TEORIJSKA I PRIMENJENA MEHANIKA 15, pp 19–30, 1989, UDK 539

CONTRIBUTION TO THE TIME DEPENDENT DEFORMATIONS ON THE STRESS (MOMENT) REDISTRIBUTION

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(Received 20.06.1988; revised 25.09.1989)

1. Introduction. In the analysis of composite beam structures concrete is considered as a linearly viscoelastic material with the aging property. The mathematical formulation of the relation between the deformation and the concrete strain, at the linear state of strain was given by Maslov [4] and McHenry [5]. The concrete creep deformation was assumed as a linear functional of the strain history, i.e. they adopted the Boltzmann-Volterra superposition principle for strain effects, modified by the concrete age, and they arrived at the integro-differential relation. In the theory of viscoelasticity with aging Mandel [6] estabilished a mathematical method where in he used the linear integro-differential operators. In the course of deriving the expressions for strain and displacement, valid for an arbitrary function of the concrete creep, Lazić [2] has introduced the linear integral operators by means of which trough algebraic operations the entire procedure of solving strains of statically determinate and indeterminate beam is developed. In the present paper the mathematical theory is applied to the analysis of the composite beam structures by the slope deflection method which was not dealt with in the hitherto literature.

In the present paper the slope deflection method will be applied, where the axial deformation of flexural members will be neglected. The structure with undisplaceable joints will be considered, so that the unknowns are only the angles of rotation of joints φ. The structure consists of "k" type members and "g" type members. The "k" type member is the one rigidly joined at both member ends. The member end are denoted by i and k and the corresponding moments by M_{ik} and M_{ki}.

The "g" type member is the one which is at one end rigidly joined and on the other end hinged. The ends of that member are denoted by i and g and the corresponding moments by M_{ig} and M_{gi}, M_{gi} being zero ($M_{gi} = 0$).

In order to determine the internal forces and the deformation in the "k" type member it is necessary that we know the rotations φ_i and φ_k of the member ends, for the "g" type member we need only to know the rotation φ_i.

If m denotes the number of joints in a structure where at least one rigid angle exists, then we can say that the number of unknown rotations φ is equal to the number m.

The angles of the rotation of joints φ for the elastic structure can in the known manner [1] be expressed though moments at the member ends M_{ik} and M_{ki} and the external load.

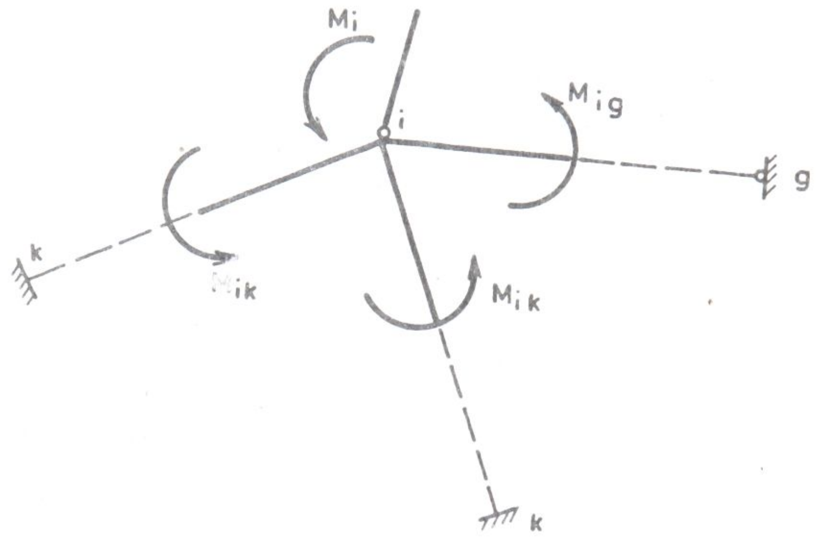

Figure 1

$$\begin{aligned} \varphi_i &= \alpha_{ik} M_{ik} - \beta_{ik} M_{ki} + \alpha_{ik,0} \\ \varphi_k &= -\beta_{ki} M_{ik} + \alpha_{ki} M_{ki} - \alpha_{ki,0}. \end{aligned} \tag{1}$$

In expressions (1) by α_{ik}, β_{ik}, β_{ki} and α_{ki} are denoted the angles of deformation at the member ends due to $M_{ik} = 1$, $M_{ki} = 1$, when the member is not loaded, and $\alpha_{ik,0}$, and $\alpha_{ki,0}$, the angles of deformation at the member ends due to external load, when $M_{ik} = M_{ki} = 0$.

2. Influence of the Time Deformations. Due to viscoelastic features of concrete and relaxation properties of prestressing steel in the analysis of composite structures, unknown quantities φ and moments at the member ends M_{ik}, M_{ki} and M_{ig} are treated as time dependent functions.

