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TIME DEPENDENT THERMO-HYDRAULICS BY BOUNDARY ELEMENTS

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1. Governing equations

The time dependent laminar motion of an isochoric viscous fluid and its energy transport are governed by the Navier-Stokes momentum equation and by mass and energy conservation laws

$$p(\partial \mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v}) = \eta\,\Delta \mathbf{v} - \nabla p + \rho \mathbf{g} \tag{1}$$

$$\nabla \mathbf{v} = 0 \tag{2}$$

$$\partial T/\partial t + \nabla(\mathbf{v}\,T) = a\,\Delta T \tag{3}$$

formulated in terms of velocity $\mathbf{v}$, pressure p and temperature T, subject to corresponding boundary and initial conditions, given as

$$\mathbf{v} = \bar{\mathbf{v}}(\text{o n }\Gamma\text{ for }t > t_0),\quad \mathbf{v} = \bar{\mathbf{v}}_0(\text{i n }\Omega\text{ a t }t = t_0) \tag{4}$$

$$T = \bar{T}(\text{o n }\Gamma_1,\ t > t_0),\ -k_0\,\partial t/\partial t\,n = \bar{q}(\text{o n }\Gamma_2, t > t_0),$$

$$-k_0\,\partial T/\partial n = h\cdot(T - T_s)(\text{o n }\Gamma_3, t > t_0),\ T = \bar{T}_0(\text{i n }\Omega,\ t = t_0) \tag{5}$$

The buoyancy body force may be approximated with Boussinesq formula

$$\rho = \rho_0\cdot(1 - \beta\cdot(T - T_0)) \tag{6}$$

where ρ_0 is some reference density at temperature T_0. With eq. (6) the momentum eq. (1) may be written as

$$\rho_0(\partial \mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v}) = \eta\,\Delta \mathbf{v} - \nabla P - \rho_0\,\beta\,(T - T_0)\,\mathbf{g} \tag{7}$$

where P is the modified pressur

$$P = p - \rho_0\,\mathbf{g}\,\mathbf{r} \tag{8}$$

and $r = (x, y, z)$ is the position vector.

2. Vorticity formulation

It is convenient to introduce the vorticity as a variable, given by

$$\mathbf{w} = \nabla \times \mathbf{v} \tag{9}$$

in order to divide the computation of the flow motion into its kinematic and kinetic parts. With the following identity for the convective term

$$(\mathbf{v}\,\nabla)\,\mathbf{v} = \nabla\, v^2/2 + \mathbf{w} \times \mathbf{v} \tag{10}$$

the momentum equation (7) can be given in the form

$$\partial\, \mathbf{v}/\partial\, t + \mathbf{w} \times \mathbf{v} = \nu\, \Delta\, \mathbf{v} - \nabla\, h - \beta\,(T - T_0)\,\mathbf{g} \tag{11}$$

where $h = P/\rho_0 + v^2/2$ is the total pressure.

It is easy to derive the formulation without pressure by taking the curl on both sides of the eq. (11)

$$\partial\, \mathbf{w}/\partial\, t + (\mathbf{v}\,\nabla)\,\mathbf{w} - (\mathbf{w}\,\nabla)\,\mathbf{v} = \nu\,\Delta\,\mathbf{w} - \nabla \times \beta\,(T - T_0)\,\mathbf{g} \tag{12}$$

For plane motion the vorticity vector has just one component, orthogonal to the plane of flow, and eq. (12) reduces to scalar form

$$\partial\, w/\partial\, t + \nabla\,(\mathbf{v}\, w) = \nu\,\Delta\, w - \beta\,(g_y, -g_x)\,\nabla\, T \tag{13}$$

3. Vector potential

The kinematics of the flow is described by eqs. (2) and (9). The vector potenial of the solenoidal velocity field can be introduced

$$\mathbf{v} = \nabla \times \psi, \quad \nabla\, \psi = 0 \tag{14}$$

and the kinematic part of the computation is presented with elliptic Poisson's equation for vector ψ

$$\Delta\, \psi = -\,\mathbf{w} \tag{15}$$

which for plane motion reduces to scalar elliptic equation for stream function ψ

$$\Delta\, \psi = -\,w \tag{16}$$

where the velocity vector is defined by

$$\mathbf{v} = (v_x, v_y) = (\partial\, \psi/\partial\, y, -\,\partial\, \psi/\partial\, x) \tag{17}$$

