UDK 539.3

THERMAL AND MECHANICAL STRESSES IN CYLINDERS AND SPHERES OBEYING POLAR AND TRANSVERSAL ANISOTROPY

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1. Introduction

The aim of this paper is to present closed form solutions of thermally induced stresses in principal directions of short and long cylinders (disks, rings, rods, tubes), and of solid or hollow spheres, made of polarly and/or transversaly anisotropic homogeneous linear elastic materials. Stresses in cylinders and spheres have been evaluated due to temperature distribution and inner/outer pressure, while body forces due to centrifugal action have been taken into account only for cylinders. Gravitational force has also been neglected. Developed formulae are extended solutions, known from Timoshenko [1], Lekhnitskii [2,5], Nowinski–Olszak [3] and Head [4].

With cylinders (n = 1) and spheres (n = 2) governing equations are, assuming radial variation of parameters (stress, strain, displacements, forces, temperatures) only:

(a) Equilibrium equation

$$d\sigma_R/dr + n(\sigma_R - \sigma_\phi)/r + (2-n)\,\rho\omega^2 r = 0 \tag{1}$$

(b) Strain-displacement relationships

$$\epsilon_R = du/dr, \ \epsilon_\phi = u/r \tag{2}$$

(c) Constitutive equations

$$\{\epsilon_R, \epsilon_\phi, \epsilon_k\} = \underline{D}\,\{\sigma_R, \sigma_\phi, \sigma_k\} + \{\epsilon_R^0, \epsilon_\phi^0, \epsilon_k^0\}\ , \ \epsilon_m^0 = \alpha_m T \tag{3}$$

or inversely

$$\{\sigma_R, \sigma_\phi, \sigma_k\} = \underline{C}(\{\epsilon_R, \epsilon_\phi, \epsilon_k\} - \{\epsilon_R^0, \epsilon_\phi^0, \epsilon_k^0\}), \ \underline{C} = \underline{D}^{-1} \tag{4}$$

where Hookean matrices are

$$\underline{D} = \begin{bmatrix} D_1 & D_2 & D_3 \\ D_2 & D_4 & D_5 \\ D_3 & D_5 & D_6 \end{bmatrix}, \qquad \underline{C} = \begin{bmatrix} C_1 & C_2 & C_3 \\ C_2 & C_4 & C_5 \\ C_3 & C_5 & C_6 \end{bmatrix} \tag{5}$$

For isotropic materials $C_1 = C_4 = C_6 = \lambda + 2\mu$, $C_2 = C_3 = C_5 = \lambda$ (6)

may be expressed by Lamé coefficients

$$\lambda = E\,\nu/((1+\nu)(1-2\nu)),\quad \mu = E/(2(1+\nu)) \tag{7}$$

In the case of a polar anisotropy elastic coefficients are

$$E_R \neq E_\phi = E_k,\ \mu_{R\phi} = \mu_{Rk} \neq \mu_{\phi k} = E_k/(2(1+\nu_{\phi k})),\ \nu_{R\phi} = \nu_{Rk} \neq \nu_{\phi k}$$

giving

$$D_1 = 1/E_R,\ D_2 = D_3 = -\nu_{Rk}/E_R,\ D_4 = D_6 = 1/E_k,\ D_5 = -\nu_{\phi k}/E_k,$$

$$C_1 = E_R(1-\nu_{\phi k})/(1-\nu_{\phi k} - 2\nu_{Rk}^2 E_k/E_R),$$

$$C_2 = C_3 = E_k\nu_{Rk}/(1-\nu_{\phi k} - 2\nu_{Rk}^2 E_k/E_R),$$

$$C_4 = C_6 = E_k/(1+\nu_{\phi k})(1-\nu_{Rk}^2 E_k/E_R)\,/\,(1-\nu_{\phi k} - 2\nu_{Rk}^2 E_k/E_R),$$

$$C_5 = E_k/(1+\nu_{\phi k})(\nu_{\phi k} + \nu_{Rk}^2 E_k/E_R)\,/\,(1-\nu_{\phi k} - 2\nu_{Rk}^2 E_k/E_R) \tag{8}$$

