TEORIJSKA I PRIMENJENA MEHANIKA pp. 139—142., 1988, UDK 539.

EFFECTS OF HALL CURRENT ON FREE CONVENCTION AND MASS TRANSFER FLOW THROUGH A POROUS MEDIUM

P. C. Ram

(Received 06. 04. 1987.)

1. Introduction

The study of magnetohydrodynamic viscous flows with Hall currents has important engineering applications in problems of magneto-hydrodynamic generators and of Hall accelerators as well as in flight magnetohydrodynamics. Raptis [2] has studied the unsteady free convection and mass transfer flow through porous medium bounded by an infinite vertical plate, by using the model of Yamamoto and Iwamura [5] for the generalized Darcy's law. Recent studies [1, 4, 3, 7] on the hydromagnetic flows with Hall currents are mainly focused upon those where the magnetic field is imposed normal to the plate.

Hence, the purpose of the present investigation is to study the effects of Hall current on free convection and mass transfer through a porous medium bounded by an infinite vertical plate when a strong magnetic field acts in a plane which makes an angle α with the plane transverse along to the plate and the plate temperature fluctuates in time about a non-zero constant mean.

2. Equations of motion

Consider the x-axis along the vertical limiting surface in the upward dirćction and y-axis normal to it. A uniform magnetic filed $\vec{H}=(0, H_0 \lambda, H_0 \sqrt{1-\lambda^2})$, where $\lambda=\cos \alpha$ is imposed on the fluid. Since the plate is infinite in extent, all physical quantitites, except pressure, are functions of W and t only. The equation of continuity $\Delta \cdot \vec{q}=0$ gives $v=-v_0$ $(v_0>0)$, where $\vec{q}=(u, v, w,)$. When the strength of magnetic field is very large, the generalized ohm's law in the absence of the electric field [6] is of the form:

$$\vec{J}+\frac{w_e T_e}{H_0}\vec{J}\, x\, \vec{H}=\sigma\left(\mu\, e\, \vec{q}\, x\, \vec{H}+\frac{1}{\mathrm{ene}}\Delta .\, \mathrm{p\, e}\right), \tag{1}$$

where σ, μe, we, Te, e, ne, and pe are the electric conductivity, the permeability, the cyclotron frequency, the electron collision time, the electric charge, the number density of electron, and the electron pressure respectively. Negleting the electron pressure, thermoelectric pressure and ion slip, we have from equation (1).

$$j_{x1} = \frac{\sigma H_0 \mu e \lambda}{1 + m^2 \lambda^2} [m \lambda u^1 - w^1],$$

$$j_{z1} = \frac{\sigma H_0 \mu e \lambda}{1 + m^2 \lambda^2} [u^1 + m \lambda w^1],$$

where $m = w_e T_e$ is the Hall parameter.

We consider further the case of short circuit problem in which the applied electric field $\vec{E} = 0$. Under these assumptions, the non-dimensional form of the equations of motion and energy reduce to:

$$\left[\frac{\partial}{\partial t} - \frac{\partial}{\partial y} - \frac{\partial^2}{\partial y^2}\right] q + m_1 q = G\theta + G^* \theta^*, \tag{2}$$

$$\frac{P \partial \theta}{\partial t} - \frac{P \partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2}, \tag{3}$$

$$\frac{Sc \, \partial \theta^*}{\partial t} - \frac{Sc \, \partial \theta^*}{\partial Y} = \frac{\partial^2 \theta^*}{\partial Y^2}, \tag{4}$$

where $y = \frac{y_1 v^0}{\nu}$, $t = \frac{t^1 v_0^2}{\nu}$, $\omega = \frac{\nu \omega^1}{v_0^2}$, $u = \frac{u^1}{v_0}$, $w = \frac{w^1}{v_0}$,

$$w = \frac{w^1}{v_0}, \quad \theta = \frac{T^1 - T^1_\infty}{T^1_w - T^1_\infty}, \quad \theta^* = \frac{c^1 - c^1_\infty}{c^1_w - c^1_\infty},$$

$$P = \frac{\rho \nu c_p}{K}, \quad Sc = \frac{\nu}{D}, \quad G = \frac{\nu g \beta (T^1_w - T^1_\infty)}{v_0^3},$$