The unit normal and the tangent to the boundary are given in plane case as

$$\mathbf{n}\,(S) = (n_x, n_y), \mathbf{t}\,(S) = (t_x, t_y) = (-\,n_y, n_x) \tag{18}$$

while the normal and the tangential velocity components to the boundary are

$$v_n\,(S) = \mathbf{v}\,(S)\,\mathbf{n}\,(S), \; v_t\,(S) = \mathbf{v}\,(S)\,\mathbf{t}\,(S) \tag{19}$$

and the x and y velocity components are

$$v_x(S)=v_n(S)\,n_x(S)-v_t(S)\,n_y(S),\, v_y(S)=v_n(S)\,n_y(S)+v_t(S)\,n_x(S) \quad (20)$$

The global mass balance equation is

$$\int \underline{\nabla}\, \mathbf{v}(S)\,d\Omega=\int \mathbf{v}(S)\,\mathbf{n}(S)\,d\Gamma=0 \quad (21)$$

4. Boundary integral equations for the kinetics

The vorticity transport equation describes the development of the vorticity field with time. It is parabolic equation of an initial boundary value problem. In addition to the initial vorticity distribution, boundary vorticity values at subsequent time levels have to be known. The boundary vorticity values are determined through the kinematic relation between the velocity and vorticity fields.

Let us consider inhomogeneous parabolic equation for the scalar w function with respect to space and time

$$\nu\,\Delta\,\bar{w}(s,t)-\partial\,w(s,t)/\partial\,t+b(s,t)=0 \qquad \text{in } \Omega \quad (22)$$

for a finite time interval $I(t_0,\ t_j)$ with corresponding boundary conditions of the first and second kind

$$w(S,t)=\bar{w}(S,t)\ \text{ on } \Gamma_1,\quad \partial\,w(S,t)/\partial\,n(S)=\bar{q}(S,t)\ \text{ on } \Gamma_2 \quad (23)$$

The initial conditions needed for the solution of eq. (22) are

$$w(s,t)=w_0(s,t_0) \qquad \text{in } \Omega \quad (24)$$

while $b(s)$ represents body forces. Using Green's theorem for scalars or by weighted residual statement the following boundary integral equation can be written

$$c(\xi)\,w(\xi,t_f)+\nu\int\int w(S,t)\,q^*(\xi,t_f;S,t)\,d\Gamma\,dt=\nu\int\int q(S,t)\ u^*\ (\xi,\ t_f;\ S,t)\,d\Gamma\,dt+$$

$$+\int\int b(s,t)\,u^*(\xi,t_f;s,t)\,d\Omega\,d\ t+\int w_{f-1}(s,t_{f-1})u^*(\xi,t_f;s,t_{f-1})\,d\Omega \quad (25)$$

where u^* $(\xi, t_f;\ S, t)$ is parabolic fundamental solution and q^* its normal derivative, with (ξ, t_j) being source space-time point, and (S, t) or (s, t) are field space-time points on the boundary or in the domain respectively. The fundamental solution and its derivative are

$$u^*(\xi,t_f;S,t)=1/4\,\pi\,\nu\,\tau.\,\exp\,(-r^2(\xi,S)/4\,\nu\,\tau),$$

$$q^*(\xi,t_f;S,t)=d(\xi,S)/8\,\pi\,\nu^2\,\tau^2.\exp\,(-r^2(\xi,S)/4\,\nu\,\tau) \quad (26)$$

where $\tau=t_f-t$ and $d(\xi,S)=(x_i(\xi)-x_i(S))\,n_i(S)$.

The coefficient $c(\xi)$ has values 0.0 (if ξ is outside of the domain), 1.0 (when ξ is in the domain), and 0.5 (if ξ lies on a smooth boundary) or $\beta/2\pi$ (when ξ is on a non-smooth boundary, with β being the internal angle).