For transverse anisotropy, elastic coefficients are

$$E_R = E_\phi \neq E_k,\ \mu_{Rk} = \mu_{\phi k} \neq \mu_{R\phi} = E_R/(2(1+\nu_{R\phi}),\ \nu_{R\phi} \neq \nu_{RK} = \nu_{\phi k}$$

giving

$$D_1 = D_4 = 1/E_R,\ D_2 = -\nu_{R\phi}/E_R,\ D_3 = D_5 = -\nu_{Rk}/E_k,\ D_6 = 1/E_k,$$

$$C_1 = C_4 = E_R(1-\nu_{Rk}^2 E_R/E_k)/((1+\nu_{R\phi})(1-\nu_{R\phi} - 2\nu_{Rk}^2 E_R/E_k)),$$

$$C_2 = E_R(\nu_{R\phi} + \nu_{Rk}^2 E_R/E_k)/((1+\nu_{R\phi})(1-\nu_{R\phi} - 2\nu_{Rk}^2 E_R/E_k)),$$

$$C_3 = C_5 = E_R\,\nu_{R\phi}/(1-\nu_{R\phi} - 2\nu_{Rk}^2 E_R/E_k),$$

$$C_6 = E_k(1-\nu_{R\phi})/(1-\nu_{R\phi} - 2\nu_{Rk}^2 E_R/E_k) \tag{9}$$

The third stress/strain component, in the k direction, options are:

- short cylinder (plane stress) $\sigma_z = 0$
- long cylinder (plane strain) $\epsilon_z = 0$ or $d\epsilon_z/dr = 0$
- sphere $\sigma_\theta = \sigma_\phi$, $\epsilon_\theta = \epsilon_\phi$.

Temperatures in long cylinders and spheres ($a < r < b$) can be determined from the following heat balance (differential) equation

$$r^{-n}\, d(r^{+n}\, k(T)\, dT/dr)/dr + q(r) = 0 \tag{10}$$

where k and q are conductivity and heat source density respectively.

For short cylinders ($a < r < b$) the following heat transfer equation has to be used

$$d^2\,\theta/dr^2 + 1/r\, d\theta/dr - \ell^2\,\theta = 0 \tag{11}$$

where $\theta = T - T_f$ and $\ell = \sqrt{2h/k\delta}$ (T_f, h, k and δ are, respectively, the temperature of a fluid, heat transfer coefficient, conductivity and thickness of the solid material).

The temperature field can thus be determined in the form of a steady state distribution:

Short hollow ring

$$T(r) = T_f + (T_a - T_f)\,\frac{K_1\,(\ell b)\, I_0\,(\ell r) + I_1\,(\ell b)\, K_0\,(\ell r)}{K_1\,(\ell b)\, I_0\,(\ell a) + I_1\,(\ell b)\, K_0\,(\ell a)} \tag{12}$$

Short solid disk

$$T(r) = T_f + (T_a - T_f)\, I_0\,(\ell r)\,, \quad a = 0 \tag{13}$$

Long (externally cooled) cylinder and sphere

$$T\,(r) = T_b + q_0/(2\,(n+1)\,k_0)\,(b^2 - r^2 + f) \tag{14}$$

where for rods and solid spheres ($a = 0$) $f = 0$
while with tubes and hollow spheres this contribution reads

$$f = 2\,a^{n+1}\,((2-n)\,\ell n\,(r/b) + (n-1)\,(1/b - 1/r)).$$

2. Isotropic case

The following isotropic body solutions can be obtained from the combined Navier–Lamé (Euler's) differential equation of the form

$$d^2u/dr^2 + n\,(1/r\, du/dr - u/r^2) + (2-n)\,\rho\omega^2\, r/C_1 = H\, dT/dr \tag{15}$$

Some of the required formulae are given in reference [1] and listed below ($H = \alpha\,(1+\nu)/(1-\nu)$, $K = 0$).

3. Polar anisotropic case

Combining the equilibrium, deformation and constitutive laws, bearing in mind the polarly anisotropic Hookean relationship, a generalised Navier–Lamé (Euler's) differential equation can be obtained in the following form

$$d^2 u/dr^2 + n\,(1/r\,du/dr - m^2 u/r^2) + (2-n)\,\rho\omega^2 r/C_1 = H dT/dr + KT/r \tag{37}$$

where the coefficients m, H and K depend on the thermoelastic coefficients, given in Section 1.

