $\beta_i - \beta_{i-1}$ is an incremental vector.

The elements of the gradient vector $\nabla J(\beta_i, T)$ and the Hessisan matrix $\nabla^2 J(\beta_i, T)$ are defined by the relation:

$$\nabla J(\beta_i, T) = \partial J(\beta_i, T)/\partial\beta_i, \quad \nabla^2 J(\beta_i, T) = H_{kl} = \partial^2 J(\beta_i, T)/\partial\beta_k\partial\beta_l.$$

Let $\beta_i - \beta_{i-1}$ be small than. the expansion in Eq (9), when higher other terms are neglected, takes the form

$$J(\beta_i, T) = J(\beta_{i-1}, T) + \nabla J^T(\beta_{i-1}, T)(\beta_i - \beta_{i-1})$$

$$+(1/2)(\beta_i-\beta_{i-1})^T\nabla^2 J(\beta_i-\beta_{i-1})=Q(d). \qquad (10)$$

The task is to determine $\beta_i-\beta_{i-1}$ so that $\beta_{i+1}+\Delta\beta=\beta_{\min}$. For fixed β_{i-1} and variable $\Delta\beta$, $J(\beta)=Q(d)$ defines a quadratic surface approximating the error surface in the neighborhood of the point $(\beta_{i-1},J(\beta_{i-1}))$. It is known that at the minimum point $(\beta_{\min},J(\beta_{\min}))$ where the slope of the error surface must be zero. If the derivative of $Q(d)$ with respect to $\Delta\beta$ vanishes, it is approximately satisfied.

When Eq (10) is used, this gives

$$\min f(\beta_i)=\min f\left[\beta(i-1)-\alpha\nabla^2 J^{-1}(\beta_{i-1},T)\nabla J(\beta_{i-1},T)\right],$$
$$\partial f/\partial\beta=0, \qquad d=-HG. \qquad (11)$$

The error function in this study is given by Eq (6). The elements of G and H for this optimization problem are obtained using finite derivative.

$$G=\nabla J(\beta,T)=\begin{Bmatrix}(J(a+\Delta a,b,c,T)-J(a-\Delta a,b,c,T))/(2\Delta a)\\(J(a,b+\Delta b,c,T)-J(a,b-\Delta b,c,T))/(2\Delta b)\\(J(a,b,c+\Delta c,T)-J(a,b,c-\Delta c,T))/(2\Delta c)\end{Bmatrix}$$

$$\Delta J=\nabla^2 J=\begin{vmatrix}\Delta_{aa}J & \Delta_{ab}J & \Delta_{ac}J\\ & \Delta_{bb}J & \Delta_{bc}J\\ & & \Delta_{cc}J\end{vmatrix}$$

where:

$$\Delta_{aa}J=\frac{J}{\Delta a^2}\left[J(a+\Delta a,b,c,T)-2J(a,b,c,T)+J(a-\Delta a,b,c,T)\right]$$

$$\Delta_{ba}J=\frac{1}{4\Delta a\Delta b}\left[J(a+\Delta a,b+\Delta b,c,T)-J(a-\Delta a,b+\Delta b,c,T)\right.$$
$$\left.-J(a+\Delta a,b-\Delta b,c,T)+J(a-\Delta a,b-\Delta b,c,T)\right].$$

In Eq (11) it is:

$$H=\nabla^2 J^{-1}(\beta_{i-1},T).$$

When we calculate the point in which $J_{\min}$ is in this direction d, the procedure is repeated until the following point β_{i+1} becomes the smallest value of the error function [4].

4. Example. A package of computer programmes has been developed for the presented theory and they are used for identification of the parameters. The experimental data used in the analysis (displacement and acceleration time histories recorded at both storey levels of the physical system) were stored in the peripherical memory of the computer and were called when it was necessary. Also, acceleration time histories, recorded on the shaking table during the performance of the corresponding test and all the other initial values of the physical variables required by the computer programme were stored.

The mathematical model was proposed with three unknown parameters which were isolated:

$$\{\beta\}=\{a,b,c\}.$$

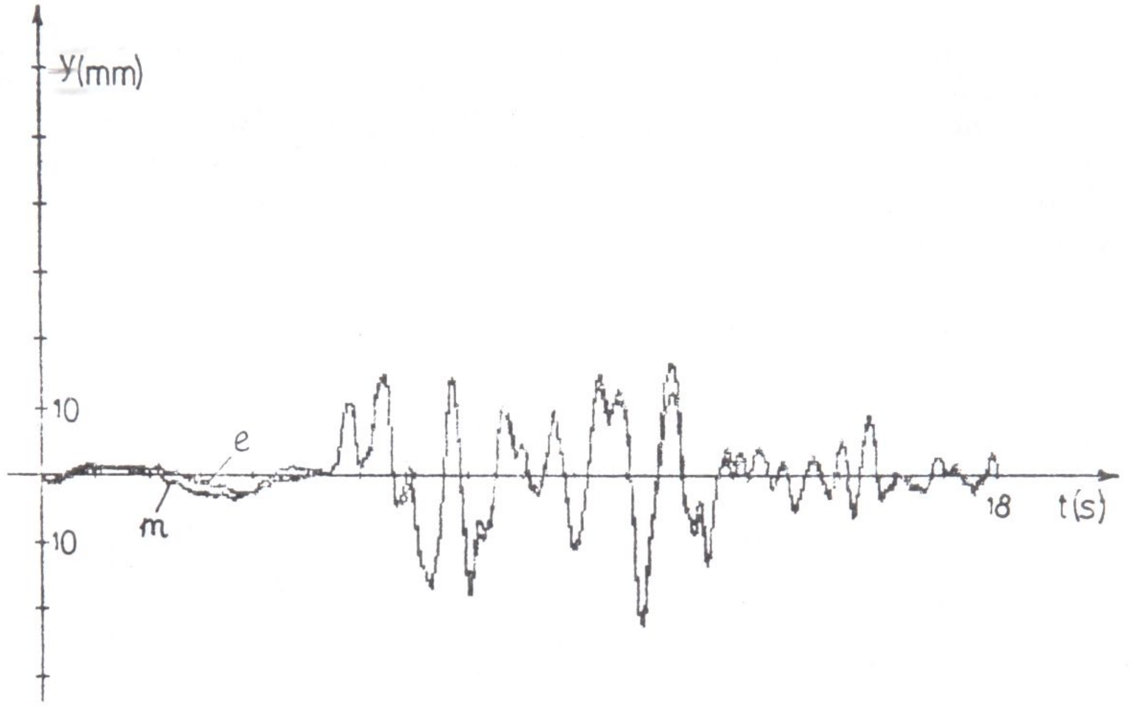

Fig. 4. Experimental (e) and mathematical (m) response in the centre of mass of steel frame for the earthquake, Petrovac 1979, span 500.

In order to assure the flow of the parameter identification process following the computer codes, it is necessary to define the initial values of vectors β. The values of this vector can be defined as random values, but in order to have a smaller number of iterations, estimate procedure vas used. The remaining initial values were selected:

$$\{\beta_0\} = \{570000,\ 400000,\ 6\}$$

The following iterations are:

$$\{\beta_1\} = \{820000,\ 580000,\ 5{,}82\}$$
$$\{\beta_2\} = \{928800,\ 629000,\ 5{,}81\}$$
$$\{\beta_3\} = \{929900,\ 629917,\ 5{,}80\}.$$

After the fourth iteration, vector has the following values:

$$\{\beta_4\} = \{930000,\ 630000,\ 5{,}8\}.$$

For the vector of parameter β_4 the responses of the mathematical model were calculated and the results were compared with the experimental ones. The results of this identification technique are given in Fig. 4. The experimental results (e) as displacement of the centre of the mass of the system and mathematical results (m) as the displacement of the same point of the frame are shown on the same figure.

It is not necessary to give comment on the differences between the responses since they are numerically qualified.

Conclusion. The parameter system identification technique which was used in the problem under investigation gives the opportuanity to quantify the effects of any parameters on the mathematical model response.

Helical springs and dashpots are durable and less sensitive to temperature changes and to physical conditions of the air. It is very easy to replace any defective helical spring or dashpot.

REFERENCES

[1] Eukhoff P.: *System Identification, Parameter and State Estimation*, John Wiley & Sons, Bristol, 1974.

[2] Distefano N., and Rath A.: *Modelling and identification in nonlinear structural dynamics — one degree of freedom models*, Report No 74–15, Berkley, California, 1974.

[3] Bekey A. G.: *System identification — An Introduction and a Survey. Simulation*; Vol. No 4, October 1970. pp. 151–166.

[4] Jovanović O.: *Matematičko modeliranje čeličnog okvirnog sistema sa baznom izolacijom sa primjenom parametarske identifikacije*, Doktorska disertacija, Skoplje, 1988.

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ СИСТЕМЫ С ИСПОЛЬЗОВАНИЕМ ПАРАМЕТРИЧЕСКОЙ ИДЕНТИФИКАЦИИ СИСТЕМ

В работе анализируется применение параметрической идентификации системы для моделирования динамического поведения нелинейной физической системы. Физическая система состоится от конструкции поставленной на вискозные затухатели и пружины как элементы базовой изоляции. В работе применяется метода Гауса-Нютона минимизации функции ошибки с высше переменных, которая представлает один из шагов в процессе параметрической идентификации. Вектор градиента и матрица Хессе формированные с помощью центральных конечных разностей. Экспериментальные результати получились в лабораторных условиях ИЗИИС — Скопле.

MATEMATIČKO MODELIRANJE SISTEMA SA BAZNOM IZOLACIJOM POMOĆU PARAMETARSKE IDENTIFIKACIJE

U radu je analizirana primjena parametarske identifikacije sistema za matematičko modeliranje dinamičkog ponašanja nelinearnog fizičkog sistema. Fizički sistem sastoji se od konstrukcije postavljene na viskozne prigušivače i opruge kao elemente bazne izolacije. U radu je primenjen Gauss-Newton-ov metod minimizacije funkcije greške više promenljivih koji predstavlja jedan od koraka u procesu parametarske identifikacije. Vektori gradijenta u Hesse-ovoj matrici su formirani preko centralnih diferencnih razlika. Eksperimentalni rezultati dobijeni su simulacijom i laboratorijskim uslovima instituta IZIIS — Skoplje.

Olivera Jovanović
Mašinski fakultet
Cetinjski put bb
81000 Titograd

Dimitar Jurukovski
Institut IZIIS
91000 Skoplje