Taking the body force term $b(s, t)$ in eq. (22) equal to convective and buoyancy terms in eq. (13), one may extend the integral formulation to vorticity parabolic transport equation

$$c(\xi) w(\xi, t_f) + \nu \int\int w(S, t) q^*(\xi, t_f; S, t) \, d\Gamma \, dt = \nu \int\int q(S, t) u^*(\xi, t_f; S, t) \, d\Gamma \, dt -$$
$$- \int\int (\nabla (\mathbf{v}(s, t) w(s, t)) + \beta (g_y, -g_x) \nabla T(s, t)) u^*(\xi, t_f; s, t) \, d\Omega \, dt +$$
$$\int w_{f-1}(s, t_{f-1}) u^*(\xi, t_f; s, t_{f-1}) \, d\Omega \tag{27}$$

The domain integrals comprise the derivatives of the vorticity and temperature, which can be eliminated in various ways. One of them is to apply Gaussian divergence theorem on domain integrals, resulting in the following boundary integral equation

$$c)\xi) w(\xi, t_f) + \nu \int\int w(S, t) q^*(\xi, t_f; S, t) \, d\Gamma \, dt = \nu \int\int q(S, t) u^*(\xi, t_f; S, t) \, d\Gamma \, dt -$$
$$- \int\int (w(S, t) v_n(S, t) + \beta g_t T(S, t)) u^*(\xi, t_f; S, t) \, d\Gamma \, dt +$$
$$+ \int\int (w(s, t) \mathbf{v}(s, t) + \beta (g_y, -g_x) T(s, t)) \nabla u^*(\xi, t_f; s, t) \, d\Omega \, dt +$$
$$+ \int w_{f-1}(s, t_{f-1}) u^*(\xi, t_f; s, t_{f-1}) \, d\Omega \tag{28}$$

where $g_t = \mathbf{g} \, \mathbf{t}$ is tangential gravity acceleration component to the boundary.

5. Boundary integral equation for the energy transport

The energy transport equation (3) is parabolic diffusion-convection equation for temperature. Owing to the formal similarity between eqs. (3) and (13) one can formulate the following boundary integral equation directly in its final form

$$c(\xi) T(\xi, t_f) + a \int\int T(S, t) q^*(\xi, t_f; S, t) \, d\Gamma \, dt = a \int\int q(S, t) u^*(\xi, t_f; S, t) \, d\Gamma \, dt -$$
$$- \int\int T(S, t) v_n(S, t) u^*(\xi, t_f; S, t) \, d\Gamma \, dt + \int\int T(s, t) \mathbf{v}(s, t) \nabla u^*(\xi, t_f; s, t) \, d\Omega \, dt +$$
$$+ \int T_{f-1}(s, t_{f-1}) u^*(\xi, t_f; s, t_{f-1}) \, d\Omega \tag{29}$$

6. Boundary integral equations for the kinematics

The kinematic relationship between the velocity and vorticity fields is given by inhomogeneous elliptic equation for scalar function $\psi(s)$

$$\Delta \psi(s) + b(s) = 0 \tag{30}$$

with Dirichlet's and Neumann's boundary conditions

$$\psi(S) = \overline{\psi}(S) \text{ on } \Gamma_1, \quad \partial \psi(S)/\partial n(S) = \overline{q}(S) = -\overline{v}_t(S) \text{ on } \Gamma_2 \tag{31}$$

The body forces term $b(s)$ vanishes for the potential flow, while for the rotational flow represents the vorticity field. Using Green's theorem for scalars one can write the following boundary integral equation

$$c(\xi)\,\psi(\xi) + \int \psi(S)\,q^*(\xi, S)\,d\Gamma = \int q(S)\,u^*(\xi, S)\,d\Gamma + \int w(s)\,u^*(\xi, s)\,d\Omega \qquad (32)$$

where $u^*(\xi, S)$ is elliptic fundamental solution and q^* its normal derivative, while ξ is the source point and S or s field points on the boundary and in the domain respectively. For plane flow case u^* and q^* are given by

$$u^*(\xi, S) = 1/2\,\pi \cdot \ln(r_0/r(\xi, S)),\quad q^*(\xi, S) = 1/2\,\pi \cdot d(\xi, S)/r^2(\xi, S) \qquad (33)$$

and the tangential derivative is $q_t^*(\xi, S) = 1/2\pi \cdot d_t(\xi, S)/r^2(\xi, S)$.