3.1 THERMAL STRESSES

In this case, taking into account nonhomogeneous thermal strains, the constitutive ceofficients are, for long cylinders and spheres

$$m = \sqrt{(C_6 + (n-1)(C_5 - C_2))/C_1} \tag{38}$$

$$H = \alpha_R + n\,\alpha_\phi\, C_2/C_1 + (2-n)\,\alpha_z\ C_2/C_1 \tag{39}$$

$$K = n\,(\alpha_R\,(C_1 - C_2)/C_1 + \alpha_\phi\,(C_2 - C_6 + (n-1)(C_2 - C_5))/C_1 + \\ + (2-n)\,\alpha_z\,(C_2 - C_5)/C_1 \tag{40}$$

while for thin disks and rings these coefficients read as follows

$$m = \sqrt{(C_6^2 - C_2^2)/(C_1 C_6 - C_2^2)} \tag{41}$$

$$H = \alpha_R + \alpha_\phi\,(C_2 C_6 - C_2^2)/(C_1 C_6 - C_2^2) \tag{42}$$

$$K = ((C_1 - C_2)\,\alpha_R - (C_6 - C_2)\,\alpha_\phi)/(C_1 - C_2^2/C_6) \tag{43}$$

The common solution of the above equation (37) for displacements is

$$u(r) = A\,r^{-q} \int_a^r T\,r^{+q}\,dr + B\,r^{+p} \int_a^r T\,r^{-p}\,dr + M\,r^{-q} + N\,r^{+p} \tag{44}$$

where

$$A = (H(n+q) - K)/(p+q)\ ,\quad B = (K - H(n-p))/(p+q),$$

$$q = (2-n)m + (n-1)(1/2 + \sqrt{1/4 + 2m^2}),$$

$$p = (2-n)m - (n-1)(1/2 - \sqrt{1/4 + 2m^2}).$$

while M and N are integration costants determined form boundary conditions, which for separately considered thermal stress case may be taken with vanishing radial stress at the surface, while at the axis of revolution $u = 0$.

Evaluated thermal stresses are:

(a) Tube and hollow sphere

$$\sigma_R = r^{-q-1}(M(nC_2 - qC_1) + I\int_a^r Tr^{+q}\,dr) + r^{+p-1}(N(nC_2 + pC_1) + J\int_a^r Tr^{-p}\,dr) \tag{45}$$

$$\sigma_\phi = r^{-q-1}\frac{n-q-1}{n}(M(nC_2 - qC_1) + I\int_a^r Tr^{+q}\,dr) + r^{+p-1}\frac{n+p-1}{n}(N(nC_2 + pC_1) + J\int_a^r Tr^{-p}\,dr) + (I+J)\,T/n \tag{46}$$

where

$$I = A(nC_2 - qC_1), J = B(nC_2 + pC_1),$$

$$M = -(I b^{-q-1}\int_a^b Tr^{+q}\,dr + J b^{+p-1}\int_a^b T r^{-p}\,dr)/ /((qC_1 - nC_2)(b^{+p-1}/a^{p+q} - 1/b^{q+1})),$$

$$N = -(I b^{-q-1}\int_a^b Tr^{+q}\,dr + J b^{+p-1}\int_a^b Tr^{-p}\,dr)/ /((pC_1 + nC_2)(b^{+p-1} - a^{p+q}/b^{q+1})).$$

(b) Rod and solid sphere

$$\sigma_R = r^{-q-1} I\int_0^r Tr^{+q}\,dr + r^{+p-1}(N_0(nC_2 + pC_1) + J\int_0^r Tr^{-p}\,dr) \tag{47}$$

$$\sigma_\phi = r^{-q-1}\,\frac{n-q-1}{n}\, I \int_0^r T r^{+q}\, dr + r^{+p-1}\,\frac{n+p-1}{n}\,(N_0\,(nC_2 + pC_1) +$$

$$+ J \int_0^r T r^{-p}\, dr) + (I+J)\, T/n) \tag{48}$$

where

$$N_0 = -(I\, b^{-q-1} \int_0^b T r^{+q}\, dr + J\, b^{+p-1} \int_0^b T r^{-p}\, dr)/$$

$$/(pC_1 + nC_2)\, b^{+p-1}) .$$

Above formulae also apply to thin disks and rings, using their own values of *m, H* and *K*.