We consider the case when since the initial time t_0 the constant moment M_{ik} acts. Also M_{ki} and external influences are being equal to zero. Therefore,

$$M_{ik}(t, t_0) = M_{ik} 1^* \tag{2}$$

where function 1^* represents the Heaviside Function i.e.:

$$1^* = 1^*(t, t_0) = H(t - t_0) = \begin{cases} 1 & \text{for } t > t_0 \\ 0 & \text{for } t \leq t_0. \end{cases}$$

Consequently, we have:

$$\varphi_i(t, t_0) = \alpha^*_{ik}(t, t_0) M_{ik,0}. \tag{4}$$

If we assume that moment M_{ik} is a continuous time dependent function, then the application of the Boltzmann-Volterra superposition principle may be written as:

$$\varphi_i(t,t_0) = \alpha^*_{ik}(t,t_0)M_{ik,0} + \int_{t_0}^{t} \alpha^*_{ik}(t,\theta)\frac{\partial M_{ik}(\theta,t_0)}{\partial\theta}\,d\theta. \tag{5}$$

By a partial integration and with the known relation $\alpha'^*_{ik} = -\alpha'_{ik}$ [2] expression (5) may be written in the form:

$$\varphi_i(t,t_0) = \alpha^*_{ik}(t,t)M_{ik}(t,t_0) + \int_{t_0}^{t} \alpha'_{ik}(t,\theta)M_{ik}(\theta,t_0)\,d\theta. \tag{6}$$

If the operator:

$$\widetilde{\alpha}'_{ik} = \alpha^*_{ik}(t,t)\widetilde{1}' + \widetilde{\alpha}'_{ik} \tag{7}$$

is introduced, expression (6) may be represented in the operator form:

$$\varphi_i(t,t_0) = \widetilde{\alpha}'_{ik}(t,t_0)M_{ik}(t,t_0). \tag{8}$$

Analogously, we can write:

$$\varphi_i = \widetilde{\alpha}'_{ik}M_{ik} - \widetilde{\beta}'_{ik}M_{ki} + \alpha^*_{ik,0}, \qquad \varphi_k = -\widetilde{\beta}'_{ki}M_{ik} + \widetilde{\alpha}'_{ki}M_{ki} - \alpha^*_{ki,0}. \tag{9}$$

Operators introduced $\widetilde{\alpha}'_{ik}$, $\widetilde{\beta}'_{ik}$, $\widetilde{\beta}'_{ki}$ and $\widetilde{\alpha}'_{ki}$ are associated to functions α'_{ik}, β'_{ik}, β'_{ki} and α'_{ki} whose integrals are given by functions $\alpha^*_{ik}(t,t_0)$, $\beta^*_{ik}(t,t_0)$, $\beta^*_{ki}(t,t_0)$ and $\alpha^*_{ki}(t,t_0)$ representing angles of deformation at the member ends due to unit moments $M_{ik} = 1^*$ and $M_{ki} = 1^*$, when the member is unloaded.

Expressions $\alpha^*_{ik,0} = \alpha^*_{ik,0}(t,t_0)$ and $\alpha^*_{ki,0} = \alpha^*_{ki,0}(t,t_0)$ presented in relation (9) represent the angels of the member ends due to the external loading.

Quantities $\alpha^*_{ik}(t,t_0)$, $\beta^*_{ik}(t,t_0)$, $\beta^*_{ki}(t,t_0)$, $\alpha^*_{ki}(t,t_0)$, $\alpha^*_{ik,0}(t,t_0)$ and $\alpha^*_{ki,0}(t,t_0)$ can be determined by known principle of virtual forces or by the conjugate-beam method.

In the slope-deflection method, applied here, the influence of the normal forces is neglected, so the expression for the curvature κ in the analysis of composite structures [3] will have the form:

$$\kappa = (1/E_uJ_i)\widetilde{F}'_{22}M. \tag{10}$$

In the expression (10) the linear integral operator has been introduced:

$$\widetilde{F}'_{22} = (1/e_{22})\widetilde{1}' + \widetilde{\Psi}'_{22} \tag{11}$$

which is associated to function F'_{22}, whose integral is function

$$F^*_{22} = (1/e_{22})1^* + \psi^*_{22} \tag{12}$$

where:

$$e_{22} = 1 + \gamma_{22}(e-1); \quad e = e(t) = \frac{E_b(t)}{E_{b0}}; \quad E_{b0} = E_b(t_0); \quad e(t_0) = 1;$$

$$\psi_{22}' = \frac{1}{\Delta\gamma}(\delta\gamma_1\gamma_1\psi_1' + \delta\gamma_2\gamma_2\psi_2'). \tag{13}$$

Through the quantities $\Delta\gamma$, $\delta\gamma_h$, γ_h, $h = 1, 2$, geometric properties of the cross-section [3] are introduced. Function ψ_h^* is the basic function. For the adopted concrete creep function F^* we can determine the value of function F_{22}^* for every considered cross-section.