Velocity components in the domain can be determined explicitly from eq. (32) by derivation, ie.

$$\partial\psi(\xi)/\partial x_i(\xi) = -\int \psi(S)\,\partial q^*(\xi, S)/\partial x_i(\xi)\,d\Gamma + \int q(S)\,\partial u^*(\xi, S)/\partial x_i(\xi)\,d\Gamma +$$

$$+ \int w(s)\,\partial u^*(\xi, s)/\partial x_i(\xi)\,d\Omega \qquad (34)$$

An alternative kinematic integral formulation can be derived using Green's theorem for vectors resulting in the following equation

$$c(\xi)\,\mathbf{v}(\xi) + \int (\nabla u^*(\xi, S)\,\mathbf{n}(S))\,\mathbf{v}(S)\,d\Gamma = \int (\nabla u^*(\xi, S) \times \mathbf{n}(S)) \times \mathbf{v}(S)\,d\Gamma +$$

$$+ \int \mathbf{w}(s) \times \nabla u^*(\xi, s)\,d\Omega \qquad (35)$$

which for plane problems decomposes into two scalar equations

$$c(\xi)\,v_x(\xi) + \int v_x(S)\,q^*(\xi, S)\,d\Gamma = \int v_y(S)\,q_t^*(\xi, S)\,d\Gamma - \int w(s)\,q_y^*(\xi, s)\,d\Omega$$

$$c(\xi)\,v_y(\xi) + \int v_y(S)\,q^*(\xi, S)\,d\Gamma = \int w(s)\,q_x^*(\xi, s)\,d\Omega - \int v_x(S)\,q_t^*(\xi, S)\,d\Gamma \qquad (36)$$

7. Boundary element approximation

For the solution of given integral equations by the numerical means of boundary elements, variables may be considered constant within individual time steps. Determining

$$U^* = \nu \int u^*(\xi, t_f; S, t)\,dt = 1/4\,\pi.\,E_1(x_{f-1})$$

$$Q^* = \nu \int q^*(\xi, t_f; S, t)\,dt = d(\xi, S)/2\,\pi\,r^2(\xi, S).\,\exp(-x_{f-1})$$

$$Q_i^* = \nu \int q_i^*(\xi, t_f; s, t)\,dt = (x_i(\xi) - x_i(s))/2\,\pi\,r^2(\xi, s).\,\exp(-x_{f-1}) \qquad (37)$$

where E_1 is exponential integral function, defined by

$$E_1(x) = \int_x^\infty e^{-t}/t\,dt \qquad (38)$$

and the argument x

$$x_{f-1}=r^2(\xi, s)/4\nu\tau \tag{39}$$

yield space integrals

$$h_e^n=\int \Phi^n U^* d\Gamma,\ g_e^n=\int \Phi^n Q^* d\Gamma,\ d_{ci}^n=\int \varphi^n \overset{*}{Q_i} d\Omega,\ b_c^n=\nu\int \varphi^n \overset{*}{u}_{f-1} d\Omega \tag{40}$$

Thus the discretized form of eq. (28) is obtained

$$c(\xi, t_f)+\sum \mathbf{h}^T \mathbf{W}_f^n=\sum \mathbf{g}^T \mathbf{Q}_f^n+1/\nu\cdot(-\sum \mathbf{g}^T \mathbf{WV}_{nf}^n-\beta\sum \mathbf{g}^T \mathbf{Tg}_{tf}^n+$$

$$+\sum \mathbf{d}_x^T(\mathbf{WV}_{xf}^n+\beta g_y \mathbf{T}_f^n)+\sum \mathbf{d}_y^T(\mathbf{WV}_{yf}^n-\beta g_x \mathbf{T}_f^n)+\sum \mathbf{b}^T \mathbf{W}_{f-1}^n) \tag{41}$$

which may be written for boundary nodes in the following abbreviated form

$$\mathbf{HW}_f=\mathbf{GQ}_f+1/\nu\cdot(-\mathbf{GWV}_{nf}-\beta\,\mathbf{GT}\,\mathbf{g}_{tf}+\mathbf{D}_x(\mathbf{WV}_{xf}+\beta g_y \mathbf{T}_f)+$$

$$+\mathbf{D}_y(\mathbf{WV}_{yf}-\beta g_x \mathbf{T}_f)+\mathbf{BW}_{f-1}) \tag{42}$$