$$G^* = \frac{\nu g \beta (c^1_w - c^1_\infty)}{v_0^3}, \quad M^2 = \frac{\sigma \mu e^2 H_0^2 \nu}{\rho v_0^2}, \quad K = \frac{v_0^2 K^1}{\nu^2},$$

and

$$q = u + i w$$

The corresponding boundary conditions reduce to

$$\begin{aligned} &u = 0, \ w = 0, \ \theta = 1 + \varepsilon e^{i\omega t} \quad at \ y = 0 \\ &\theta^* = 1 + \varepsilon e^{i\omega t} \\ &u = 0, \ w = 0, \ \theta \to 0, \ \theta^* \to 0 \quad as \ y \to \infty \end{aligned} \tag{5}$$

To solve equations (2)—(4) under the boundary conditions (5), we now represent the velocity, temperature and concentration in the neighbourhood of the plate as

$$q = q_0 + \varepsilon\, q_1\, e^{i\omega t} + 0\,(\varepsilon^2),$$
$$\theta = \theta_0 + \varepsilon\, \theta_1\, e^{i\omega t} + 0\,(\varepsilon^2), \tag{6}$$
$$\theta^* = \theta_0{}^* + \varepsilon\, \theta_1{}^*\, e^{i\omega t} + 0\,(\varepsilon^2),$$

where $\varepsilon \ll 1$

substituting (6) in equations (2)—(4) and equating harmonic and non-harmonic terms, we have

$$q_0'' + q_0' - M_1\, q_0 + GQ_0 + G^*\, \theta_0{}^* = 0 \tag{7}$$

$$q_1'' + q_1' - (M_1 + i\,\omega)\, q_1 + G\,\theta_1 + G^*\,\theta_1^* = 0 \tag{8}$$

$$\theta_0'' + P\,\theta_0' = 0, \tag{9}$$

$$\theta_0'' + Sc\,\theta_0^{*\prime} = 0, \tag{10}$$

$$\theta_1'' + P\,\theta_1' - i\,\omega\, P\,\theta_1 = 0, \tag{11}$$

$$\theta_1^{*\prime\prime} + Sc\,\theta_1^{*} - i\,\omega\, Sc\,\theta_*^1 = 0, \tag{12}$$

where the primes denote differentiation with respect to y, and

$$M_1 = \left[\frac{M^2\lambda^2(1 - i\,m\,\lambda)}{1 + \lambda^2 m^2} + \frac{1}{K}\right]$$

The appropriate boundary conditions are

$$q_0 = q_1 = 0,\quad \theta_0 = \theta_0^* = 1,\quad \theta_1 = \theta_1^* = 1 \quad at\ \lambda = 0$$

$$q_0 = q_1 = 0,\quad \theta_0 = \theta_0^* = 0,\quad \theta_1 = Q_1^* = 0 \quad as\ \lambda \to \infty \tag{13}$$

The solutions of equations (7)—12) subject to the boundary conditions (13) are the following:

$$q_0 = \frac{G(\bar{e}^{hy} - \bar{e}^{py})}{P^2 - P - M_1} + \frac{G^*(\bar{e}^{hy} - \bar{e}^{scy})}{Sc^2 - Sc - M_1}, \qquad \theta_0 = \bar{e}^{py}, \qquad \theta_0^* = \bar{e}^{Sc}\,y,$$

$$q_1 = \frac{G(\bar{e}^{h_2 y} - \bar{e}^{h_2 y})}{h_2^1 - h_1 - (M_1 + i\,w)} + \frac{G^*(\bar{e}^{h_2 y} - (\bar{e}^{h_1{}^* y})}{h_1^{*2} - h_1^* - (M_1 + i\,w)} \quad \theta_1 = \bar{e}^{h_1 y}, \quad \theta_1^* = \bar{e}^{h_1{}^* y},$$

where

$$h = \frac{1}{2}\,[1 + \surd(1 + 4\,M_1)], \qquad h_1 = \frac{1}{2}\,[P + \surd(P^2 + 4\,i\,\omega\,P)]$$

$$h^* = \frac{1}{2}\,[Sc + \surd(Sc^2 + 4\,i\,\omega\,Sc)[, \qquad h_2 = \frac{1}{2}\,[1 + \surd\{1 + 4\,(M_1 + i\,\omega)\}]$$

3. Results

The dimensionless velocity profiles (the transient primary velocity u and the transient secondary velocity w) are plotted in figures 1 and 2 ($P=0.71$, $Sc=0.7$, $G=5.0$, $G^*=5.0$, $\omega=5.0$, $\varepsilon=0.01$, $\omega t=\pi/2$, $M^2=10$) for different va.ues of α and m. It is observed that for constant values of M and m, both u and ω increases with the increase of α where as for fixed value of α primary velocity u increases and secondary velocity w decreases with the increase of Hall parameter m. For $\lambda=1$, $M=0$ and $m=0$, the velocity distributions coincide with results of Raptis [2].