Using relation (10) and the principle of virtual forces the joint rotation may be expressed in the following manner:

$$\varphi_i(t,t_0) = \int_z \widehat{M}(z,s)\kappa(z,t,t_0)\,dz = \int_z \widehat{M}(z,s)\frac{1}{E_u J_i}\widetilde{F}_{22}'(z,t,\theta)M(z,\theta,t_0)\,dz. \tag{14}$$

If we consider the rotation due to moment M_{ik}:

$$M_{ik} = 1^* f(z) \tag{15}$$

M_{ki} and the external influences being equal to zero, then:

$$\begin{aligned} \alpha_{ik}^*(t,t_0) &= \int_z \widehat{M}(z,s)\frac{1}{E_u J_i}\widetilde{F}_{22}'(z,t,\theta)1^*(\theta,t_0)f(z)\,dz \\ &= \int_z \widehat{M}(z,s)\frac{1}{E_u J_i}F_{22}^*(z,t,t_0)f(z)\,dz. \end{aligned} \tag{16}$$

We will consider the rotation due to external influences H: $(M_{ik} = M_{ki} = 0)$.

a) $H = q$ distributed dead load

$$\alpha_{ik,q}^*(t,t_0) = \int_z \widehat{M}(z,s)\frac{1}{E_u J_i}\widehat{F}_{22}'(z,t,\theta)M_q(z,\theta,t_0)\,dz. \tag{17}$$

b) $H = c$ given displacement of the support

The rotation angle of the member $\psi_{ik,c}$ will be a time-independent value.

$$\alpha_{ik,c}^*(t,t_0) = \psi_{ik,c}1^*(t,t_0). \tag{18}$$

The system of relations (9) with relations (17) and (18) considering various effects H can be written in the following form:

$$\begin{aligned} \varphi_{i,H} &= \widetilde{\alpha}_{ik}' M_{ik,H} - \widetilde{\beta}_{ik}' M_{ki,H} + \alpha_{ik,H}^*, \\ \varphi_{k,H} &= -\widetilde{\beta}_{ki}' M_{ik,H} + \widetilde{\alpha}_{ki}' M_{ki,H} - \alpha_{ki,H}^* \end{aligned}, \qquad H = q, c. \tag{19}$$

The operators appearing in the system of equations (19) are comutative, and the system of equations can then be formally solved by applying the Cramer rule.

The system of equations (19) has the solution given in the form:

$$\begin{aligned} M_{ik,H} &= \widetilde{A}_{ik}'\varphi_{i,H} + \widetilde{B}_{ik}'\varphi_{k,H} + \mathcal{M}_{ik,H}^*, \\ M_{ki,H} &= \widetilde{B}_{ki}'\varphi_{i,H} + \widetilde{A}_{ki}'\varphi_{k,H} + \mathcal{M}_{ki,H}^* \end{aligned} \tag{20}$$

For the "g" type member we obtain analogously the expressions for moments:

$$M_{ig,H} = \widetilde{D}'_{ig}\varphi_{i,H} + \mathcal{M}^*_{ig,H}. \tag{21}$$

Operators in relations (20) and (21) will have the following form:

$$\begin{aligned}\widetilde{A}'_{ik} &= (\widetilde{\alpha}'_{ik}\widetilde{\alpha}'_{ki} - \widetilde{\beta}'_{ik}\widetilde{\beta}'_{ki})^{-1}\cdot\widetilde{\alpha}'_{ki}, \quad \widetilde{B}'_{ik} = \widetilde{B}'_{ki} = (\widetilde{\alpha}'_{ik}\widetilde{\alpha}'_{ki} - \widetilde{\beta}'_{ik}\widetilde{\beta}'_{ki})^{-1}\cdot\widetilde{\beta}'_{ik}, \\ \widetilde{A}'_{ki} &= (\widetilde{\alpha}'_{ik}\widetilde{\alpha}'_{ki} - \widetilde{\beta}'_{ik}\widetilde{\beta}'_{ki})\cdot\widetilde{\alpha}'_{ik}, \quad \widetilde{D}'_{ig} = (\widetilde{\alpha}'_{ig})^{-1}.\end{aligned} \tag{22}$$