The application of a collocation method to the eq. (29) leads to the following discretized formulation

$$c(\xi)\mathbf{T}(\xi, t_f)+\sum \mathbf{h}_t^T \mathbf{T}_f^n=\sum \mathbf{g}^T \mathbf{Q}_f^n+1/a\cdot(-\sum \mathbf{g}^T \mathbf{TV}_{nf}^n+\sum \mathbf{d}^T \mathbf{TV}_{xf}^n+$$

$$+\sum \mathbf{d}^T \mathbf{TV}_{yf}^n+\sum \mathbf{b}^T \mathbf{T}_{f-1}^n) \tag{43}$$

which reads for boundary nodes as follows in the abbreviated form

$$\mathbf{HT}_f=\mathbf{GQ}_f+1/a\cdot(-\mathbf{GTV}_{nf}+\mathbf{D}_x \mathbf{TV}_{xf}+\mathbf{D}_y \mathbf{TV}_{yf}+\mathbf{BT}_{f-1}) \tag{44}$$

After rearranging columns and rows due to the application of known boundary conditions, the above system can be written as

$$\mathbf{AX}=\mathbf{F}_0+\mathbf{F}_N(\mathbf{T}, \mathbf{v}) \tag{45}$$

While discretizing the kinematic eq. (36) space integrals are required

$$h_e^n=\int \Phi^n q^* d\Gamma,\ h_{et}^n=\int \Phi^n \overset{*}{q_t} \varphi d\Gamma,\ d_{ci}^n=\int \varphi^n \overset{*}{q_i} d\Omega \tag{46}$$

yielding the two equations for the plane case

$$c(\xi)v_x(\xi)+\sum \mathbf{h}^T \mathbf{V}_x^n=\sum \mathbf{h}_t^T \mathbf{V}_y^n-\sum d_y^T \mathbf{W}^n$$

$$c(\xi)v_y(\xi)+\sum \mathbf{h}^T \mathbf{V}_y^n=\sum \mathbf{d}_x^T \mathbf{W}^n-\sum \mathbf{h}_t^T \mathbf{V}_x^n \tag{47}$$

which become for boundary nodes in the abbreviated notation

$$\mathbf{c}(\xi)\mathbf{V}_x(\xi)+\mathbf{HV}_x=\mathbf{H}_t \mathbf{V}_y-\mathbf{D}_y \mathbf{W}$$

$$\mathbf{c}(\xi)\mathbf{V}_y(\xi)+\mathbf{HV}_y=\mathbf{D}_x \mathbf{W}-\mathbf{H}_t \mathbf{V}_x \tag{48}$$

or

$$\mathbf{HV}_x = \mathbf{H}_t \mathbf{V}_y - \mathbf{D}_y \mathbf{W}$$

$$\mathbf{HV}_y = \mathbf{D}_x \mathbf{W} - \mathbf{H}_t \mathbf{V}_x \qquad (49)$$

Velocities in the domain can now be obtained by setting $c(\xi)=1$, ie.

$$\mathbf{V}_x(\xi) = -\mathbf{HV}_x + \mathbf{H}_t \mathbf{V}_y - \mathbf{D}_y \mathbf{W}$$

$$\mathbf{V}_y(\xi) = -\mathbf{HV}_y - \mathbf{H}_t \mathbf{V}_x + \mathbf{D}_x \mathbf{W} \qquad (50)$$

8. Solution procedure

The above coupled nonlinear equations can be solved in a step-by-step procedure, requiring the following parts within each time increment, starting with some initial guess for vorticity and temperature.

Kinematics: — solution for boundary vorticity values,
— explicit calculation of domain velocities,

Energy: — solution for unknown boundary temperatures and heat fluxes,
— explicit calculation of internal temperatures,

Kinetics: — solution for unknown boundary vorticity flux values,
— explicit calculation of internal vorticity values.

The reversed sweep is used in computation of internal temperature and vorticity values. With this the convergence of the solution is accelerated and greater stability of the iterative scheme achieved.

9. Case studies

In addition to our previously published results for steady state (ref. [13] to [19]) a nonsteady case has been run for a cavity, in which there is a thermally driven buoyancy flow due to the temperature difference between left and right vertical wall. Results are presented in Fig's 1 to 3 for various values of Rayleigh number ($Ra=Gr \cdot Pr$) at several time steps during the transient phenomenon.