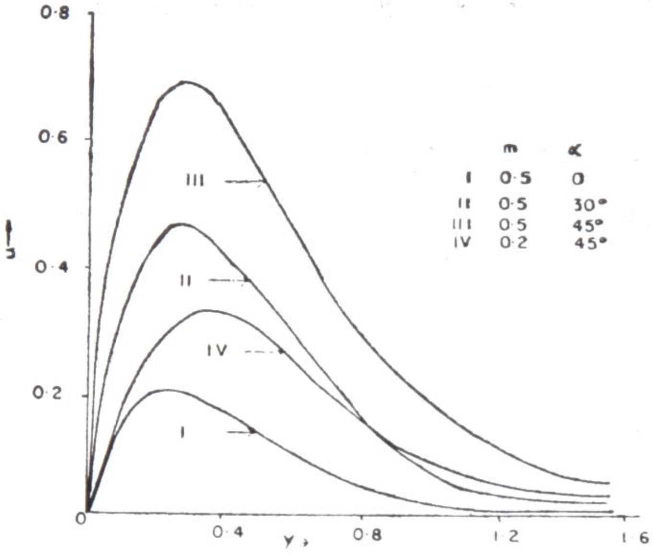

Fig Profiles of non-dimensional primary velocity u

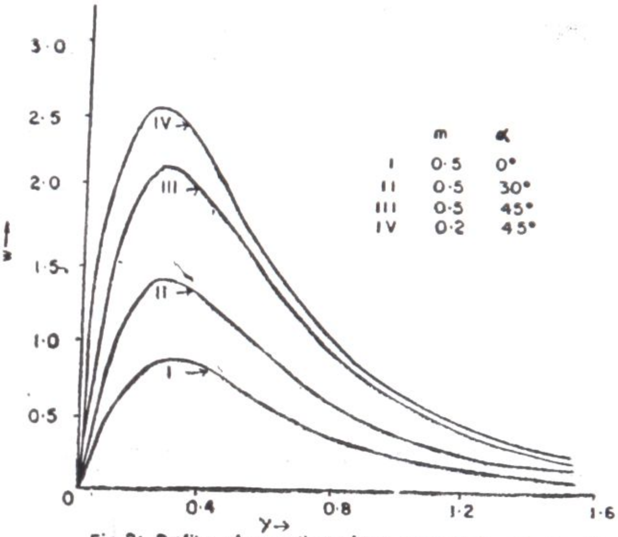

Fig 2: Profiles of non-dimensional secondary velocity w

REFERENCES

[1] A. B. Vatazhin, *Magnetohydrodynamic boundary layer growth of a compressible fluid in an impulsive motion (in Russian)*. J. Appl. Mech. and Techn. Phys. No. 2, 3 (1965).

[2] A. Raptis, *Unsteady mass transfer and free convection through porous medium*. Teorijska I Primenjena MEHANIKA. No. 9, 105 (1983).

[3] I. Pop, *The effect of Hall currents on hydromagnetic Flow near an accelated plate*. J. Math and Phys. Sci. 5, 375 (1971).

[4] I. Pop, *Der Einfluss von Hall-Stromen auf die Rayleigh-Grenzschiecht*. Z. angew. Math. Mech. (ZAMM). 55, T/149 (1975).

[5] K. Yamamoto and N. Iwamura, *Flow with convective acceleration through a porous medium*, Journal of Eng. Mathematics 10, 41 (1976).

[6] T. G. Cowling, *Magnetohydrodynamics*, Interscience Publications. (1957).

[7] N. Datta, R. N. Jana, *Oscillatory magnetohydrodynamic flow past a flat plate with Hall effects*. P. Phys. Soc. Japan, 40, 1469 (1976).

Dr. P. C. Ram
Department of Mathematics
Kenyatta University
Kenya