The freee terms in expressions (20) $\mathcal{M}^*_{ik,H}$, $\mathcal{M}^*_{ki,H}$ and $\mathcal{M}^*_{ig,H}$ depend on external influences $H = q, c$:

a) $H = q$ distributed dead load

$$\begin{aligned}\mathcal{M}^*_{ik,q} &= -\widetilde{A}'_{ik}\alpha^*_{ik,q} + \widetilde{B}'_{ik}\alpha^*_{ki,q}, \\ M^*_{ki,q} &= \widetilde{A}'_{ki}\alpha^*_{ik,q} + \widetilde{B}'_{ik}\alpha^*_{ki,q}, \quad M^*_{ig,q} = -\widetilde{D}'_{ig}\alpha^*_{ig,q}.\end{aligned} \tag{23}$$

b) $H = c$ given displacement of the support

$$\mathcal{M}^*_{ik,c} = -\widetilde{C}'_{ik}\psi_{ik,c}1^*, \quad M^*_{ki,c} = -\widetilde{C}'_{ki}\psi_{ik,c}1^*, \quad M^*_{ig,c} = -\widetilde{D}_{ig}\psi_{ig,c}1^* \tag{24}$$

where:

$$\widetilde{C}'_{ik} = \widetilde{A}'_{ik} + \widetilde{B}'_{ik}, \qquad \widetilde{C}'_{ki} = \widetilde{A}'_{ki} + \widetilde{B}'_{ki}. \tag{25}$$

In the structure with undisplaceable joints the unknowns are angles of rotation of joints φ. When writing the equations for determing the unknowns we start from the equilibrium conditions of the joint i:

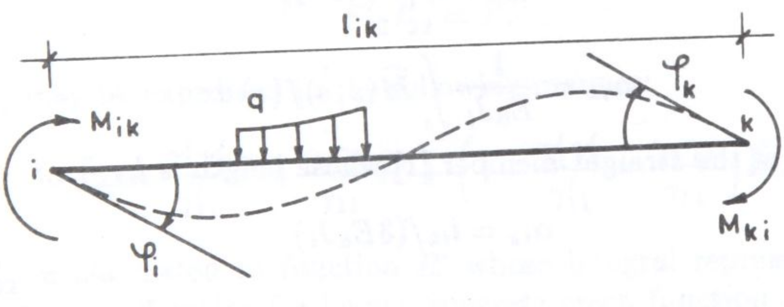

Figure 2

$$\sum_k M_{ik} + \sum_g M_{ig} + M_i = 0. \tag{26}$$

When expressions (20) and (21) are introduced into expression (26) i.e.

$$\sum_k(\widetilde{A}'_{ik}\varphi_{i,H} + \widetilde{B}'_{ik}\varphi_{k,H} + \mathcal{M}^*_{ik,H}) + \sum_g(\widetilde{D}'_{ig}\varphi_{i,H} + \mathcal{M}^*_{ig,H}) + \mathcal{M}^*_i = 0 \tag{27}$$

$$i = 1, 2, \ldots, m; \quad H = q, c$$

we obtain m integral equations with m unknowns φ. These equations are called the equations of joint rotaton or shorter joint equations.

If we introduce operators:

$$\widetilde{\mathcal{A}}'_{ii} = \sum_k \widetilde{\mathcal{A}}'_{ik} + \sum_g \widetilde{\mathcal{D}}'_{ig}, \quad \widetilde{\mathcal{A}}'_{ik} = \widetilde{\mathcal{B}}'_{ik} \tag{28}$$

and function

$$\mathcal{A}^*_{i,H} = \sum_k \mathcal{M}^*_{ik,H} + \sum_g \mathcal{M}^*_{ig,H} + \mathcal{M}^*_i \tag{29}$$

then the joint equations may be written in the form:

$$\widetilde{\mathcal{A}}'_{ii}\varphi_{i,H} + \sum_k \widetilde{\mathcal{A}}'_{ik}\varphi_{k,H} + \mathcal{A}^*_{i,H} = 0, \qquad i = 1, 2, \dots, m; \quad H = q, c. \tag{30}$$

Joint equations (30) make a system of m integral equations with respect to unknowns φ.

For the rate of flow and rate of creep ($E_b =$ const.) of concrete creep function F^* ther system of integral equations (30) may be solved by applying the Laplace transforms. In other cases those equations may be solved by numerical procedures.

3. Special Case-Members of Constant Cross-Sections. Expression (16) may be writrten in the following manner:

$$\alpha^*_{ik}(t, t_0) = \left[\frac{1}{E_u J_i}\int_z \widehat{M}(z, s) f(z)\, dz\right] F^*_{22}(t, t_0) \tag{31}$$

or shorter:

$$\alpha^*_{ik}(t, t_0) = \alpha_{ik} F^*_{22} \tag{32}$$

where

$$\alpha_{ik} = \frac{1}{E_u J_i}\int_z \widehat{M}(z, s) f(z)\, dz \tag{33}$$

known constant of the straight member [1] whose length is l_{ik}, i.e.:

$$\alpha_{ik} = l_{ik}/(3E_u J_i). \tag{34}$$

Then:

$$\widetilde{\alpha}'_{ik} = \alpha_{ik}\widetilde{F}'_{22}. \tag{35}$$

Analogously we can write:

$$\widetilde{\beta}'_{ik} = \widetilde{\beta}'_{ki} = \beta_{ik}\widetilde{F}'_{22}, \qquad \widetilde{\alpha}'_{ki} = \alpha_{ki}\widetilde{F}'_{22} \tag{36}$$

where

$$\beta_{ik} = \beta_{ki} = l_{ik}/(6E_u J_i), \qquad \alpha_{ik} = \alpha_{ki} = l_{ik}/(3E_u J_i). \tag{37}$$