3.2 CENTRIFUGAL STRESSES

Alternatively to the use of Navier–Lamé (Euler's) differential equation for displacements, the following Euler's equation for radial stress may directly be used when evaluating stresses of solid body revolution:

$$d^2\, \sigma_R/dr^2 + 3/r\, d\sigma_R/dr - (m^2 - 1)\, \sigma_R/r^2 + \rho\omega^2 L = 0 \tag{49}$$

where for thin disks and rings

$$m = \sqrt{D_1/D_6}\ ,\ L = (3D_6 - D_2)/D_6 ,$$

while for long rods and tubes

$$m = \sqrt{(D_1 + D_2^2/D_6)/(D_6 - D_5^2/D_6)}\ ,\ L = (3D_6 - 3D_5^2/D_6 - D_2 + D_2 D_5/D_6)/$$

$$/(D_6 - D_5^2/D_6$$

Using this option the desired stresses have been calculated:

(a) Ring

$$\sigma_R = \rho\omega^2\, (3\, D_6 - D_2)/(9D_6 - D_1)\, (((b^{3+m} - a^{3+m})\, r^{m-1} -$$

$$- (b^{3-m} - a^{3-m})\, (ab)^{2m}/r^{m+1})/(b^{2m} - a^{2m}) - r^2) \tag{50}$$

$$\sigma_\phi = \rho\omega^2\, (3D_6 - D_2)/(9D_6 - D_1)\, (((b^{3+m} - a^{3+m})\, r^{m-1} +$$

$$+ (b^{3-m} - a^{3-m})\, (ab)^{2m}/r^{m+1})\, m/(b^{2m} - a^{2m}) +$$

$$+ r^2\, (3D_2 - D_1)/(3D_6 - D_2)) \tag{51}$$

(b) Disk

$$\sigma_R = \rho\omega^2 (3D_6 - D_2)/(9D_6 - D_1) r^2 ((b/r)^{3-m} - 1) \tag{52}$$

$$\sigma_\phi = \rho\omega^2 (3D_6 - D_2)/(9D_6 - D_1) r^2 (m (b/r)^{3-m} + (3D_2 - D_1)/(3D_6 - D_2)) \tag{53}$$

(c) Tube

$$\sigma_R = M r^{+m-1} + N r^{-m-1} + \rho\omega^2 L/(9 - m^2) r^2 \tag{54}$$

$$\sigma_\phi = m (M r^{+m-1} - N r^{-m-1}) + \rho\omega^2 (1 - 3L/(9 - m^2)) r^2 \tag{55}$$

where

$$M = \rho\omega^2 L/(9 - m^2)(b^{3+m} - a^{3+m})/(b^{2m} - a^{2m})$$

and

$$N = \rho\omega^2 L/(9 - m^2)(a^{3-m} - b^{3-m})(ab)^{2m}/(b^{2m} - a^{2m})$$

(d) Rod

$$\sigma_R = \rho\omega^2 L/(9 - m^2) r^2 ((b/r)^{3-m} - 1) \tag{56}$$

$$\sigma_\phi = \rho\omega^2 L/(9 - m^2) r^2 (m (b/r)^{3-m} - 3) + \rho\omega^2 r^2 \tag{57}$$

3.3 INTERNAL/EXTERNAL PRESSURE INDUCED STRESSES

For material obeying polar anisotropy, pure pressure load stresses are in a long tube and hollow sphere given by

$$\sigma_R = R r^{p-1} - S/r^{q+1} \tag{58}$$

$$\sigma_\phi = P R r^{p-1} - Q S/r^{q+1} \tag{59}$$

where

$$P = (2 - n) m + (n - 1)(C_6 + C_5 + pC_2)/(2C_2 + pC_1),$$

$$Q = (n - 2) m + (n - 1)(C_6 + C_5 - qC_2)/(2C_2 - qC_1),$$

$$R = (p_a a^{1+q} - p_b b^{1+q})/(b^{p+q} - a^{p+q}), \quad \text{and}$$

$$S = (p_a a^{1-p} - p_b b^{1-p})(ab)^{p+q}/(b^{p+q} - a^{p+q}),$$

while in a long rod and solid sphere the stresses are given by

$$\sigma_R = -p_b (r/b)^{p-1} \tag{60}$$

$$\sigma_\phi = -p_b P (r/b)^{p-1} \tag{61}$$

4. Transversaly anisotropic case

Within the scope of transversaly anisotropic bodies, thermal stresses may be expressed as shown in ref. [6] for rods, tubes and spheres respectively. They can be summarised as follows:

(a) Tubes and hollow spheres

$$\sigma_R = \frac{(C_1 - C_2)(C_1 + nC_2)}{C_1 (n+1)/n} \Big(\int_a^r K T \frac{dr}{r} - \frac{b^{n+1} (r^{n+1} - a^{n+1})}{(b^{n+1} - a^{n+1}) r^{n+1}} \int_a^b K T \frac{dr}{r} -$$

$$- 1/r^{n+1} \int_a^r H T r^n \, dr + \frac{r^{n+1} - a^{n+1}}{r^{n+1} (b^{n+1} - a^{n+1})} \int_a^b H T r^n \, dr \Big) \tag{62}$$

$$\sigma_\phi = \frac{(C_1 - C_2)(C_1 + nC_2)}{C_1 (n+1)/n} \Big(\int_a^r K T \frac{dr}{r} - \frac{b^{n+1} (r^{n+1} + a^{n+1}/n)}{(b^{n+1} - a^{n+1}) r^{n+1}} \int_a^b K T \frac{dr}{r} +$$

$$+ 1/(nr^{n+1}) \int_a^r H T r^n \, dr + \frac{r^{n+1} - a^{n+1}/n}{r^{n+1} (b^{n+1} - a^{n+1})} \int_a^b H T r^n \, dr -$$

$$- \alpha_\phi T (n+1)/n - (2-n) \alpha_z T \, 2C_3/(C_1 + C_2) \Big) \tag{63}$$

where

$$H = \alpha_R + n \alpha_\phi + (2-n) \, 2C_3/(C_1 + C_2) \alpha_z, \quad K = \alpha_R - \alpha_\phi$$

(b) Rods and solid spheres

$$\sigma_R = \frac{(C_1 - C_2)(C_1 + nC_2)}{C_1 (n+1)/n} \Big(\int_b^r K T \frac{dr}{r} - 1/r^{n+1} \int_0^r H T r^n \, dr +$$

$$+ 1/b^{n+1} \int_0^b H T r^n \, dr \Big) \tag{64}$$

$$\sigma_\phi = \frac{(C_1 - C_2)(C_1 + nC_2)}{C_1 (n+1)/n} \Big(\int_b^r K T \frac{dr}{r} + 1/(nr^{n+1}) \int_0^r H T r^n \, dr +$$

$$+ 1/b^{n+1} \int_0^b H T r^n \, dr - \alpha_\phi T (n+1)/n - (2-n) \alpha_z T \, 2C_3/(C_1 + C_2) \Big) \tag{65}$$

With internal/external pressures induced stresses are given by

$$\sigma_R = (p_b b^{n+1} (a^{n+1} - r^{n+1}) - p_a a^{n+1} (b^{n+1} - r^{n+1}))/(r^{n+1} (b^{n+1} - a^{n+1})) \qquad (66)$$

$$\sigma_\phi = (p_a a^{n+1} (b^{n+1}/n + r^{n+1}) - p_b b^{n+1} (a^{n+1}/n + r^{n+1}))/(r^{n+1} (b^{n+1} - a^{n+1})) \qquad (67)$$

in tubes and hollow spheres, while in rod and solid spheres external pressure induced stresses are trivially

$$\sigma_R = \sigma_\phi = -p_b \qquad (68)$$

In tubes and rods there are also axial stresses. These can be expressed (plane strain, under $\int \sigma_z r\, dr = 0$ assumption) by:

(a) Tube

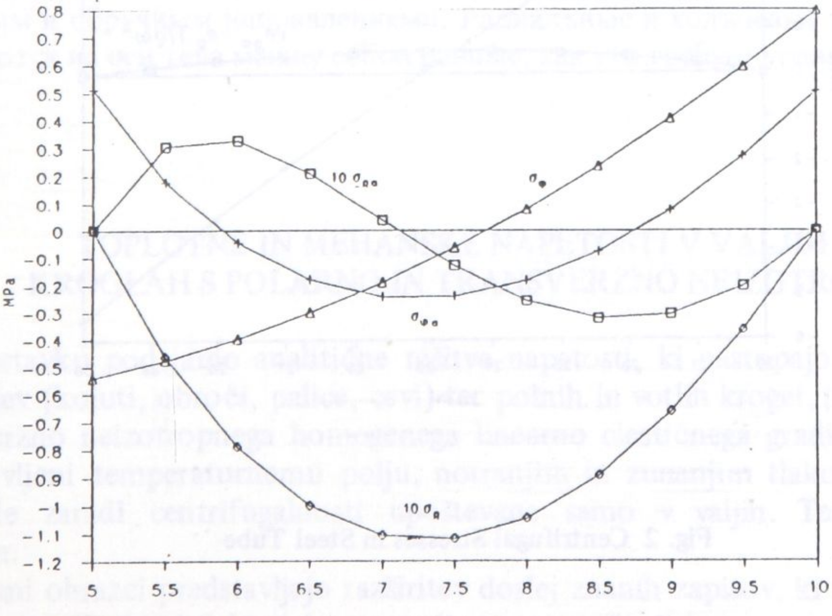
$$\sigma_z = C_3/(C_1 + C_2)(\sigma_R + \sigma_\phi + 2(p_b b^2 - p_a a^2)/(b^2 - a^2)) -$$
$$- (2C_3^2/(C_1 + C_2) - C_6)((2/(b^2 - a^2)) \int \alpha_z T r\, dr - \alpha_z T) \qquad (69)$$

(b) Rod

$$\sigma_z = C_3/(C_1 + C_2)(\sigma_R + \sigma_\phi + 2p_b) -$$
$$- (2C_3^2/(C_1 + C_2) - C_6)(2/b^2 \int \alpha_z T r\, dr - \alpha_z T) \qquad (70)$$

5. Conclusion

Known formulae for polarly and transversaly anisotropic cylinders and spheres, available hitherto only for pressure loaded tubes and spheres (Lekhnitskii [2]), thermally loaded tubes (Nowinski and Olszak [3] and Head [4]), and of rotating disks (Lekhnitskii



[5]) have been supplemented by thermal stresses in all kinds of solid and hollow rolls and balls (i.e. plane stress and plane strain cases of radial problems).

Two examples of numerical results are given in this paper, showing tubes made of materials obeying polar anisotropy. Fig. 1 illustrates thermal stresses in graphite, while Fig. 2 yields centrifugal stresses in steel (actual values and deviation from the isotropic level of radial and hoop stresses). Some more examples (transverse anisotropy) have been given in reference [6].