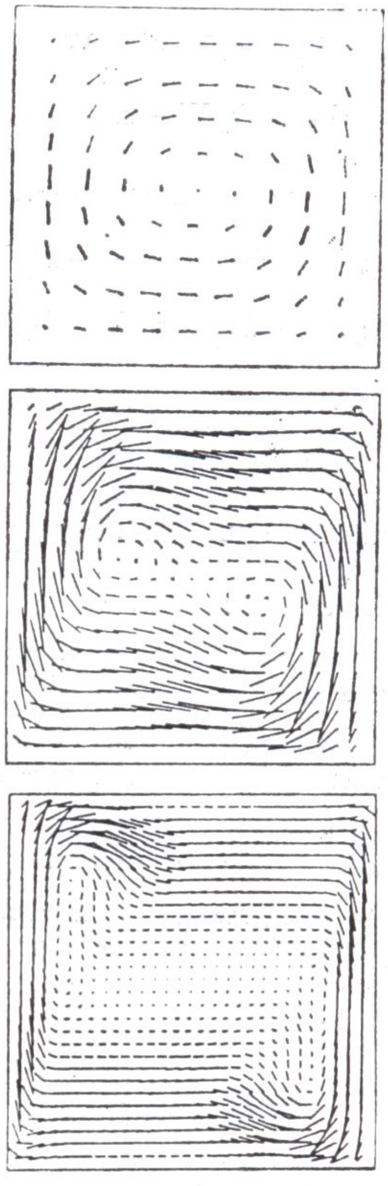

Fig. 1 Thermally driven cavity flow, steady state.
Velocity fields for (a) $Ra=10^4$, (b) $Ra=10^5$, (c) $Ra=10^6$

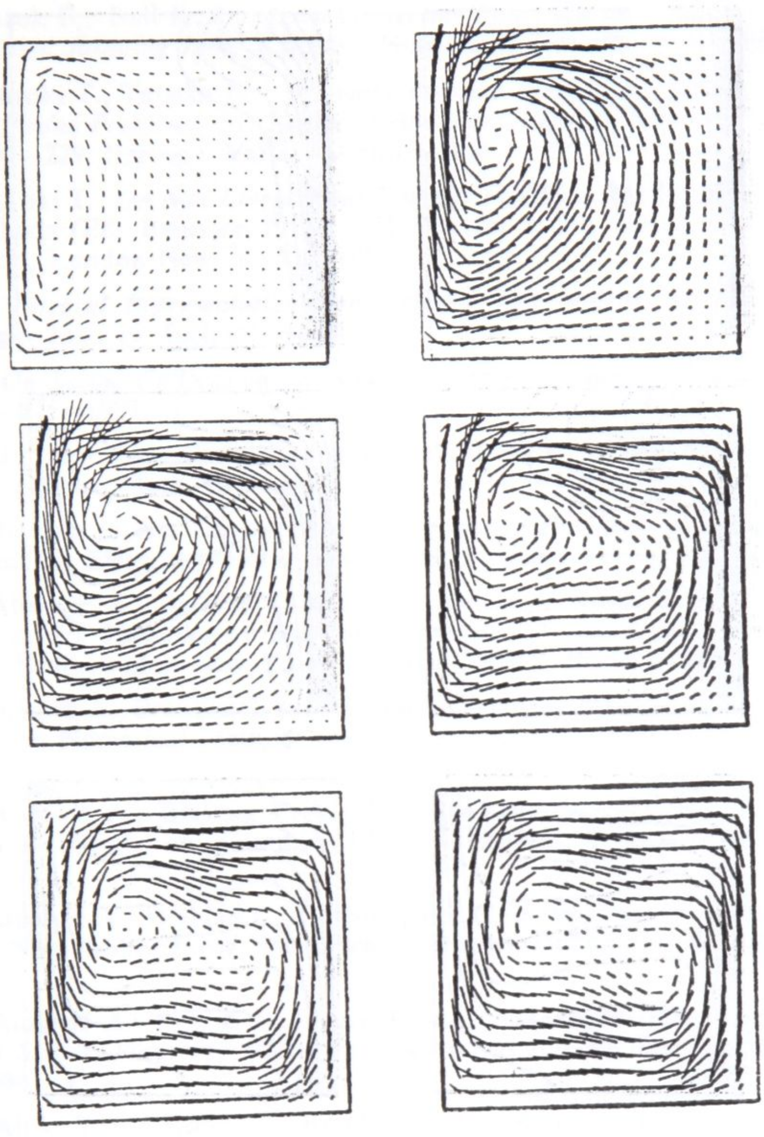

Fig. 2 Thermally driven cavity flow, transient states.
Velocity fields for $Ra=10^5$ at $t=0.2, 0.8, 1.2, 3.2, 6.4, 40.$ secs.