We will consider the rotation due to external influences H: ($M_{ik} = M_{ki} = 0$)

a) $H = q$ distributed dead load, assumed constant since the initial time t_0:

$$M = M_q 1^*(t, t_0). \tag{38}$$

Using relation(17) the rotation may be written in the following way:

$$\alpha^*_{ik,q}(t,t_0) = \left[\frac{1}{E_u J_i}\int_z \widehat{M}(z,s)M_q(z)\,dz\right]\cdot F^*_{22}(t,t_0) \tag{39}$$

or

$$\alpha^*_{ik,q}(t,t_0) = \alpha_{ik,q}F^*_{22} \tag{40}$$

where:

$$\alpha_{ik,q} = \frac{1}{E_u J_i}\int_z \widehat{M}(z,s)M_q(z)\,dz \tag{41}$$

b) $H = c$

Under the influence c, the given displacement of the support will be considered. The rotation angle of the member $\psi_{ik,c}$, will be a time-independent value, i.e.:

$$\alpha^*_{ik,c} = \psi_{ik,c}1^*. \tag{42}$$

The operators $\widetilde{A}'_{ik}$, $\widetilde{B}'_{ik}$, $\widetilde{A}'_{ki}$, and $\widetilde{D}'_{ig}$ appearing in the relation (22) for the members of constant cross-section will have the following form:

$$\widetilde{A}'_{ik} = \widetilde{A}'_{ki} = a_{ik}\widetilde{I}'_{22} = a_{ki}\widetilde{I}'_{22},\quad \widetilde{B}'_{ik} = \widetilde{B}'_{ki} = b_{ik}\widetilde{I}'_{22},\quad \widetilde{D}'_{ig} = d_{ig}\widetilde{I}'_{22}, \tag{43}$$

where the constants are of the form:

$$a_{ik} = a_{ki} = \alpha_{ik}/(\alpha_{ik}\alpha_{ki} - \beta^2_{ik}) = 4E_u J_i/l_{ik}, \tag{44}$$

$$b_{ik} = b_{ki} = \beta_{ik}/(\alpha_{ik}\alpha_{ki} - \beta^2_{ik}) = 2E_u J_i/l_{ik},\quad d_{ig} = 1/\alpha_{ig} = 3E_u J_i/l_{ik}.$$

The operator $\widetilde{I}'_{22}$ appearing in expressions (43) is introduced as:

$$\widetilde{F}'_{22}\widetilde{I}'_{22} = \widetilde{1}'. \tag{45}$$

Operator $\widetilde{I}'_{22}$ may be expressed in the following manner:

$$\widetilde{I}'_{22} = \frac{\gamma'_1\gamma'_2}{\gamma'_{11}}\widetilde{1}' + \frac{\gamma_1\gamma_2}{\gamma_{11}}\widetilde{R}' + \left(1 - \frac{\gamma'_1\gamma'_2}{\gamma'_{11}} - \frac{\gamma_1\gamma_2}{\gamma_{11}}\right)\widetilde{B}'. \tag{46}$$

Operator $\widetilde{I}'_{22}$ is associated to function R' whose integral represents the concrete relaxation function R^* which for known concrete creep function F^* may be determined. Operator $\widetilde{B}'$ is associated to function B' whose integral represents function B^* which is determined by solving the unhomogenous integral equation:

$$\widetilde{\mathcal{K}}'B^* = 1^*.$$

Function $\mathcal{K}^*$ depends linearly on concrete creep function F^* and on geometric properties of the cross-section, i.e.

$$\mathcal{K}^* = \gamma_{11}1^* + \gamma'_{11}F^*. \tag{48}$$

Coefficients γ_{11}, γ'_{11} and γ_h, γ'_h $(h = 1,2)$ depend on geometric properties of the cross-section [3].

For the adopted concrete creep function F^* we may always determine function I^*_{22} representing the integral of function I'_{22} associated to operator $\widetilde{I}'_{22}$.

The free terms $\mathcal{M}^*_{ik,H}$ and $\mathcal{M}^*_{ki,H}$ in the system of equations (20) depend on external influences.