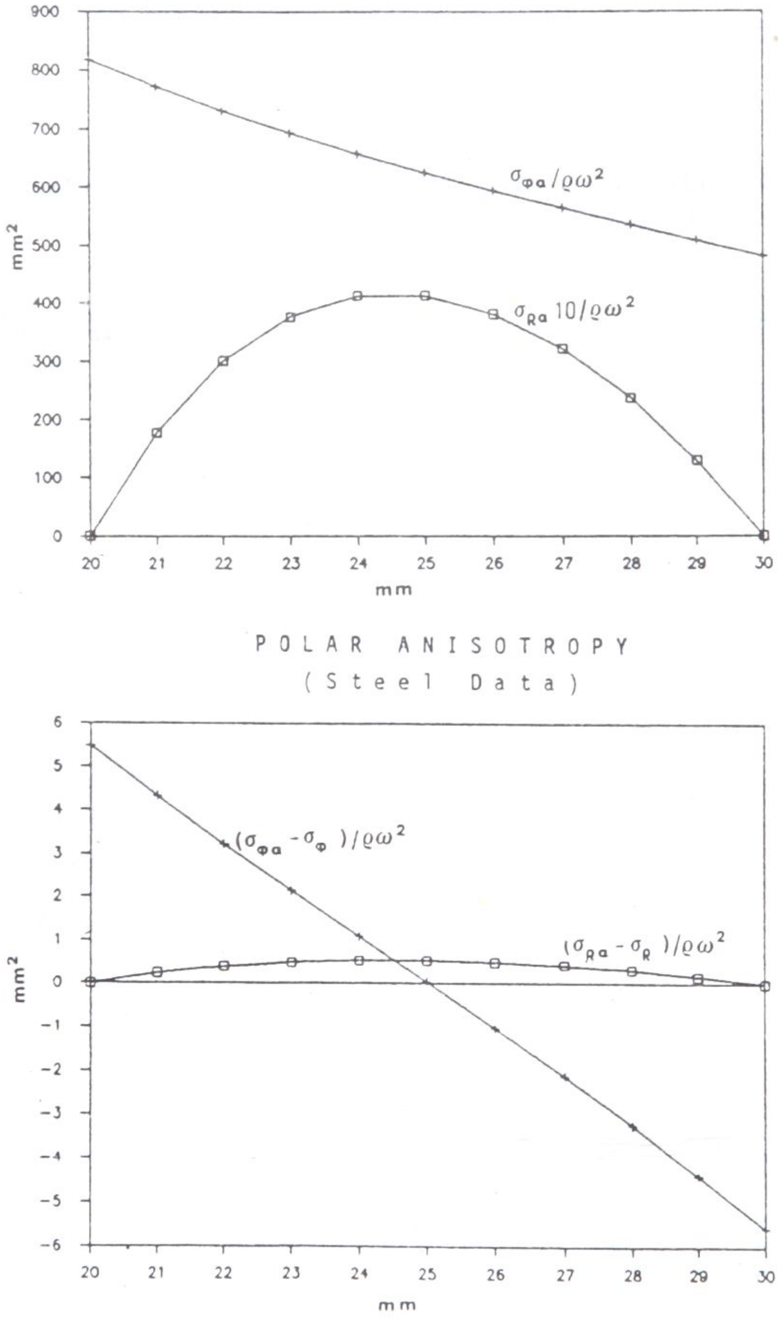


Fig. 2 Centrifugal Stresses in Steel Tube

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ТЕРМИЧЕСКИЕ И МЕХАНИЧЕСКИЕ НАПРЯЖЕНИЯ ВАЛОВ И ШАРОВ С ПОЛЯРНОЙ И ПОПЕРЕЧНОЙ АНИЗОТРОПИИ

В статъе приводим аналитические разрешения напряжений которые наступают в корпусе тонких и длинных валов (диски, колъца, трости, трубы) и сплошних и полых шаров, сделанных из полярно или поперечно анизотропного однородного линейно-упругого материала. Валы и шары подвергаются температурному полю, внутренним и внешним давлениям а корпусные силы из за центробежности приняты во внимание только в валах. Сила тяжести тоже не принята в внимание.

Опомянутые образцы представляют расширение известных записей, которые были известны для трубы и шара с поверхностными давлениями, а отчасти и для вращателъных дисков.

Поступок проведения мы исполнили с помощ уравнения Навиер-Ламе, которое для радиалъной проблемы вала и шара лишъ одно, но содержит упругие параметры, зависимые от вида обсуждаемой анизотропии. Математические взято это дифференциалъное уравнение Эйлера, однородная частъ которого решает краевые условия, а неоднородние доли содержает влияние температур и вращения тела.

Партикулярное решение содержит сингулярностъ на оси тела вращения (полное тело), которую возможно устранит приняв во внимание, что там нет разницы между радиалъным и обручным направлениями. Радиалъные и колъцевые напряжения из за того являются на оси тела между собой равные, как это требует уравнение равновесия.

TOPLOTNE IN MEHANSKE NAPETOSTI V VALJIH IN KROGLAH S POLARNO IN TRANSVERZNO NEIZOTROPIJO

V sestavku podajamo analitične rešitve napetosti, ki nastopajo v telesu tankih in dolgih valjev (koluti, obroči, palice, cevi) ter polnih in votlih krogel, izdelanih iz polarno ali transverzno neizotropnega homogenega linearno elastičnega gradiva. Valji in krogle so izpostavljeni temperaturnemu polju, notranjim in zunanjim tlakom, medtem ko so telesne sile zaradi centrifugalnosti upoštevane samo v valjih. Težnost pravtako ni upoštevana.

Podani obrazci predstavljajo razširitev doslej znanih zapisov, ki so bili znani za cev in kroglo s površinskimi tlaki, delno pa tudi za rotirajoče diske.

Postopek izlepljave smo izvedli s pomočjo Navier–Laméjeve enačbe, ki je za radialni problem valja in krogle ena sama, vsebuje pa elastične koeficiente, odvisne od vrste obravnavane anizotropije. Matematično vzeto je to Eulerjeva diferencialna enačba, katere homogeni del reši robne pogoje, nehomogeni deleži pa vsebujejo vpliv temperatur in vrtenja telesa.

Partikularna rešitev vsebuje singularnost na osi vrtenine (polno telo), ki pa jo lahko odpravimo z razmislekom, da tam ni razlike med radialno in obročno smerjo. Radialne in cirkumferenčne napetosti so zato na osi telesa med seboj enake, kot to zahteva ravnovesna enačba.

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