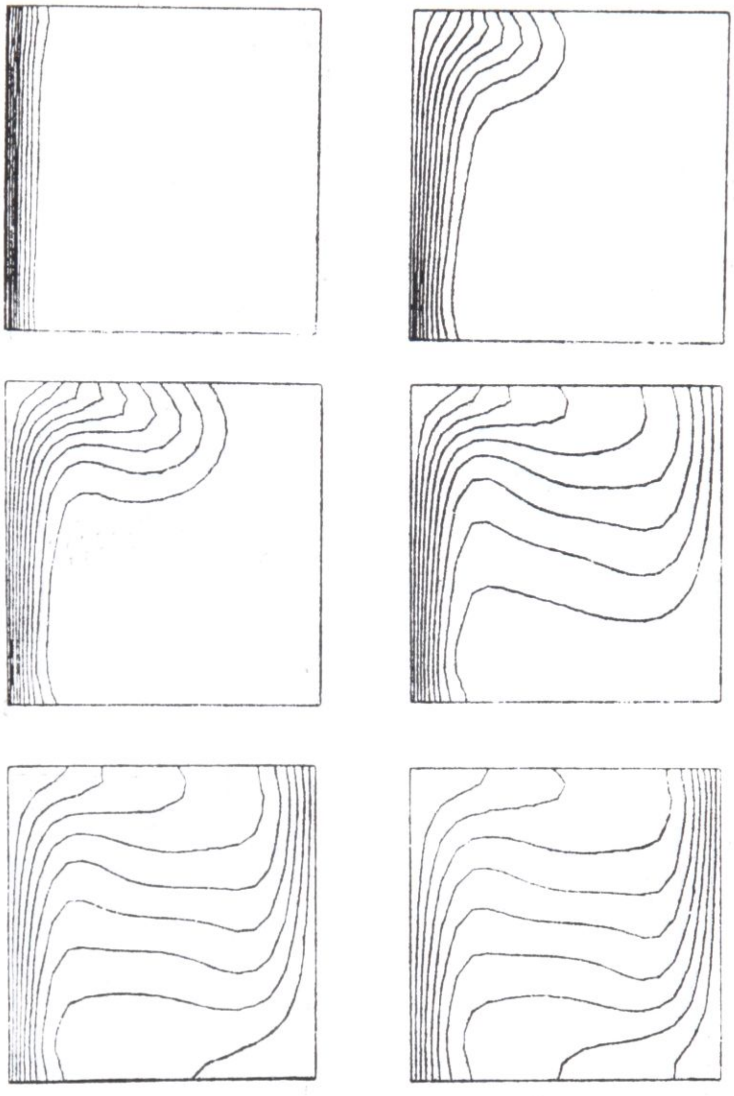

Fig. 3 Thermally driven cavity flow, transient states.
Isothermal lines $Ra=10^5$ at $t=0.2$, 0.8, 1.2, 3.2, 6.4, 40. secs

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ČASOVNO ODVISNA TERMO-HIDRAVLIKA Z ROBNIMI ELEMENTI

V prispevku obravnavamo nestacionarno obnašanje laminarne viskozne izohorne tekočine z numeričnim postopkom robnih elementov. Problem obsega toplotno, kinetsko in kinematsko obravnavo toka. Zaradi konvektivnih členov je poleg robne diskretizacije potrebna tudi notranja integracija v območju, kar nekoliko zmanjšuje odlike postopka.

Kot konkreten zgled smo ovrednotili vzgonski tok v kotanji pravokotne oblike zaradi temperaturne razlike med levo in desno navpično steno. Rezultati so podani za različne vrednosti Rayleigh števila ($Ra=Gr \cdot Pr$) med odvijanjem prehodnega pojava.

TIME DEPENDENT THERMO-HYDRAULICS BY BOUNDARY ELEMENTS

The aim of this paper is to deal with nonsteady behaviour of laminar viscous isochoric fluid by a numerical boundary element method. The problem is composed of thermal, kinetic and kinematic descriptions. Due to the presence of convective therms there is also a need for internal integration cells in the domain in addition to boundary discretization, what in a way reduces the advantages of the procedure.

As a case study example buoyancy flow has been dealt with in a square cavity, caused by a temperature difference between the left and right vertical walls. Results are presented for variable Rayleigh number values ($Ra=Gr \cdot Pr$) during the transient phenomenon.

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