For the constant cross-section members the solution of the system of equations (19) may be obtained as:

$$\mathcal{M}^*_{ik,H} = \widetilde{I}'_{22}\left[a_{ik}\alpha^*_{ik,H} + b_{ik}\alpha^*_{ki,H}\right], \quad \mathcal{M}^*_{ki,H} = \widetilde{I}'_{22}\left[a_{ki}\alpha^*_{ki,H} - b_{ik}\alpha^*_{ik,H}\right]. \tag{49}$$

For the considered loading cases H ($H = q, c$) by using expressions (45), (40) and (42) we may write:

a) $H = q$

$$\mathcal{M}^*_{ik,q} = -a_{ik}\alpha^*_{ik,q} + b_{ik}\alpha^*_{ki,q} = \mathfrak{M}^*_{ik,q}, \quad \mathcal{M}^*_{ki,q} = a_{ki}\alpha^*_{ki,q} - b_{ik}\alpha^*_{ik,q} = \mathfrak{M}^*_{ki,q}. \tag{50}$$

$\mathfrak{M}^*_{ik,q}$ and $\mathfrak{M}^*_{ki,q}$ are the moments of fully fixed ends due to dead load.

b) $H = c$

$$\begin{aligned} \mathcal{M}^*_{ik,c} &= \widetilde{I}'_{22}(-c_{ik}\psi_{ik,c}1^*) = I^*_{22}\mathfrak{M}^*_{ik,c}, \\ \mathcal{M}^*_{ki,c} &= \widetilde{I}'_{22}(-c_{ki}\psi_{ik,c}1^*) = I^*_{22}\mathfrak{M}^*_{ki,c} \end{aligned} \tag{51}$$

where:

$$c_{ik} = a_{ik} + b_{ik}; \qquad c_{ki} = a_{ki} + b_{ki}. \tag{52}$$

$\mathfrak{M}^*_{ik,c}$ and $\mathfrak{M}^*_{ki,c}$ are the moments of fully fixed ends due to given displacement of the support.

This paper presents the calculation of the composite beam with undisplaceable joints by the approximate slope-deflection method.

When analyzing the composite beam by the accurate slope-deflection method, with the aim to encompass the influence of axial forces and derive all necessary expressions without mathematical negligences, considerable difficulties of mathematical nature occur. They may be understood by solving step by step, starting from the most simple model towards more complex. That was the reason for adopting the present approach solution of the beam with undisplaceable joints by neglecting the influence of axial forces to the deformation.

4. Example. For the composite beam shown in Fig. 3 the support moments will be determined due to given loading cases:

a) constant distributed loading $q = 20\,\mathrm{kN/m}$

b) lowering of middle support $c = 0{,}01\,\mathrm{m}$

The cross sections are inhomogeneous and contain concrete part, denoted by (b) and the steel part, denoted by (n). The respective areas are F_b and F_n. The geometric properties of the cross section will be reduced by the following factors [2]:

$$\nu_k = E_k/E_u, \qquad k = b, n, \qquad E_u = E_n \tag{a}$$

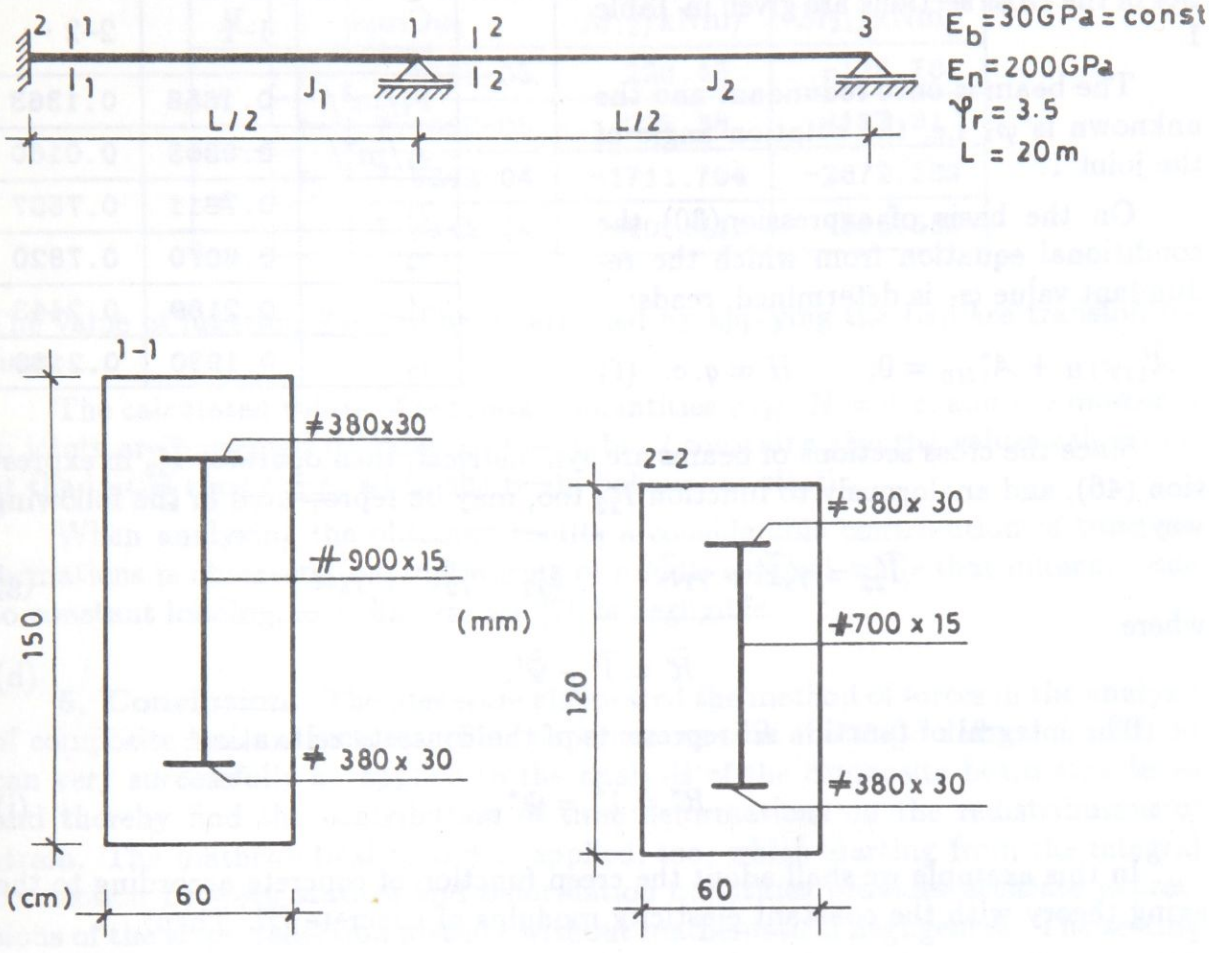

Figure 3

The areas of the idealized cross section are:

$$F_i = \sum_k F_{ki} = \sum_k \nu_k F_{kr}; \qquad k = b, n. \tag{b}$$

The moments of inertia of the idealized areas F_i, with respect to the axis x are equal to the sum of centroidal moments of inertia J_{kr} of the reduced areas F_{kr} with respect to axis x, i.e.:

$$J_i = \sum_k J_{kr}, \qquad k = b, n. \tag{c}$$

The elements γ_{hl} of the symmetric matrix $\|\gamma_{hl}\|_{2,2}$ of the reduced geometric properties of the cross section [2] will have the form:

$$\gamma_{11} = F_{br}/F_i, \quad \gamma_{22} = J_{br}/J_i, \quad \gamma_{12} = \gamma_{21} = S_{br}/S_i = 0; \quad S = \sqrt{F_i J_i'}. \tag{d}$$

The eigen values of the matrix $\|\gamma_{hl}\|$ are denoted by γ_1 and γ_2. The eigen values of symmetric matrix $\|\gamma'_{hl}\|$ of the reduced geometric properties are:

$$\gamma'_1 = 1 - \gamma_1; \qquad \gamma'_2 = 1 - \gamma_2. \tag{e}$$

The values of the geometric properties of the cross sections are given in Table 1.

Table 1

Cross section	1–1	2–2
$F_i/\mathrm{m}^2/$	0.1658	0.1363
$J_i/\mathrm{m}^4/$	0.0303	0.0160
γ_1	0.7811	0.7557
γ_2	0.8070	0.7820
γ_1'	0.2189	0.2443
γ_2'	0.1930	0.2180

The beam is once redundant and the unknown is φ_1 i.e. the rotation angle of the joint 1.

On the basis of expression(30) the conditional equation from which the redundant value φ_1 is determined, reads:

$$\widetilde{\mathcal{A}}'_{11}\varphi_{1H} + \mathcal{A}^*_{1H0} = 0, \qquad H = q, c. \quad \text{(f)}$$

Since the cross sections of beams are symmetrical, then operator $\widetilde{I}'_{22}$ in expression (46), and analogously to function I^*_{22} too, may be represented in the following way:

$$\widetilde{I}'_{22} = \gamma_2'\widetilde{1}' + \gamma_2\widetilde{R}', \qquad I^*_{22} = \gamma_2'1^* + \gamma_2 R^* \tag{g}$$

where

$$\widetilde{R}' = \widetilde{1}' - \widetilde{\Psi}'. \tag{h}$$

The integral of function R' represents of the concrete relaxation:

$$R^* = 1^* = \Psi^*. \tag{i}$$

In this example we shall adopt the creep function of concrete according to the aging theory with the constant elasticity modulus of concrete [2]. Then:

$$\psi^* = 1 - e^{-\varphi_r}, \qquad \psi' = e^{-\varphi_r}. \tag{j}$$

Using the mentioned expressions the integral expression (k) will be solved by the application of the Laplace transformation. Redundant value φ_{1H} may then be written in the form:

$$\varphi_{1H} = K_0 + K_H e^{p_2\varphi_r}, \qquad H = q, c \tag{k}$$

where K_0 and K_H constans obtained by the application of the inverse Laplace transformation.

The moments in joints will be determined on the basis of expression (20) i.e.:

$$M_{ik,H} = \widetilde{A}'_{ik}\varphi_{1H} + \mathcal{M}^*_{ikH,0}, \qquad H = q, c \tag{l}$$

where on the basis of expressions (43), (44), (g) and (h) expression (1) then reduced to following form:

$$M_{ik,H} = a_{ik}\widetilde{1}'\varphi_{1H} - a_{ik}\gamma_2\widetilde{\psi}'\varphi_{1H} + \mathcal{M}^*_{ikH,0}, \qquad H = q, c. \tag{m}$$

We shall introduce the following function:

$$Z_H = \widetilde{\psi}'\varphi_{1H}, \qquad H = q, c. \tag{n}$$

Table 2

H	t	φ_1	M_{12}/kNm/	M_{21}/kNm/
q	t_0	2.468E-05	226.45	-136.78
	t	7.436E-05	225.38	-137.31
c	t_0	7.934E-04	-1711.704	-2672.589
	t	7.614E-04	-401.412	-595.553

The value of function Z_H will be determined by applying the Laplace transformation.

The calculated values of redundant quantities φ_{1H}, $H = q, c$, and the moments in joints are presented in Table 2. The Table 2 constains also the values calculated at the initial time $t = t_0$ when the beam behaves as elastic.

When analyzing the obtained results a considerable contribution of time deformations is observed, due to lowering of middle support while that influence due to constant loading, as is known, see [7], is negligible.

5. Conclusion. The literature elaborated the method of forces in the analysis of composite beam structures. This paper shows that the slope deflection method can very successfully be applied to the analysis of the composite beam structures and thereby find the contribution of time deformations on the redistribution of strain. The mathematical theory is applied, too, which starting from the integral connection between statical and deformation quantities provides accurate expressions of the slope deflection method without mathematical negligences. The setting of such a theory is justigied from the practical point of view because it opens the possibility to evaluate the accuracy of the existing approximative methods, approximation nature as well as the competency of their application in practice.

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ВКЛАД ДЕФОРМАЦИЙ ЗАВИСИМЫХ ОТ ВРЕМЕНИ НА ПЕРЕРАСПРЕДЕЛЕНИЕ НАПРЯЖЕНИЙ (МОМЕНТОВ)

Для сопряженных конструкций произведенны основные уравнения методам деформаций. Влияние нормальных сил на деформацию пренебрегатеся. Рассматривается конструкция с неподвижными узлами, которая составленна из стержней типа "k" и типа "g". Деформационные неопределимые будут только углы вращения φ концов стержней. Рассматриваются два случая внешних влияний, то есть: равномерно распределённая нагрузка и смещение одного конца стержня в направлении перпендикулярно к осы стержня.

Имея в виду вязко-упругие особенности бетона и реляксацию высококачественной стали для предварительного напряжения бетона, деформационные неопределимые величины φ и моменты на концах стержней, рассматриваются как функция времени. Поэтому для представления всех основных соотношений использованны интегральные уравнения, символически линейными интегральными операторами. Таким образом устанавливается аналогия с известными алгебраическими реляциями метода деформаций для конструкций от упругого материала.

Применяя метод деформации в численном примере рассчитанны деформационно неопределённые величины и моменты в опорах времени t и t_0. Сравнивая эти величины замечается значительный вклад временных деформаций на их перераспределение.

DOPRINOS DEFORMACIJA ZAVISNIH OD VREMENA NA PRERASPODELU NAPONA (MOMENATA)

Za spregnute konstrukcije izvedene su osnovne jednačine metode deformacija. Uticaj normalnih sila na deformaciju se zanemaruje. Posmatran je nosač sa nepomerljivim čvorovima sastavljen od štapova tipa "k" i tipa "g". Deformacijske nepoznate su samo uglovi obrtanja φ krajeva štapova. Razmatrana su dva slučaja spoljašnjih uticaja, i to: zadato raspodeljeno opterećenje q i zadato pomeranje jednog kraja štapa upravno na njegovu osu.

Zbog visoko elastičnih osobina betona, deformacijskih neodređene veličine φ i momenti na krajevima štapova posmatrani su kao funkcije vremena. Otuda su sve osnovne relacije prikazane preko integralnih jednačina, simbolički preko linearnih integralnih operatora. Tako je uspostavljena analogija sa poznatim algebarskim izrazima približne metode deformacija za nosače sa nepomerljivim čvorovima od elastičnog materijala. Svi potrebni izrazi izvedeni su bez matematičkih zanemarenja.

Primenjujući metodu deformacije u brojnom primeru sračunate su deformacijski neodređene veličine i momenti nad osloncima u trenutku t i t_0. Upoređujući te veličine, uočava se znatan doprinos vremenskih deformacija na njihovu preraspodelu.

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